11
Objects: Tokens for (Eigen-)Behaviors*

A seed, alas, not yet a flower, for Jean Piaget to his 80th birthday from Heinz von Foerster with admiration and affection.

I shall talk about notions that emerge when the organization of sensorimotor interactions (and also that of central processes (cortical-cerebellar-spinal, cortico-thalamic-spinal, etc.)) is seen as being essentially of circular (or more precisely of recursive) nature. Recursion enters these considerations whenever the changes in a creature's sensations are accounted for by its movements ($s_i = S(m_k)$), and its movements by its sensations ($m_k = M(s_j)$). When these two accounts are taken together, then they form "recursive expressions," that is, expressions that determine the states (movements, sensations) of the system (the creature) in terms of these very states ($s_i = S(M(s_j)) = SM(s_j)$; $m_k = M(S(m_i) = MS(m_i))$.

One point that with more time, effort and space could be made rigorously and not only suggestively as it has been made here, is that what is referred to as "objects" (GEGEN-STAENDE = "against-standers") in an observer-excluded (linear, open) epistemology, appears in an observer-included (circular, closed) epistemology as "tokens for stable behaviors" (or, if the terminology of Recursive Function Theory is used, as "tokens for Eigen-functions").

Of the many possible entries into this topic the most appropriate one for this occasion appears to me the (recursive) expression that forms the last line on page 63 of J. Piaget's *L'Equilibration des Structures Cognitives* (1975):

$$\text{Obs.O} \rightarrow \text{Obs.S} \rightarrow \text{Coord.S} \rightarrow \text{Coord.O} \rightarrow \text{Obs.O} \rightarrow \text{etc.}$$

This is an observer's account of an interaction between a subject S and an object (or a set of objects) O. The symbols used in this expression (defined on page 59 *op. cit.*) stand for (see also Fig. 1):

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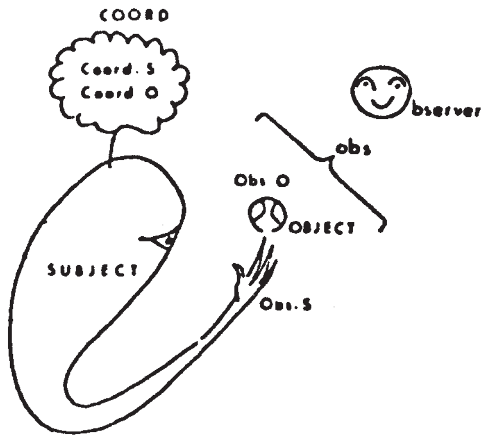


FIGURE 1.

Obs.S	"observables relatifs a l'action du sujet"
Obs.O	"observables relatifs aux objets"
Coord.S	"coordinations inferentielles des actions (ou operations) du sujet"
Coord.O	"coordinations inferentielles entre objets"
"etc."	"the (syntactic) injunction to iterate (with no limits specified) the sequence of these operations (HVF)"

For the sake of brevity (lucidity?) I propose to compress the symbolism of before even further, compounding all that is observed (i.e. Obs.O and Obs.S) into a single variable

obs,

and compounding coordinating operations that are performed by the subject (i.e. Coord.S and Coord.O) into a single operator

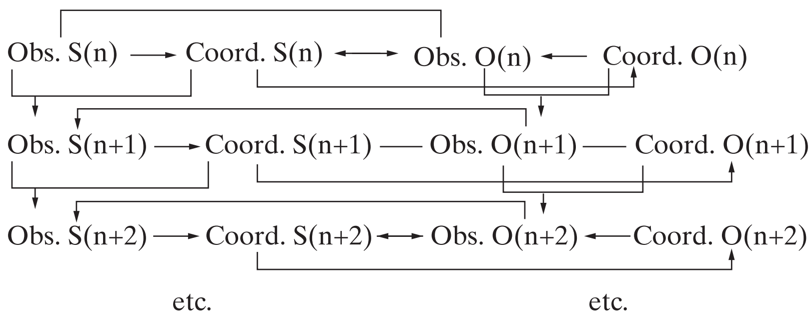
COORD.

COORD transforms, rearranges, modifies etc., the forms, arrangements, behaviors, etc., observed at one occasion (say, initially obs_o, and call it the "primary argument") into those observed at the next occasion, obs_1. Express the outcome of this operation through the equality:[1]

[1] By replacing the arrow "→", whose operational meaning is essentially to indicate a one-way (semantic) connectedness (e.g., "goes to," "implies," "invokes," leads to," etc.) between adjacent expressions, with the equality sign provides the basis for a calculus. However, in order that legitimate use of this sign can be made, the variables "obs_1" must belong to the same domain. The choice of domain is, of course left to the observer who may wish to express his observations in form of, for instance, numerical values, of vectors representing arrangements or geometrical configurations, or his observations of behaviors in form of mathematical functions (e.g., "equations of motion," etc.), or by logical propositions (e.g., McCulloch-Pitts" "TPE's" 1943 (i.e., Temporal Propositional Expressions), etc.).

$$obs_1 = COORD(obs_o).$$

While some relational fine structure is (clearly) lost in this compression, gained, however, may be an easiness by which the progression of events, suggested on the last lined page of 62 *op. cit.* and copied here can now be watched.



Allow the operator COORD to operate on the previous outcome to give

$$obs_2 = COORD(obs_1) = COORD(COORD(obs_o)) \tag{2}$$

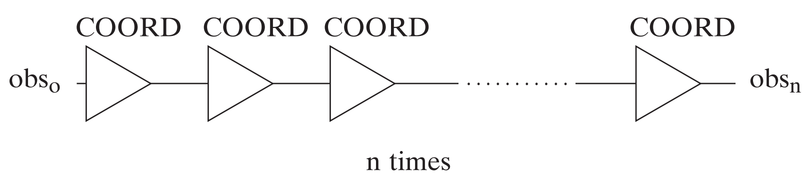
and (recursively) after n steps

$$obs_n = \underbrace{COORD(COORD(COORD(\ldots\ldots\ldots\ldots (obs_o)))..)}_{n \text{ times}} \quad \left|n \text{ times}\right| \tag{3}$$

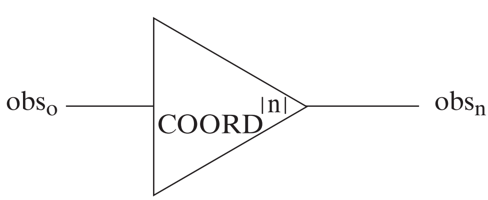
or by notational abbreviation

$$obs_n = COORD^{(n)}(obs_o). \tag{4}$$

By this notational abbreviation it is suggested that also functionally



can be replaced by



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Let n grow without limit ($n \to \infty$):

$$obs_\infty = \lim_{n\to\infty} COORD^{(n)}(obs_o) \tag{5}$$

or:

$$obs_\infty = COORD(COORD(COORD(COORD. . . . \tag{6}$$

Contemplate the above expression (6) and note:

(i) that the independent variable obs_o, the “primary argument” has disappeared (which may be taken as a signal that the simple connection between independent and dependent variables is lost in indefinite recursions, and that such expressions take on a different meaning).

(ii) that, because obs_∞ expresses an indefinite recursion of operators COORD onto operators COORD, any indefinite recursion within that expression can be replaced by obs_∞:

$$obs_\infty =$$

$$COORD(COORD(COORD(COORD(. . . .$$

$$obs_\infty$$

$$obs_\infty$$

$$obs_\infty$$

(iii) Hence:

$$obs_\infty = obs_\infty \tag{7.0}$$
$$obs_\infty = COORD(obs_\infty) \tag{7.1}$$
$$obs_\infty = COORD(COORD(obs_\infty)) \tag{7.2}$$
$$obs_\infty = COORD(COORD(COORD(obs_\infty))) \tag{7.3}$$
etc.

Note that while in this form the *horror infinitatis* of expression (6) has disappeared (all expressions in COORD are finite), a new feature has emerged, namely, that the dependent variable obs_∞ is, so to say, “self-depending” (or “self-defining,” or “self-reflecting,” etc., through the operator COORD).

Should there exist values $obs_{\infty i}$ that satisfy equations (7), call these values

“Eigen-Values”

$$obs_{\infty i} \equiv Obs_i \tag{8}$$

(or “Eigen-Functions,” “Eigen-Operators,” “Eigen-Algorithms,” “Eigen-Behaviors,” etc., depending on the domain of obs) and denote these “Eigen-Values by capitalizing the first letter. (For examples see Appendix A).

Contemplate expressions of the form (7) and note:

(i) that Eigenvalues are discrete (even if the domain of the primary argument obs_o is continuous).

This is so because any infinitesimal perturbation $\pm\in$ from an Eigenvalue Obs_i (i.e., $Obs_i \pm \in$) will disappear, as did all other values of obs, except those for which obs = Obs_i, and obs will be brought either back to Obs_i (*stable* Eigenvalue), or to another Eigenvalue Obs_j (*instable* Eigenvalue Obs_i).

In other words, Eigenvalues represent *equilibria*, and depending upon the chosen domain of the primary argument, these equilibria may be equilibrial values ("Fixed Points"), functional equilibria, operational equilibria, structural equilibria, etc.

(ii) that Eigenvalues Obs_i and their corresponding operators COORD stand to each other in a complementary relationship, the one implying the other, and vice versa; there the Obs_i represent the externally observable manifestations of the (introspectively accessible) cognitive computations (operations) COORD.

(iii) that Eigenvalues, because of their self-defining (or self-generating) nature imply topological "closure" ("circularity") (see Figures 2 and 3):

This state of affairs allows a symbolic re-formulation of expression (5);

$$\lim_{n\to\infty} \mathrm{COORD}^{(n)} \equiv \mathrm{COORD}$$

that is, the snake eating its own tail: cognition computing its own cognitions.

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Let there be, for a given operator COORD, at least three Eigenvalues

$$Obs_1, Obs_2, Obs_3,$$

FIGURE 2.

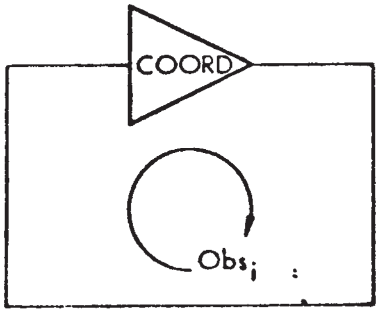


FIGURE 3.

and let there be an (albegraic) composition "*" such that

$$Obs_1 * Obs_2 = Obs_3, \tag{10}$$

then the coordinating operations COORD appear to coordinate the whole (i.e., the composition of the parts) as a composition of the apparent coordinations of the parts (see proof in Appendix B):

$$COORD(Obs_1 * Obs_2) = COORD(Obs_1) * COORD(Obs_2). \tag{11}$$

In other words, the coordination of compositions (i.e., the whole) corresponds to the composition of coordinations.

This is the condition for what may be called the "principle of cognitive continuity" (e.g., breaking pieces of chalk produces pieces of chalk).

This may be contrasted with the "principle of cognitive diversity" which arises when the Obs_i and the composition "*" are *not* the Eigenvalues and compositions complementing the coordination COORD′:

$$COORD'(Obs_1 * Obs_2) \neq COORD'(Obs_1) * COORD'(Obs_2), \tag{12}$$

and which says that the whole is neither more nor is it less than the sum of its parts: it is *different*. Moreover, the formalism in which this sentiment appears (expression (12)) leaves little doubt that it speaks neither of "wholes," nor of "parts" but of a subject's distinction drawn between two states of affairs which by an (other) observer may be seen as being not qualitatively, but only quantitatively distinct.

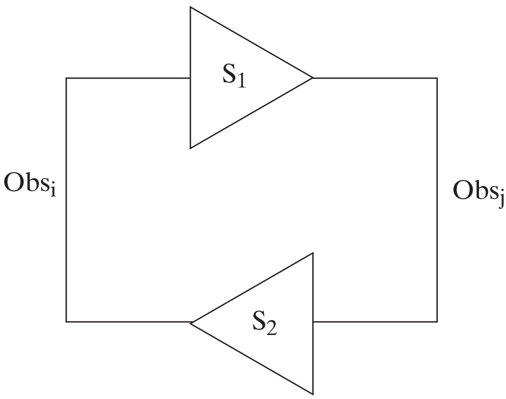
* * * * *

Eigenvalues have been found ontologically to be discrete, stable, separable and composable, while ontogenetically to arise as equilibria that determine themselves through circular processes. Ontologically, Eigenvalues and objects, and likewise, ontogenetically, stable behavior and the manifestation of a subject's "grasp" of an object cannot be distinguished. In both cases "objects" appear to reside exclusively in the subject's own experience of his sensori-motor coordinations; that is, "objects" appear to be exclusively subjective? Under which conditions, then, do objects assume "objectivity?"

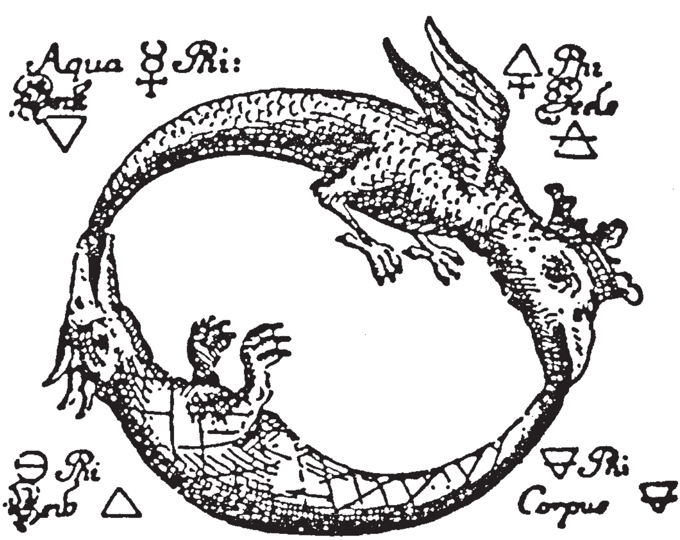
Apparently, only when a subject, S_1, stipulates the existence of another subject, S_2, not unlike himself, who, in turn, stipulates the existence of still another subject, not unlike himself, who may well be S_1.

In this atomical social context each subject's (observer's) experience of his own sensori-motor coordination can now be referred to by a token of this experience, the "object," which, at the same time, may be taken as a token for the externality of communal space.

With this I have returned to the topology of closure



where equilibrium is obtained when the Eigenbehaviors of one participant generate (recursively) those for the other (see, for instance, Appendix Example A 2); where one snake eats the tail of the other as if it were its own, and where cognition computes its own cognitions through those of the other: here is the origin of ethics.



Appendix A

Examples:

A1. Consider the operator (linear transform) Op_1:

$$Op_1 = \text{“divide by two and add one”}$$

and apply it (recursively) to x_0, x_1, etc., (whose domains are the real numbers).

Choose an initial x_0, say $x_0 = 4$.

$$\begin{aligned}
x_1 &= Op_1(4) = \frac{4}{2} + 1 = 2 + 1 = 3;\\
x_2 &= Op_1(3) = 2.500;\\
x_3 &= Op_1(2.500) = 2.250;\\
x_4 &= Op_1(2.250) = 2.125;\\
x_5 &= Op_1(2.125) = 2.063;\\
x_6 &= Op_1(2.063) = 2.031;\\
x_{11} &= Op_1(x_{10}) = 2.001;\\
x_\infty &= Op_1(x_\infty) = 2.000
\end{aligned}$$

Choose another initial value; say $x_0 = 1$

$$\begin{aligned}
x_1 &= Op_1(1) = 1.500;\\
x_2 &= Op_1(1.500) = 1.750;\\
x_3 &= Op_1(1.750) = 1.875;\\
x_8 &= Op_1(x_7) = 1.996;\\
x_{10} &= Op_1(x_9) = 1.999;\\
x_\infty &= Op_1(x_\infty) = 2.000
\end{aligned}$$

And indeed:

$$\frac{1}{2} \cdot 2 + 1 = 2$$

$$Op_1(2) = 2$$

i.e., “2” is the (only eigenvalue of Op_1.

A2. Consider the operator Op_2:

$$Op_2 = \exp(\cos \quad).$$

There are three eigenvalues, two of which imply each other (“bi-stability”), and the third one being instable:

$$\left.\begin{aligned}
Op_2(2.4452\ldots) &= 0.4643\ldots\\
Op_2(0.4643\ldots) &= 2.4452\ldots
\end{aligned}\right\} \text{ stable}$$

$$Op_2(1.3029\ldots) = 1.3092\ldots \quad \text{instable}$$

This means that:

$$Op_2^{(2)}(2.4452...) = 2.4452 \text{ stable}$$

$$Op_2^{(2)}(0.4643...) = 0.4643 \text{ stable}$$

A3. Consider the differential operator Op_3:

$$Op_3 = \frac{d}{dx}.$$

The eigenfunction for this operator is the exponential function "exp:"

$$Op_3(\exp) = \exp$$

i.e.,

$$\frac{de^x}{dx} = e^x$$

The generalizations of this operator are, of course, all differential equation, integral equations, integro-differential equations, etc., which can be seen at once when these equations are re-written in operator form, say:

$$F(Op_3^{(n)}, Op_3^{(n-1)}. \ldots, f) = 0$$

Of course, these operators, in turn, may be eigenvalues (eigen-operators) of "meta-operators" and so on. This suggests that COORD, for instance, may itself be treated as an eigen-operator, stable within bounds, and jumping to other values whenever the boundary conditions exceed its former stable domain:

$$Op(COORD_i) = COORD_i.$$

One may be tempted to extend the concept of a meta-operator to that of a "meta-meta-operator" that computes the "eigen-meta-operators," and so on and up a hierarchy without end. However, there is no need to invoke this escape as Warren S. McCulloch has demonstrated years ago in his paper (1945): "A Heterarchy of Values Determined by the Topology of Nervous Nets."

It would go too far in this presentation to demonstrate the construction of heterarchies of operators based on their composability.

A4. Consider the (self-referential) proposition:

"THIS SENTENCE HAS . . . LETTERS"

and complete it by writing into the appropriate space the word for the number (or if there are more than one, the numbers) that make this proposition true.

Proceeding by trial and error (comparing what this sentence says (abscissa) with what it is (ordinate)): one finds two eigenvalues "thirty-one" and "thirty-three." Apply the proposition above to itself: "this sentence has thirty-one

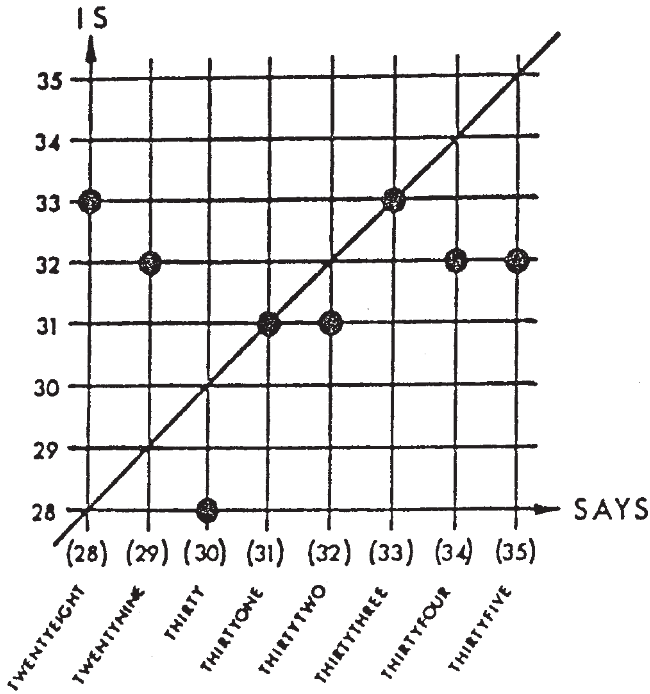


FIGURE 4.

letters' has thirty-one letters." Note that, for instance, the proposition: "this sentence consists of . . . letters" has only one eigenvalue (thirty-nine); while the proposition: "This sentence is composed of . . . letters" has none!

Appendix B

B1. Proof of Expression (11):

$$\begin{aligned}&\mathrm{COORD}(\mathrm{Obs}_1 * \mathrm{Obs}_2) = \mathrm{COORD}(\mathrm{Obs}_3)\\ &\quad = \mathrm{Obs}_3 = \mathrm{Obs}_1 * \mathrm{Obs}_2 = \mathrm{COORD}(\mathrm{Obs}_1) * \mathrm{COORD}(\mathrm{Obs}_2)\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{Q.E.D.}\end{aligned}$$

The apparent distributivity of the operator COORD over the composition "*" should not be misconstrued as "*" being a linear composition. For instance, the fixed points $u_i = \exp(2\pi\lambda i)$, (for $i = 0, 1, 2, 3 \ldots$) that complement the operator Op(u):

$$\mathrm{Op}(u) = u \tan\left(\frac{\pi}{4} \pm \frac{1}{\lambda}\lambda n\ u\right),$$

with λ an arbitrary constant, compose multiplicatively:

$$Op(u_i * u_j) = Op(u_i) * Op(u_j).$$

etc.

References

McCulloch, W. S., *A Heterarchy of Values Determined by the Topology of Nervous Nets*, *Bulletin of Mathematical Biophysics*, 1945, 7: 89–93.

McCulloch, W. S. and Pitts, W. H., *A Logical Calculus of the Ideas Immanent in Nervous Activity*, *Bulletin of Mathematical Biophysics*, 1943, 5: 115–133.

Piaget, T., *L'Equilibration des Structures Cognitives*. Presses Univ. France, Paris, 1975.