Kinetic Art: Application of Abstract Algebra to Objects with Computer-Controlled Flashing Lights and Sound Combinations
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KINETIC ART: APPLICATION OF ABSTRACT ALGEBRA TO OBJECTS WITH COMPUTER-CONTROLLED FLASHING LIGHTS AND SOUND COMBINATIONS

Vladimir Bonačić*

Abstract—*The author describes three schemes for using digital computers for artistic purpose. He is sceptical of the attempts to produce computer art with commercially available display equipment and through play with randomness and by the deliberate introduction of errors in programs prepared for nonartistic ends. He favors the approach in which art objects are controlled by a computer with feedback from a viewer to the object or to the computer or both.*

He describes his kinetic object with flashing white lights in which the sequence of patterns is controlled by a special purpose computer for generating fields of abstract algebra or Galois fields.

The second object he describes is more complex in that the frontal panel of flashing lights is made as a relief; colored light is used; the sequence of Galois field patterns can be changed in rhythm; and sound combinations corresponding to the patterns can be produced by the object.

The mathematics of Galois fields generated by polynomial equations that was used for determining the sequence of patterns in the objects is described. One of the most interesting aspects of this work is the demonstration of the different visual appearance of the patterns resulting from the polynomials that had not been noted before by mathematicians who have studied Galois fields.

I. INTRODUCTION

Up to the present time art produced with the aid of digital computers has depended mainly on the use of commercially available display equipment such as line printers, plotters and cathode ray tubes [1–12] (Fig. 1(a)). I find that this is akin to an artist being limited to the use of only two or three colours in a painting. It is true that much can be done with such equipment but one can hope that ways will be found to take better advantage of computers. I am especially sceptical of the attempts to produce computer art through play with randomness and the deliberate introduction of errors in programs prepared for non-artistic purposes. Dedalus, in James Joyce's novel, *Portrait of the Artist as a Young Man*, debates the question of whether an object made by hacking in fury at a piece of wood results in an object of art. Even if one grants the possibility of arriving at an art object through the use of chance effects or of the arbitrary limitations of available computer programs, it appears to me that this is neither the best method nor even a promising approach for artists.

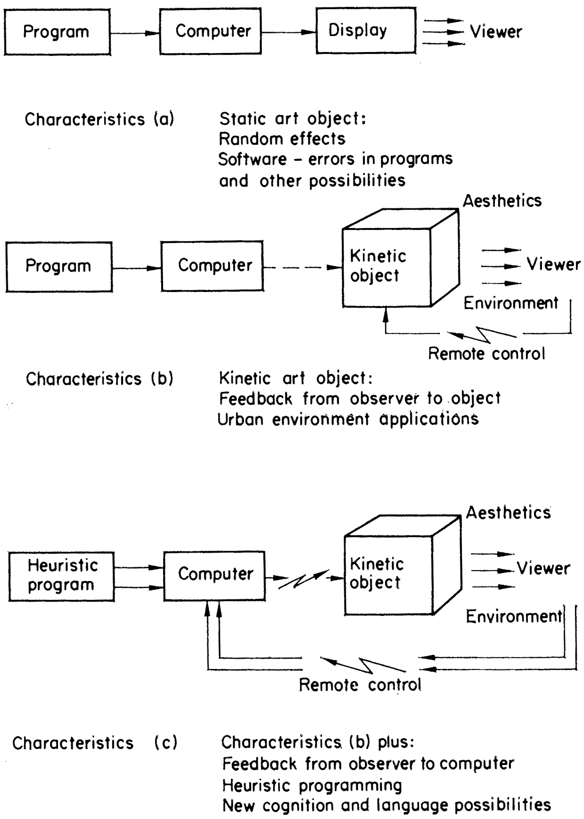

Fig. 1. Three schematic diagrams of the possible use of a digital computer for the production of visual art.

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Another way of making use of a computer in art is shown schematically in Fig. 1(b). Instead of using the presently available display equipment, the object is built and then controlled with the help of a computer [13–16]. The interaction of a viewer with an art object may be done by, for example, remote control. The fact that the dimension of time is a key factor in objects of a kinetic art type makes it possible to give artistic expression to visual and auditory experiences, perhaps of more significance than those that can be made by computer display methods that have been commonly used (Fig. 1(a)).

A further possible approach to computer art is shown in Fig. 1(c). Through additional feedback connections, for example, an electroencephalographic technique of tapping brain waves, it may be possible to obtain an even more subtle interaction with a heuristic program in addition to those obtained by means of the usual visual and auditory inputs. The feedback loop might be closed with an aesthetic output to an art object, which would then provide semantically relevant information to a viewer. I believe that such interactions will add to cognition, which will be reflected in language and perhaps provide improved means of communication [17, 18]. Discussions of the use of brain waves for artistic purposes can be found in Refs. 19 and 20.

It should be noted that the patterns of information that are semantically significant to the human nervous system do not always bear a readily verfiable mathematical counterpart in conventional symbolic notation. I have used in the work described in this article two types of pictorial representation instead of conventional symbolic methods generally used in mathematics [21, 22]. Some of these patterns (Figs. 2 and 3) represent *orbits* of finite fields of abstract algebra or Galois fields. E. Galois studied them by means of polynomial equations (cf. Appendix I) [23–28]. These polynomials are generators of Galois fields. If one compares in Fig. 3 patterns a to d, on the left, with those designated e to h, on the right, one notices that they have a clearly different visual appearance. This difference in the visual character of the orbits is difficult to anticipate from a consideration of the polynomials that symbolically express them and they were not, to the best of my knowledge, noted by mathematicians who study Galois fields.

II. APPLICATION OF GALOIS FIELDS TO KINETIC COMPUTER-ART OBJECTS

Kinetic objects making use of flashing lights have been described in *Leonardo* [20, 29]. If the flashing rate exceeds about 60 per second, the observer

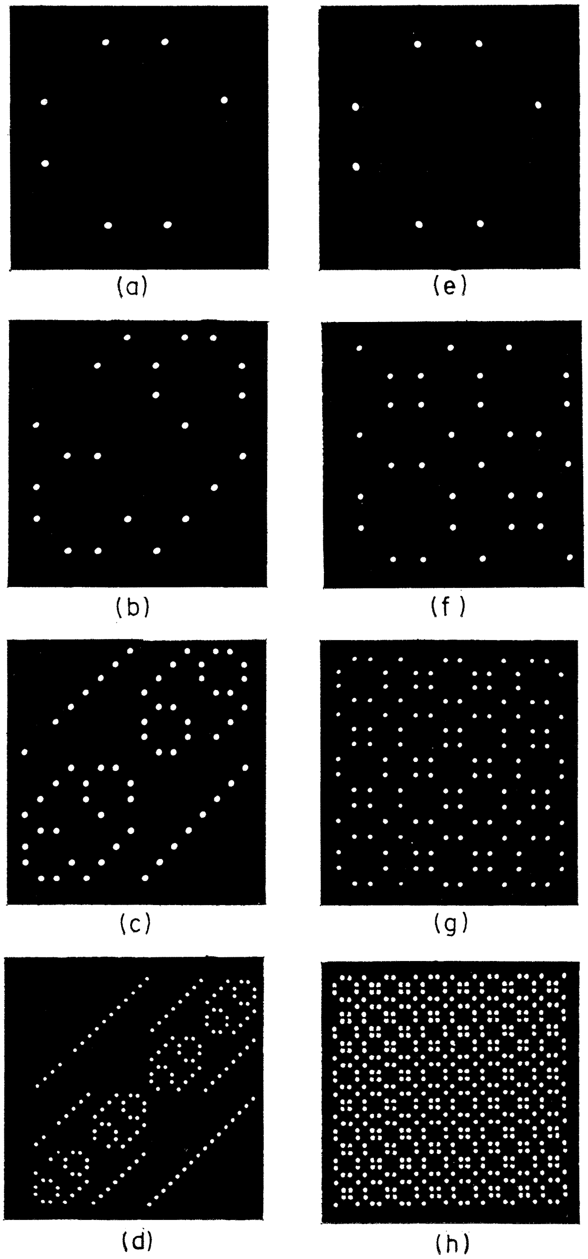

Fig. 2. Examples of patterns representing characteristic orbits of Galois fields.

cannot obtain a meaningful perception of images or patterns presented. [If the flashing rate is near the frequency of Alpha brain waves, the observer undergoes visual illusionary effects and those suffering from epilepsy are greatly affected [20]. Ed.]

1. 'Dynamic Object GF.E 32–S69/70'

Four consecutive symmetrical patterns generated by this object are shown in Fig. 4 (cf. Appendix II). The frontal panel is built of a 32 × 32 matrix of squares, which are illuminated with electric light bulbs. The synchronous selective flashing of the 1024 squares produces the patterns. The squares represent Galois field elements. The flashing is controlled by a Galois field generator with whose

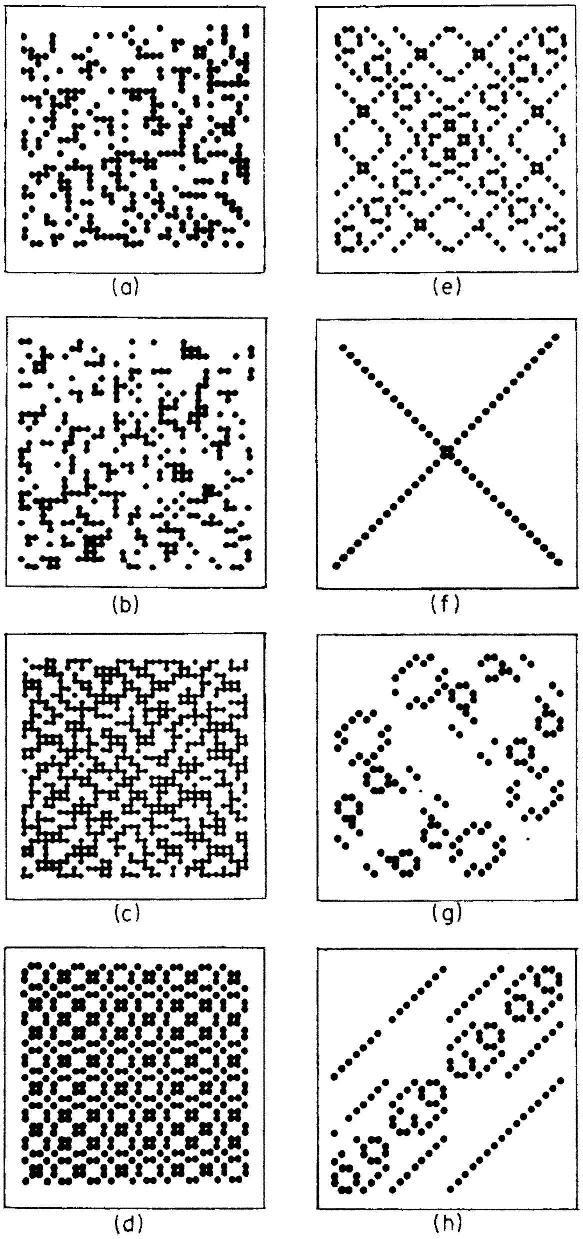

Fig. 3. *Examples of patterns representing characteristic orbits of Galois fields.*

help maximal difference between successive patterns is maintained. This maximization of differences enables one to distinguish between the successive patterns in the sequence, which change at the rate of one pattern every two seconds. The total number of symbolic structures is 2^{32}. At the rate of change used, the same pattern will repeat only after 274 years, 140 days, 12 hours, 32 minutes and 20 seconds.

The dimensions of the object are 64 × 64 × 12 cm. It has electric light bulbs whose lifetime has been significantly prolonged by the use of special electronic circuitry. The field generators are part of the special purpose computer located inside the object. The unit is self-contained and performs the generation of the Galois fields. The object has a

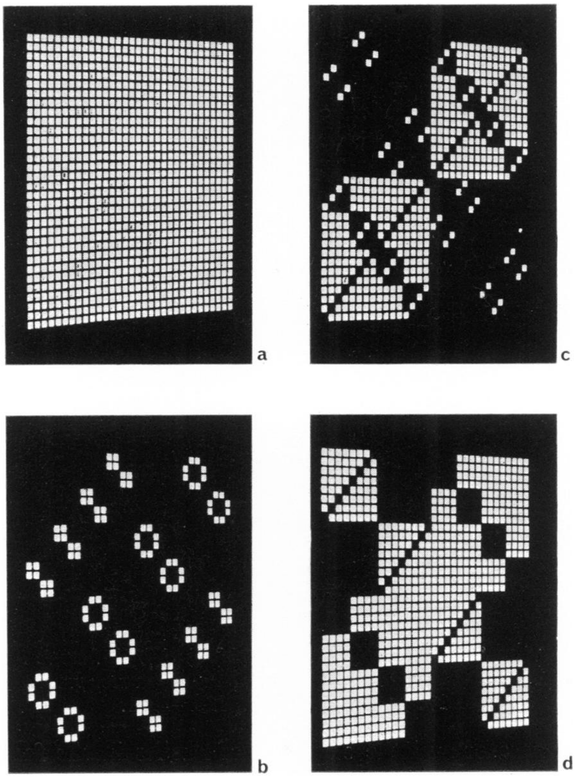

Fig. 4. *Four consecutive patterns generated by 'Dynamic Object GF. E32–S69/70'; flashing lights controlled by a computer program,* 64 × 64 × 12 cm., 1969–1970.

variable clock that allows the observer to control the speed of the pattern changes with periods ranging between 0·1—5 sec. On the rear of the object there are manual controls to start, stop, control the speed and for selecting or reading out any of the patterns. With the binary notation, 32 light indicators and 32 push buttons enable any pattern from the sequence to be read or set.

2. 'Dynamic Object GF.E (16, 4) 69/71'

A more complex artistic experience than provided by the first object described above can be provided if the frontal panel is made as a relief, colored light is used, the patterns are flashed rhythmically and sound combinations are incorporated. This I have done in 'Dynamic Object GF.E (16, 4) 69/71', which also makes use of light patterns controlled by a Galois field generator. This object is shown in Fig. 5.

The frontal panel, a relief structure, is a 32 × 32 matrix of 4·5 × 4·5 cm. squares. The overall dimensions are 178 × 178 × 20 cm. and the object weighs about one-half ton. The squares are transparent colored glass at the end of tubes of the length 8, 12, 16 and 20 cm. and they are illuminated by flashing electric light bulbs within the tubes (Fig. 6, cf. color plate). There are 16 different colors and 16 squares of the same color flash simultaneously in one plane. The sequence of flashes from plane to plane is controlled by a Galois field

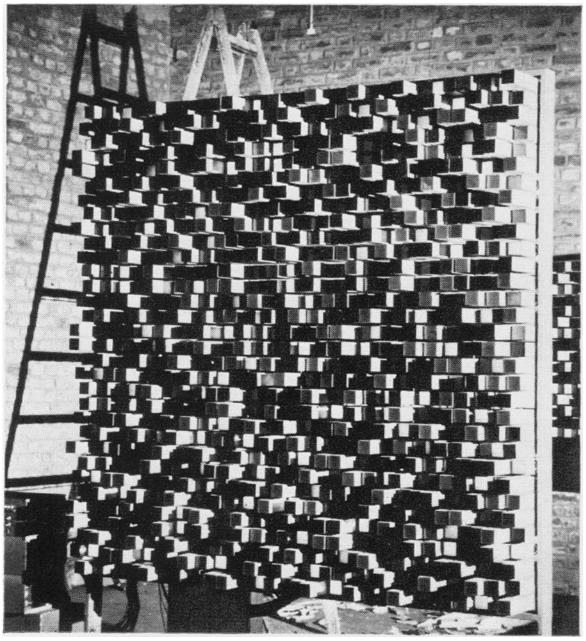

Fig. 5. 'Dynamic Object GF.E (16, 4) 69/71', kinetic art object with flashing lights controlled by a computer program, 178 × 178 × 20 cm., 1969–1971.

generator and the sequence of flashes in the four planes is controlled by another generator. A light will flash only when directed to do so by both generators. The rhythm of the changing patterns is controlled by a third Galois field generator.

For each color in a given plane a tone oscillator is provided. Since there are 16 colors in each plane, there are also 16 different tones and there are two octaves of tones in each of the four planes (Table 1). The field generator that controls the sequence of flashes in a plane also controls the sounding of the tones and the same generator that controls flashes from plane to plane also controls the sounding of tones separated by octaves in the four planes. A tone will be sounded by an oscillator only when directed to do so by both generators, as in the case of the light flashes. The maximalization of differences between visual patterns takes place also in the case of combinations of tones. The patterns and combinations are, therefore, intrinsically interwoven.

So far a satisfactory notation for electronic music has not been developed. I have, therefore, developed a notation that is inherent for the program used. Instead of the octaves of the tempered scale, I have simply doubled frequencies of each succeeding octave. The tone combinations are controlled by sophisticated mathematical relations, which can be expressed by the notation selected. Hearers of the sounds produced by the object have told me that they find them pleasant.

The total number of visual patterns and of tonal combinations that can be produced is $2^{16} \times 2^4$, where the exponents correspond to the 16 × 16 matrix in each plane and to the four planes, respectively. If the rhythm generator, which controls both the lights and the sound oscillators, causes changes every two seconds on the average, then the same visual pattern and its corresponding combination of tones will repeat only every 24 days, 6 hours, 32 minutes and 32 seconds.

A general purpose computer was used to help design the special purpose computer built within the object for controlling the patterns and tone combinations and their sequences. The electronic arithmeto-logical units for generating the Galois fields are physically separate from the object.

The object can be controlled by a viewer either manually or by remote control to the extent of the volume of the sound, the speed of change from pattern to pattern, the replay of sequences, the rhythm of the patterns and the exclusion of some of the production of tones. The patterns and their sequence cannot be altered.

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TABLE 1.

TABLE 1. 64 DIFFERENT SOUND TONE FREQUENCIES IN THE FOUR PLANES OF PATTERNS CONTROLLED BY COMPUTER ('DYNAMIC OBJECT GF.E (16, 4) 69/71'.

	ΔF	Frequency *F* (Cycles per second)							
Plane	8	64	72	80	88	96	104	112	120
I	16	128	144	160	176	192	208	224	240
Plane	32	256	288	320	352	384	416	448	480
II	64	512	576	640	704	768	832	896	960
Plane	128	1024	1152	1280	1408	1536	1664	1792	1920
III	256	2048	2304	2560	2816	3072	3328	3584	3840
Plane	512	4096	4608	5120	5632	6144	6656	7168	7680
IV	1024	8192	9216	10240	11264	12288	13312	14336	15360

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APPENDIX I

GALOIS FIELD GF (2^n)

In abstract algebra finite fields are known as *Galois fields*, named after E. Galois and studied by him in connection with his work on the roots of polynomial equations. It can be shown that for each prime number p and each positive integer n there is one and only one field with p^n elements and that fields of such composition are the only finite fields that exist. The field of order p^n is called the *Galois field of order* p^n and is denoted by GF(p^n) [23–28, 30, 31].

The Galois field of 2^n elements GF(2^n) may be formed as the field of polynomials over $GF(2)$, modulo $P_n(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \ldots + a_0$ [28, 29]. For example, if $n = 4$ and if the polynomial $P_n(x) = x^4 + x + 1$, which is irreducible and primitive, then the coefficients of the 15 polynomials that are 15 nonzero field elements are as follows (Table 2):

TABLE 2.

1.	1000	9.	1010
2.	0100	10.	0101
3.	0010	11.	1110
4.	0001	12.	0111
5.	1100	13.	1111
6.	0110	14.	1011
7.	0011	15.	1001
8.	1101		

In Table 3, below, the Galois field (2^4) is formed as the field of polynomials over $GF(2)$, module reducible polynomial $x^4 + x^2 + 1$. The 15 nonzero elements are listed as three subsets or *orbits*, starting with elements 1000, 1100 and 1110, respectively. Pictorial representations of orbits of $GF(2^n)$ modulo $P_n(x)$ are shown in Figs. 2 and 3.

TABLE 3.

1.	1000		
2.	0100		
3.	0010		
4.	0001		
5.	1010		
6.	0101		
		7.	1100
		8.	0110
		9.	0011
		10.	1011
		11.	1111
		12.	1101
13.	1110		
14.	0111		
15.	1001		

→ 16

0	14	5	11
9	7	12	2
11	5	14	0
2	12	7	9
15	1	10	
6	8	3	
4	10	1	
13	3	8	
7	9		
14	0		
12	2		
5	11		
8	6		
1	15		
3	13		
10	4		

→ 16

0	2	1	3
1	3	0	2
3	1	2	0
2	0	3	1
3	1	2	
2	0	3	
0	2	1	
1	3	0	
3	1		
2	0		
0	2		
1	3		
0	2		
1	3		
3	1		
2	0		

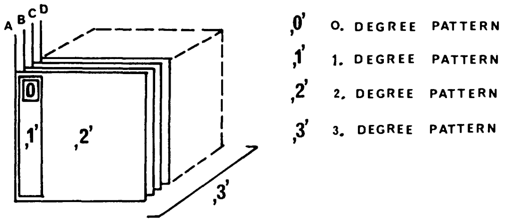


Fig. 7. Organization of patterns that determine the distribution of colors.

For the given orbit $A = [a_j]_{n\times s}$ of s elements, modulo $P_n\ (x)$, the coordinates $Z_j\ (\underline{x}'_j\ ,\ \underline{y}'_j)$, for $j = 1, 2, 3, \ldots, s$, in pictorial representation are obtained as [21]:

$$\underline{X}' = \underline{X}\ .\ \underline{A} = [\underline{x}_i]_{\ q_1\times n}\cdot[\underline{a}_j]_{\ n\times s} = [\underline{x}'_j]_{\ q_1\times s}$$
$$\underline{Y}' = \underline{Y}\ .\ \underline{A} = [\underline{y}_i]_{\ q_2\times n}\cdot[\underline{a}_j]_{\ n\times s} = [\underline{y}'_j]_{\ q_2\times s}$$

where $q_1 = \text{n} - q_2$

$q_2 = \text{integer}\ (n/2)$

Example: For an orbit of $GF(2^4)$, modulo $x^4 + x^2 + 1$, starting with element 1000 (Table 3) and for $\underline{X} = [\ \underline{I}\ ,\ \underline{O}\ ]\ ,\ \underline{Y} = [\ \underline{O}\ ,\ \underline{I}\ ]$, one obtains:

$$\underline{X}' = \begin{bmatrix}1000\\0100\end{bmatrix}\cdot\begin{bmatrix}100010\\010001\\001010\\000101\end{bmatrix} = \begin{bmatrix}100010\\010001\end{bmatrix}$$

$$\underline{Y}' = \begin{bmatrix}0010\\0001\end{bmatrix}\cdot\begin{bmatrix}100010\\010001\\001010\\000101\end{bmatrix} = \begin{bmatrix}001010\\000101\end{bmatrix}$$

An orbit of the first six field elements in Table 3 is shown represented in Fig. 10. For the same orbit another arrangement may be obtained by choosing different values of $\underline{X}$ and $\underline{Y}$.

Figure 3 shows pictorial representations for some polynomials P_n(x), where $n = 10$. From the total

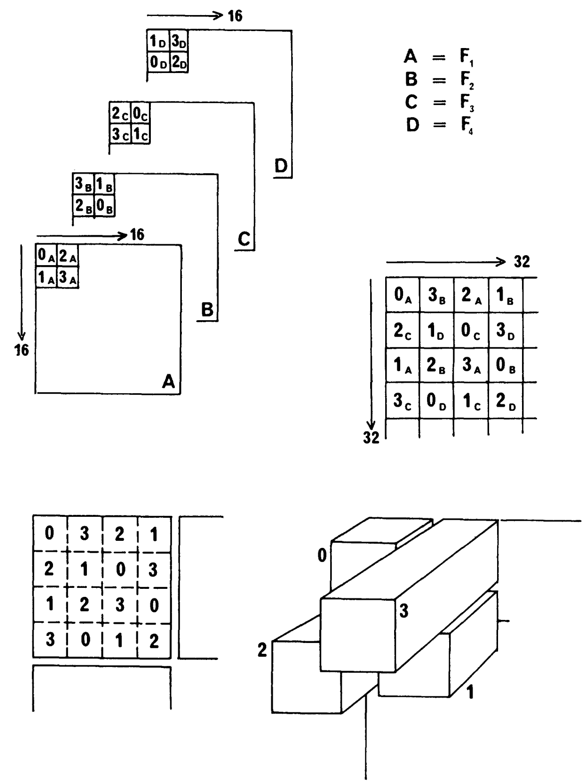


Fig. 8 Organization of patterns that determine relief.

number of 2^9 different polynomials the great majority have the structures as those shown in Fig. 3a, b, c and d, while a few of them have symmetrical structure as shown in Fig. 3e, f, g and h. Fig. 3e and f show orbits modulo $x^{10} + x^7 + x^2 + 1$, while Fig. 3g and h show orbits modulo $x^{10} + x^5 + x^4 + x^3 + x^2 + x + 1$. $\underline{X}$ and $\underline{Y}$ used were $\underline{X} = [\ \underline{I}\ ,\ \underline{O}\ ]\ ,\ \underline{Y} = [\ \underline{O}\ ,\ \underline{I}\ ]$.

Some characteristic symmetrical structures for polynomials $P_n(x)$, $n = 4, 6, 8, 10$, are given in Fig. 2. It is expected that more information about fields, structures and their relations can be obtained by means of pictorial representation [21].

APPENDIX II

FURTHER DISCUSSION OF THE ART OBJECTS

In the objects I have described, the relations among distinct residue classes of polynomials from $GF(2^m)$, module polynomial $P_n(x)$, can be observed. The relations are presented by means of two-dimensional patterns. The patterns are at maximal distance one from another; this is obtained by forming a field of polynomials $GF(2^{2^n})$, over $GF(2)$, modulo irreducible and primitive polynomial $P_{2^n}(x)$ [28, 30, 31].

The polynomials $D_m(x)_{j_1}$, $D_m(x)_{j_2} \ldots D_m(x)_{j_e}$ are in the same residue class, module polynomial:

$$P_n(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$$

if

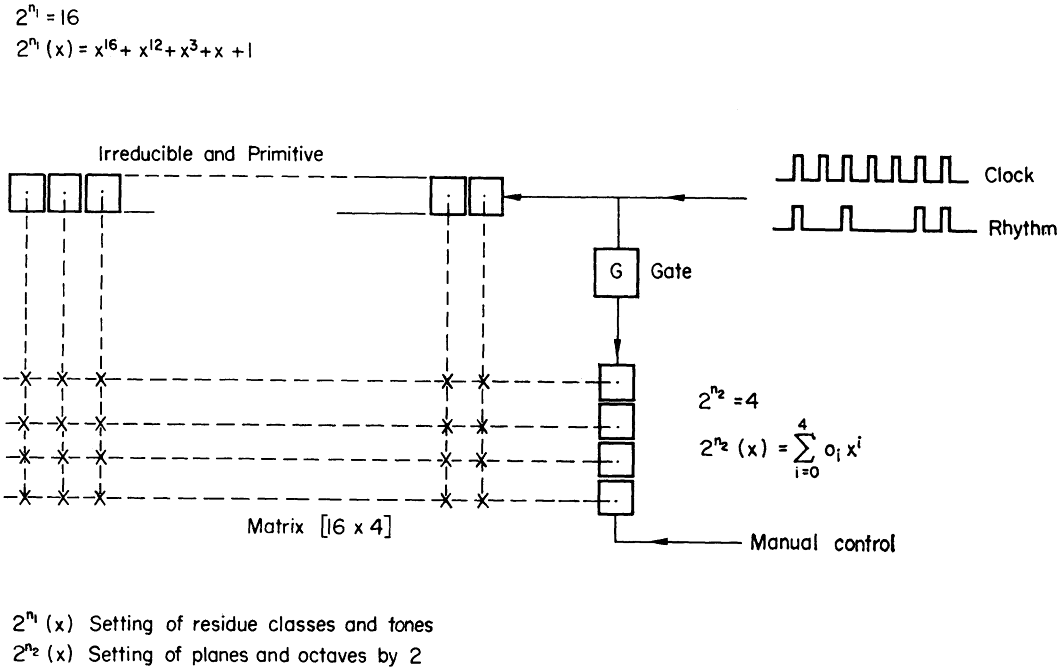


Fig. 9. Setting of residue classes, tones and planes by 2.

$D_m(x)_{j_i} \cdot S(x) = Q(x)_{j_i} \cdot P_n(x) + R(x)_{j_i}$ for $i = 1, 2, \ldots, e$

and if

$R(x)_{j_1} = R(x)_{j_2} = \ldots = R(x)_{j_e}$.

For any polynomial $P_n(x)$ there is the associated mask $\underline{M}_p = [\underline{k}_i]_{nxm}$ [32, 33], by which one can describe multiplication and division procedures for all the polynomials from $GF(2^m)$ in matrix notation:

$\underline{R} = \underline{M}_p \cdot \underline{D} = [\underline{k}_1, \ldots, \underline{k}_m] \cdot [\underline{d}_1, \ldots, \underline{d}_{2^m}] = [\underline{r}_1, \ldots, \underline{r}_{2^m}]$

where

$\underline{D} = [\underline{d}_j]_{n \times 2^m}$ and $\underline{d}_j \in GF(2^m)$ following in the natural order and

$\underline{R} = [\underline{r}_j]_{n \times 2^m}$ and $\underline{r}_j \in GF(2^n)$

1. 'Dynamic Object GF.E 32 – S 69/70'

The raster for a frontal panel is accomplished as the two-dimensional presentation of the 2nd degree pattern and can be described as [22]:

$$\underline{F} = [\underline{f}_i]_{2^n x 2^n} = \begin{bmatrix} \underline{r}_1 & \underline{r}_{2^n+1} & \cdots & \\ \underline{r}_2 & \underline{r}_{2^n+2} & & \\ \vdots & \vdots & & \\ \underline{r}_{2^n} & \underline{r}_{2^{n+1}} & \cdots & \underline{r}_{2^m} \end{bmatrix}$$

An arbitrary field element from $GF(2^{2^n})$ is a polynomial of the form:

$T(x) = t_{2^n-1} \cdot x^{2^n-1} + \ldots + t_1 \cdot x + t_0$ where $t_i \in GF(2)$

If $t_i = 1$ then the elements are set on the positions of those $\underline{r}_j$ for which

$\underline{r}_{j_1} = \underline{r}_{j_2} = \ldots = \underline{r}_{j_e} = i$

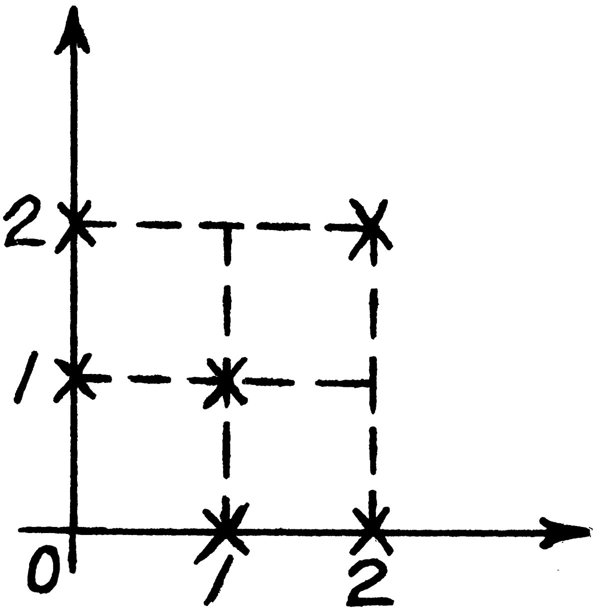


Fig. 10.

For this particular object, I had:

$n = 5$, $m = 10$	
$a_5 = a_0 = 1$; $a_4 = a_3 = a_2 = a_1 = a_0 = 0$	(reducible)
$\underline{M}_p = [\underline{I}, \underline{I}]_{5 \times 10}$	
$\underline{D} = [\underline{0}, \underline{1}, \underline{2}, \ldots, \underline{1023}]$	(decimal)
$0 \leqslant r_j \leqslant 31$	(decimal)
$P_{2^n}(x) = x^{32} + x^{22} + x^2 + x + 1$	(irreducible and primitive).

The raster and some sequential patterns that are at maximal distance one from another are shown in Fig. 4. The bulbs that correspond to the elements of a particular residue class, being set by t_i, will light during the time determined by a clock. Inside the object there is an arithmetical unit that performs arithmetical operations (modulo an irreducible binary polynomial). The arithmetical unit has a variable clock ranging from 0·1 to 5 seconds.

2. 'Dynamic Object GF.E (16, 4) 69/71'

The raster for a frontal panel is accomplished as the two-dimensional presentation of one quarter of the 3rd degree pattern (Fig. 7) and can be described as [22]:

$$\underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 + \underline{F}_4 = [\underline{f}_i]_1 + [\underline{f}_i]_2 + [\underline{f}_i]_3 + [\underline{f}_i]_4 =$$

$$= \begin{bmatrix} \underline{r}_1 & 0 & \cdots & & \\ \bar{0} & 0 & & & \\ \underline{r}_2 & 0 & & & \\ \bar{0} & 0 & & & \\ \vdots & & & & \\ \underline{r}_{2n} & 0 & & \underline{r}_{22n} & 0 \\ \bar{0} & 0 & \cdots & \bar{0} & 0 \end{bmatrix}_{2^{n+1} \times 2^{n+1}} +$$

$$+ \begin{bmatrix} 0 & \underline{r}_{2^{2n}+1} & \cdots & & \\ 0 & 0 & & & \\ \vdots & & & & \\ 0 & \underline{r}_{2^{2n}+2^n} & & 0 & \underline{r}_{2.2^{2n}} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 & \cdots & & 0 \\ \underline{r}_{2.2^{2n}+1} & 0 & & & \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 \\ \underline{r}_{2.2^{2n}+2^n} & 0 & \cdots & \underline{r}_{3.2^{2n}} & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 & \cdots & & \\ 0 & \underline{r}_{3.2^{2n}+1} & & & \\ \vdots & & & & \\ 0 & 0 & & 0 & 0 \\ 0 & \underline{r}_{3.2^{2n}+1} & & 0 & \underline{r}_{4.2^{2n}} \end{bmatrix}$$

where

$$\underline{D} = [\underline{D}_1, \underline{D}_2, \underline{D}_3, \underline{D}_4] = \left[[\underline{d}_j]_1, [\underline{d}_j]_2, [\underline{d}_j]_3, [\underline{d}_j]_4\right]_{m \times 2^m}$$

and $\underline{d}_j \in GF(2^m)$ following in a natural order and

$$\underline{R} = [\underline{R}_1, \underline{R}_2, \underline{R}_3, \underline{R}_4] = \left[[\underline{r}_j]_1, [\underline{r}_j]_2, [\underline{r}_j]_3, [\underline{r}_j]_4\right]_{n \times 2^m}$$

and $\underline{r}_j \in GF(2^n)$

The relief $\underline{H} = [h_{ij}]_{2^{n+1} \times 2^{n+1}}$ of the frontal panel (Fig. 8) is obtained by transforming each element from $\underline{F}$, so that an arbitrary element in the i-th row and the j-th column can be described:

$$\underline{h}_{ij} = \underline{M} \cdot \underline{r}_{ij} = [m_{ij}]_{v \times n} \cdot [\rho i 1]_{n \times 1}$$

The rhythm of the change of patterns is accomplished by linear recurring sequence $\{a_c\}$ which satisfies the linear recurrence [34]:

$$\sum_{i=0}^{n} p_{n-i} \cdot a_{c-i} = 0$$

It is possible to choose a short or a long sequence.

An arbitrary field element from $GF(2^{2^{n_1}})$, which controls light and sound tone, is a polynomial of the form:

$$T(x) = t_{2^{n_1}-1} \cdot x^{2^{n_1}-1} + \ldots + t_1 \cdot x + t_0$$

where $t_{i_1} \in GF(2)$

If $t_{i_1} = 1$ then:

(a) the elements are set on the position of those r_j for which
$r_{j_1} = r_{j_1} = \ldots = r_{j_e} = i_1$

(b) the i_1-th tones in all octaves are set.

An arbitrary field element from $GF(2^{2^{n_2}})$ for panel selection and 'octaves by 2' control is also a polynomial:

$$O(x) = o_{2^{n_2}-1} \cdot x^{2^{n_2}} + \ldots + o_1 \cdot x + o_0$$

where $o_{i_2} \in GF(2)$

If $o_{i_2} = 1$ then:

(a) the previously set elements are prepared for lighting only in planes $\underline{F}_{i_2}$

(b) the previously set tones are ready for activation only in o_{i_2} 'octaves by 2'.

For this particular object (Fig. 9) I had:

$n = 4, m = 10$

$a_4 = a_3 = a_0 = 1 ; a_2 = a_1 = 0$ (irreducible and primitive)

$$\underline{M}_p = \begin{bmatrix} 1111010110 \\ 0111101011 \\ 0011110101 \\ 1110101100 \end{bmatrix}_{4 \times 10}$$

$\underline{D}_1 = [\underline{0}, \ldots, \underline{255}]$ $\underline{D}_2 = [\underline{256}, \ldots, \underline{511}]$

$\underline{D}_3 = [\underline{512}, \ldots, \underline{767}]$ $\underline{D}_4 = [\underline{768}, \ldots, \underline{1023}]$

$0 \leqslant r_j \leqslant 15$ (decimal)

$v = 2, n_1 = 4, n_2 = 2$

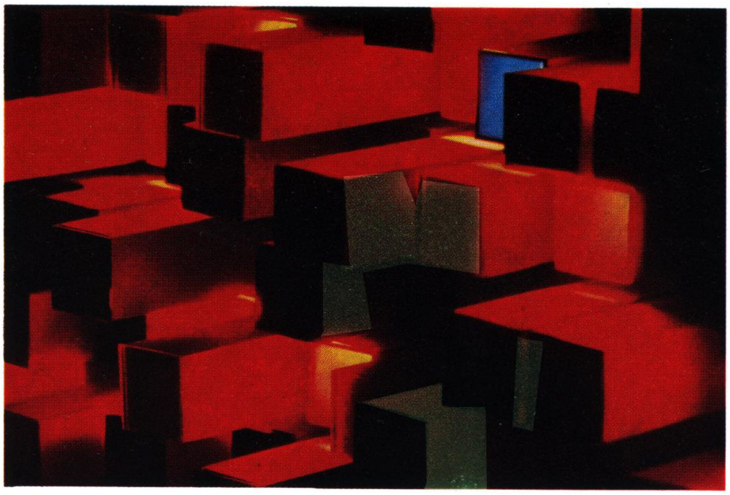
$\underline{M} = [\underline{I}, \underline{O}]_{2 \times 4}$

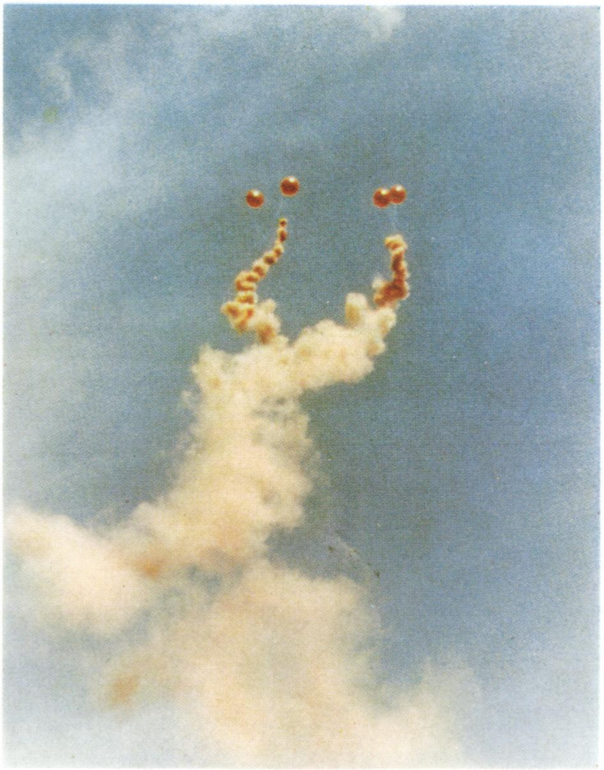
$h \leqslant 0 \leqslant 3$ (decimal)

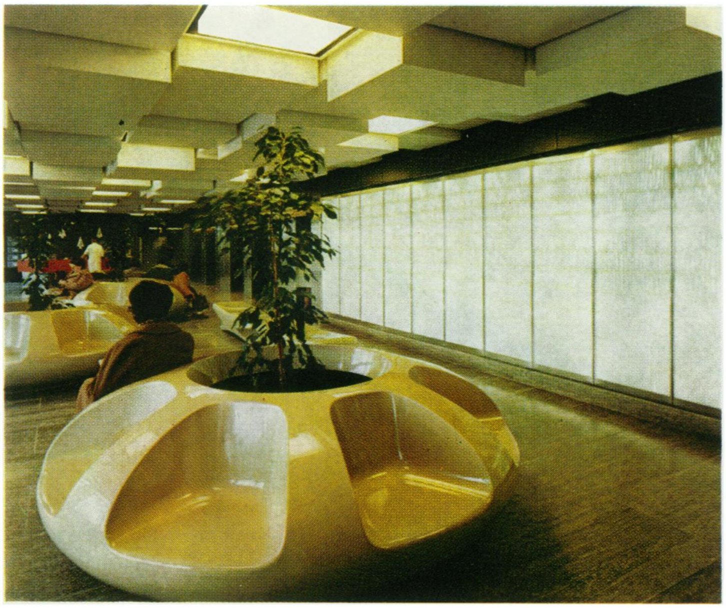
$T_{2^{n_1}}(x) = x^{16} + x^{12} + x^3 + x + 1$ (irreducible and primitive)

$O_{2^{n_2}}(x) = \sum_{i=0}^{4} o_i \cdot x^i$ (variable)

The frequencies of the tones in the four planes are listed in Table 1 in the main text.







Top, left: *Vladimir Bonačić, View of part of a pattern generated by 'Dynamic Object GF.E (16, 4) 69/71', flashing lights controlled by a computer program*, 178 × 178 × 20 cm., 1969–71. (Fig. 6, cf. page 195.)

Top, right: *Howard Woody, 'Twin Trail', two units, Nylon seine twine*, four 40 in. *Neoprene balloons, two red aniline-dye smoke cannisters*, 1972. (Fig. 4, cf. page 210.)

Bottom, left: *Nino Calos, Lumino-kinetic mural panel, Lumidyne system*, 2.80 × 12.00 m., 1972–73. (In the St. Antoniushove Hospital, Leidschendam, The Hague, Holland.) (Photo: *Lighting Review*, Amsterdam, Holland.) (Fig. 1, cf. page 253.)

Bottom, right: *William Vazan, 'Rainbow Grounded', acrylic on Masonite slats placed on a rock outcropping*, 1968–72. (Fig. 1, cf. page 203.)