APPENDIX 1

Prime Factorisation of the first 200 Integers, with Primes and Prime Powers shown in bold:

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APPENDIX 2

Sieve Listings
Pour la Paix (1981)

Pour Maurice (1982)
Chant des soleils (1983)

Idmen A (1985)
Tracées (1987)
APPENDIX 3

Full Scores and Score Excerpts

Note: Occasional marks on the scores are not related to the present analysis. They are performance notes and, unfortunately, could not be removed.
Keqrops (1896, for piano and orchestra)

Bars 121-141 & 160-165
À l’île de Gorée (1896, for amplified harpsichord and ensemble)

Full Score
à l’île de Gorée

pour clavecín amplifié et orchestre

Iannis XENAKIS
Horos (1986, for orchestra)

Bars 46-51
Akea (1986, for piano and string quartet)

Full Score
*Ata* (1987, for orchestra)

Bars 1-9 & 64-75
Echange (1989, for bass clarinet and ensemble)

Bars 1-9
ECHANGE

PAR CLARINETTE BASSE ET ENSEMBLE INSTRUMENTAL

J. XENAKIS 1989

DEDIE A HENRY SPARRARY
ET A L'ASCO ENSEMBLE

COMMANDE DE : "STICHTING AMSTERDAM FONDS VOOR DE KUNST" POUR L'ENSEMBLE ASKO

INSTRUMENTATION : 1 clarinette basse en soliste (4.3.3.3)
1 flute, 1 hautbois, 1 clarinette en si, 1 basson, 1 cuire, 1 trompette en si,
1 trombone, 1 tuba, 2 violons, 1 alto, 1 violoncelle, 1 contrebasse.

NOTATION EN NOTES NATURELLES SAUF A LA CONTREBAIX QUI DOIT AVOIR "*" PAS DE VIBRATO !

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EDITIONS SALABERT S.A. 22, rue Clauzel 75009 PARIS
Tetora (1990, for string quartet)

Full Score
Figure 3.1: The Sieve of Eratosthenes, with $n = 50$. 

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Figure 3.2: Matrix for Moduli 4 and 3.

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Figure 3.3: (3, 0) + (4, 0).

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Figure 3.4: Major Diatonic Scale.

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Figure 3.5: Matrix for moduli 5 and 12.
Figure 3.6: $(3, 0) + (4, 3)$.

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Figure 3.7: $(3, 0) + (4, 1)$.

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Figure 3.8: $(3, 0) + (4, 0) + (3, 1)·(4, 1) + (3, 2)·(4, 2)$.

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Figure 3.9: $(4, 0) + (3, 0)\cdot(4, 2)$.

Figure 3.10: Matrices for the Mode of D and the Major Diatonic Scale.
Figure 3.11: Harmonic minor.

3.12: Sieve of Jonchaies.
Prime                  Composite

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Figure 3.13: Types of Sieves according to their Symmetry and Periodicity.

Figure 3.14: Cyclic Transposition of the Major Diatonic Scale, $T_{+10}(\text{mod}12)$, that results in the Mode of D.
Figure 3.15: $r_2 = r_1 + r_2 + 1$

Figure 3.16: $m_2 = m_2 + 2$
Figure 4.1: Sieve of Nekuña.

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Figure 4.2: Decomposed Matrix of the Sieve of Nekuña.
| M, I, R | 0 | 2 | 3 | 4 | 7 | 9 | 10 | 13 | 14 | 16 | 17 | 21 | 24 | 25 | 29 | 30 | 34 | 35 | 38 | 39 | 43 |
|---------|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 24, 0, 3 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 14, 2, 6 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 22, 3, 3 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 31, 4, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 28, 7, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 29, 9, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 19, 10, 4 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 25, 13, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 24, 14, 3 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 26, 17, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 23, 21, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 24, 10, 3 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 30, 9, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 35, 17, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 29, 24, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 32, 25, 1 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 30, 29, 1 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 26, 21, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 30, 17, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 31, 16, 2 | + |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

Figure 5.1: Simplified Matrix of the sieve of Nekuia.
| M. I. R | 44 | 47 | 48 | 52 | 53 | 57 | 58 | 59 | 62 | 63 | 66 | 67 | 69 | 72 | 73 | 77 | 78 | 82 | 86 | 87 | 88 |
|--------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 24, 0, 3 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 14, 2, 6 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 22, 3, 3 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 31, 4, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 28, 7, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 29, 9, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 19, 10, 4 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 25, 13, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 24, 14, 3 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 26, 17, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 23, 21, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 24, 10, 3 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 30, 9, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 35, 17, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 29, 24, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 32, 25, 1 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 30, 29, 1 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 26, 21, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 30, 17, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 31, 16, 2 | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |

Figure 5.1: Simplified Matrix of the sieve of Nekuña, cont.
Figure 5.2: Sieve of Akea.

Figure 5.3: Decomposed matrix for the sieve of Akea.
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<td>8\textsuperscript{ve} + 5\textsuperscript{th}</td>
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Figure 5.4: Sketches of Akea, Simplified Formula of the Sieve.
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Figure 5.5: Xenakis’s Calculation of the Simplified Formula for the sieve of Akea.
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Figure 5.6: Simplified Matrix of the Sieve of *Akea* from the Formula as found in the Pre-compositional Sketches.
Figure 5.7: The Inner Periodicities of 2·8ve (24, 7, 3) and Perfect 4th (5, 40, 8) in the Sieve of Akea.

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Figure 5.8: Moduli Congruent with the Perfect 4th in the Sieve of Akea.
Figure 5.9: Sieve of Akea and its Moduli Congruent Modulo the Perfect 4th.
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Figure 5.10: Simplified Formula of the Sieve of Akea, suggested by Program B.
Figure 5.11: Simplified Matrix of the Sieve of Akea, from the Formula suggested by Program B.
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Figure 5.12: The Sieve of Akea and the two versions of the Algorithm (1986 and 1990).
Figure 5.13: The size of $M$ as $I$ increases, in the Formulae of the Sieve of $Akea$ by the two versions of the Algorithm.
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Figure 5.15: Simplified Matrix of the Inversion of the Sieve of *Akea*. 
Figure 5.16: The Minimum and Maximum Values of $M$, according to the Condition of Inner Symmetry, as $I$ Increases in the Sieve of $Akeu$. 
Figure 5.17: The Maximum Values of $I$ as $M$ Increases, when $n = 80$. 
Figure 5.18: The Condition of Inner Symmetry and the Sieve of *Akea*.
Figure 5.19: The values of $R$ and $n \pmod{M}$ as $M$ increases, when $n = 80$. 
Figure 5.20: Inner Symmetry Chart for the Sieve of Akea.
Figure 5.21: Inner Symmetry Chart for (2, 0), with $n = 80$. 
Figure 5.22: Inner Symmetry Chart for \((3, 1) + (4, 0)\), with \(n = 80\).
Figure 5.23: Inner Symmetry Chart for (5, 0) + (6, 1) + (7, 2), with $n = 80$. 
Figure 6.1: Sketches of Palimpsest, sieve of Jonchaies $T_1$

Figure 6.2: Sketches of Palimpsest, Sieve of Jonchaies $T_7$ and with perfect 4th instead of m3rd.

Figure 6.3: Sketches of Palimpsest, Sieve of Jonchaies $T_{-1}$ and omitting the last element.
Figure 6.4: Original Sieve of Mists.

Figure 6.5: Final Version of the Sieve of Mists.
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</table>

Figure 6.6: Decomposed Matrix for the Sieve of Mists.

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Figure 6.7: Sub-matrix for the Sieve of Mists.
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<th>Interval</th>
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<tbody>
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<td>25, 16, 2</td>
<td>2·8ve + semitone</td>
</tr>
<tr>
<td></td>
<td>28, 10, 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28, 16, 2</td>
<td>2·8ve + M3rd</td>
</tr>
<tr>
<td></td>
<td>28, 23, 2</td>
<td></td>
</tr>
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<td>29, 23, 2</td>
<td>2·8ve + 4th</td>
</tr>
<tr>
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<td>30, 8, 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30, 21, 2</td>
<td>2·8ve + tritone</td>
</tr>
<tr>
<td></td>
<td>30, 27, 2</td>
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<tr>
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<td>32, 30, 1</td>
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<td>33, 10, 2</td>
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<td>33, 10, 2</td>
<td>2·8ve + M6th</td>
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<td>5·7</td>
<td>35, 23, 1</td>
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<tr>
<td></td>
<td>35, 23, 1</td>
<td>2·8ve + M7th</td>
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<td>41, 8, 2</td>
<td>3·8ve + 4th</td>
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<td>47, 2, 1</td>
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<tr>
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<td>47, 2, 1</td>
<td>3·8ve + M7th</td>
</tr>
</tbody>
</table>

Figure 6.8: Simplified Formula of the Sieve of Mists.
Figure 6.9: Inner Symmetry Chart for the Final Sieve of *Mists*. 
Figure 6.10: Sieve of Mists, $T_7(\text{mod}90)$.
Figure 6.11: Sieve of Mists, $T_d(\text{mod}90)$. 
Figure 6.12: Sieve of Mists, $T_{10}(\text{mod}90)$. 
Figure 6.13: Sieve of Mist, \( T_{15}(\text{mod}90) \).
Figure 6.14: Sieve of Mists, $T_{20}(\text{mod}90)$. 
Figure 6.15: Sieve of Mists, $T_{24}(\text{mod}90)$. 
Figure 6.16: Sieve of $Mists, T_{29} (\text{mod} 90)$. 
Figure 6.17: Sieve of Mists, $T_{30} \pmod{90}$. 
Figure 6.18: Sieve of Mists, $T_{36}(\text{mod90})$. 
Figure 6.19: Sieve of $M_{38}(\text{mod}90)$. 
Figure 6.20: Sieve in the sketches of Aïs that was not finally used.

Figure 6.21: Original Sieve of Aïs (not finally used).

Figure 6.22: Sieve of Aïs (T_{+(7)} of the Original).
<table>
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<tr>
<th>Canonical Form</th>
<th>M, I, R</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>3·7</td>
<td>21, 9, 3</td>
<td>8ve + M6th</td>
</tr>
<tr>
<td></td>
<td>21, 19, 3</td>
<td></td>
</tr>
<tr>
<td>2·11</td>
<td>22, 20, 3</td>
<td>8ve + m7th</td>
</tr>
<tr>
<td>2³·3</td>
<td>24, 16, 2</td>
<td>2·8ve</td>
</tr>
<tr>
<td></td>
<td>24, 23, 2</td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td></td>
</tr>
<tr>
<td>5²</td>
<td>25, 0, 3</td>
<td>2·8ve + semitone</td>
</tr>
<tr>
<td></td>
<td>25, 12, 2</td>
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</tr>
<tr>
<td>2·13</td>
<td>26, 19, 2</td>
<td>2·8ve + tone</td>
</tr>
<tr>
<td></td>
<td>26, 20, 2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2²·7</td>
<td>28, 6, 2</td>
<td>2·8ve + M3rd</td>
</tr>
<tr>
<td></td>
<td>29, 0, 2</td>
<td>2·8ve + 4th</td>
</tr>
<tr>
<td>2·3·5</td>
<td>30, 20, 2</td>
<td>2·8ve + tritone</td>
</tr>
<tr>
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<td>31, 16, 2</td>
<td>2·8ve + 5th</td>
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<td>2·8ve + m6th</td>
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<td>33, 6, 2</td>
<td>2·8ve + M6th</td>
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<td>33, 20, 2</td>
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<tr>
<td>2·17</td>
<td>34, 3, 2</td>
<td>2·8ve + m7th</td>
</tr>
<tr>
<td>2³·3</td>
<td>36, 0, 2</td>
<td>3·8ve</td>
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<tr>
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Figure 6.23: Simplified Formula of the Final Sieve of Aïs.
Figure 6.24: Inner Symmetry Chart for the Final Sieve of *Aiš*. 
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<th>Interval</th>
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<td>$8\text{ve} + \text{M6th}$</td>
</tr>
<tr>
<td></td>
<td>$21, 20, 3$</td>
<td></td>
</tr>
<tr>
<td>$2\cdot 11$</td>
<td>$22, 11, 3$</td>
<td>$8\text{ve} + \text{m7th}$</td>
</tr>
<tr>
<td></td>
<td>$22, 14, 3$</td>
<td></td>
</tr>
<tr>
<td>$5$</td>
<td>$23, 14, 3$</td>
<td>$8\text{ve} + \text{M7th}$</td>
</tr>
<tr>
<td></td>
<td>$5^2$</td>
<td>$2\cdot 8\text{ve} + \text{semitone}$</td>
</tr>
<tr>
<td></td>
<td>$28, 16, 2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$28, 21, 2$</td>
<td>$2\cdot 8\text{ve} + \text{M3rd}$</td>
</tr>
<tr>
<td></td>
<td>$28, 27, 2$</td>
<td></td>
</tr>
<tr>
<td>$10$</td>
<td>$30, 0, 2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2\cdot 3\cdot 5$</td>
<td>$2\cdot 8\text{ve} + \text{tritone}$</td>
</tr>
<tr>
<td></td>
<td>$30, 3, 2$</td>
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<tr>
<td></td>
<td>$30, 28, 1$</td>
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<td>$31, 7, 2$</td>
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<td>$15$</td>
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<td>$2\cdot 8\text{ve} + \text{5th}$</td>
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<td>$2\cdot 8\text{ve} + \text{m6th}$</td>
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<tr>
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<td>$33, 20, 2$</td>
<td>$2\cdot 8\text{ve} + \text{M6th}$</td>
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Figure 6.25: Simplified Formula of the Original Sieve of *Aïs*. 
Figure 6.26: Inner Symmetry Chart for the original sieve of Aïs.
Figure 6.27: Inner Symmetry Chart of the $T_7(\text{mod}86)$ Transposition of the original sieve of $Aïs$. 
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<td>19, 10, 4</td>
<td>8ve + 5th</td>
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<tr>
<td>2·11</td>
<td>22, 3, 3</td>
<td>8ve + m7th</td>
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<tr>
<td>23</td>
<td>23, 21, 2</td>
<td>8ve + M7th</td>
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<td>5</td>
<td>24, 0, 3</td>
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<td>24, 10, 3</td>
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</tr>
<tr>
<td>5²</td>
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<td>2·8ve + semitone</td>
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<td>2·13</td>
<td>26, 17, 2</td>
<td>2·8ve + tone</td>
</tr>
<tr>
<td></td>
<td>26, 21, 2</td>
<td></td>
</tr>
<tr>
<td>2³·7</td>
<td>28, 7, 2</td>
<td>2·8ve + M3rd</td>
</tr>
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<td>29</td>
<td>29, 9, 2</td>
<td>2·8ve + 4th</td>
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<tr>
<td></td>
<td>29, 24, 2</td>
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</tr>
<tr>
<td>15</td>
<td>30, 9, 2</td>
<td>2·8ve + tritone</td>
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<tr>
<td>2·3·5</td>
<td>30, 17, 2</td>
<td>2·8ve + tritone</td>
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<tr>
<td></td>
<td>30, 29, 1</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>31, 4, 2</td>
<td>2·8ve + 5th</td>
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<tr>
<td></td>
<td>31, 16, 2</td>
<td></td>
</tr>
<tr>
<td>2³</td>
<td>32, 25, 1</td>
<td>2·8ve + m6th</td>
</tr>
<tr>
<td>20</td>
<td>35, 17, 2</td>
<td>2·8ve + M7th</td>
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Figure 6.28: Simplified Formula of the Sieve of Nekuïa.
Figure 6.29: Inner Symmetry Chart for the Sieve of Nekaia.


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<th>Effective Index mod88</th>
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<td>$s_i + 11_j$</td>
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<td>$s_i + 11_{j+8}$</td>
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<tr>
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</tr>
<tr>
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<tr>
<td>$s_i + 11_{j+1}$</td>
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<tr>
<td>$s_{i+5} + 11_j$</td>
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<tr>
<td>$s_i + 11_{j+3}$</td>
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Figure 6.30: Cyclic Transpositions in Nekuia.
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<th>Interval</th>
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<td>8ve + tone</td>
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<tr>
<td>19</td>
<td>19, 16, 3</td>
<td>8ve + 5th</td>
</tr>
<tr>
<td>2·11</td>
<td>22, 9, 3</td>
<td>8ve + m7th</td>
</tr>
<tr>
<td>23</td>
<td>23, 4, 3</td>
<td>8ve + M7th</td>
</tr>
<tr>
<td>23, 22, 2</td>
<td>2·8ve</td>
<td></td>
</tr>
<tr>
<td>24, 6, 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2·3</td>
<td>24, 16, 2</td>
<td>2·8ve</td>
</tr>
<tr>
<td>24, 20, 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2·5</td>
<td>25, 15, 2</td>
<td>2·8ve + semitone</td>
</tr>
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<td>25, 19, 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2·13</td>
<td>26, 23, 2</td>
<td>2·8ve + tone</td>
</tr>
<tr>
<td>2·7</td>
<td>27, 9, 2</td>
<td>2·8ve + m3rd</td>
</tr>
<tr>
<td>28, 13, 2</td>
<td>2·8ve + M3rd</td>
<td></td>
</tr>
<tr>
<td>30, 5, 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2·3·5</td>
<td>30, 19, 2</td>
<td>2·8ve + tritone</td>
</tr>
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<td>30, 23, 2</td>
<td></td>
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</tr>
<tr>
<td>31</td>
<td>31, 10, 2</td>
<td>2·8ve + 5th</td>
</tr>
<tr>
<td>31, 22, 2</td>
<td>2·8ve + 5th</td>
<td></td>
</tr>
<tr>
<td>31, 27, 1</td>
<td></td>
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<tr>
<td>2·7</td>
<td>32, 27, 1</td>
<td>2·8ve + m6th</td>
</tr>
<tr>
<td>2·3·3</td>
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<td>3·8ve</td>
</tr>
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</table>

\[ T_{33} \text{mod}88 \]

\[ T_{40} \text{mod}88 \]

\[ T_{50} \text{mod}88 \]

Figure 6.31: Simplified Formulae of the Cyclic Transpositions of the Sieve of Nekuia.
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<th>Interval</th>
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</thead>
<tbody>
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<td>8ve + 5th</td>
</tr>
<tr>
<td>2·5</td>
<td>20, 4, 4</td>
<td>8ve + m6th</td>
</tr>
<tr>
<td>2·11</td>
<td>22, 13, 3</td>
<td>8ve + m7th</td>
</tr>
<tr>
<td>2·3</td>
<td>24, 19, 2</td>
<td>2·8ve</td>
</tr>
<tr>
<td>5²</td>
<td>25, 4, 3</td>
<td>2·8ve + semitone</td>
</tr>
<tr>
<td>2·13</td>
<td>26, 18, 2</td>
<td>2·8ve + tone</td>
</tr>
<tr>
<td>3²</td>
<td>27, 25, 2</td>
<td>2·8ve + m3rd</td>
</tr>
<tr>
<td>2·7</td>
<td>28, 0, 3</td>
<td>2·8ve + M3rd</td>
</tr>
<tr>
<td>29</td>
<td>29, 9, 2</td>
<td>2·8ve + 4th</td>
</tr>
<tr>
<td>2·3·5</td>
<td>30, 18, 2</td>
<td>2·8ve + tritone</td>
</tr>
<tr>
<td>31</td>
<td>31, 1, 2</td>
<td>2·8ve + 5th</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Canonical Form</th>
<th>M, I, R</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>19, 12, 4</td>
<td>8ve + 5th</td>
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<tr>
<td>23</td>
<td>23, 8, 3</td>
<td>8ve + M7th</td>
</tr>
<tr>
<td>2·3</td>
<td>24, 1, 3</td>
<td>2·8ve</td>
</tr>
<tr>
<td>5²</td>
<td>25, 0, 3</td>
<td>2·8ve + semitone</td>
</tr>
<tr>
<td>2·13</td>
<td>26, 4, 3</td>
<td>2·8ve + tone</td>
</tr>
<tr>
<td>2·7</td>
<td>28, 17, 2</td>
<td>2·8ve + M3rd</td>
</tr>
<tr>
<td>29</td>
<td>29, 1, 3</td>
<td>2·8ve + 4th</td>
</tr>
<tr>
<td>2·3·5</td>
<td>30, 0, 2</td>
<td>2·8ve + tritone</td>
</tr>
<tr>
<td>31</td>
<td>31, 3, 2</td>
<td>2·8ve + 5th</td>
</tr>
<tr>
<td>2²</td>
<td>32, 21, 2</td>
<td>2·8ve + m6th</td>
</tr>
<tr>
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**Figure 6.31**: Simplified Formulae of the Cyclic Transpositions of the Sieve of Nekuia, cont.
Figure 6.32: Sieve of Nekuïa, $T_8(\text{mod} 88)$. 
Figure 6.33: Sieve of Nekuiña, $T_{33}(\text{mod}88)$. 
Figure 6.34: Sieve of Nekuïa, T_{a0}(mod88).
Figure 6.35: Sieve of Nekaia, $T_{36} \pmod{88}$.
Figure 6.36: Sieve of Nekuña, $T_{77}(\mod 88)$. 
Figure 6.37: Sieve of Nekuña, $T_{80}(\text{mod}88)$. 
Figure 6.38: First Version of the Sieve of Nekuña.
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Figure 6.39: Simplified Formula of the First Version of the sieve of Nekuya.
Figure 6.40: Inner Symmetry Chart of the First Version of the Sieve of Nekuiša.
Figure 6.41: Inner Symmetry Chart of the sieve segment used in Naama.
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Figure 6.45: Fourth version of the sieve of Nekuia.
Figure 6.46: Inner Symmetry Chart for the Segment of the Sieve used in *Horos*.
Figure 6.47: Complement of the Sieve of Akea.
Figure 6.48: Fifth Version of the sieve of Nekuña.
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Figure 6.50: Opening Sieve of Komboï.
Figure 6.51: Sieve of Komboï.
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Figure 6.53: Inner Symmetry Chart for Sieve $\alpha$ of Shaar.
**Figure 6.54: Intervallic Permutations in Sieve δ of Shaar.**

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**Figure 6.55: Intervallic Permutations in Sieve γ of Shaar.**

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Figure 6.56: Sieve of Tetras.
Figure 6.57: Opening Sieve of Khal Perr.
Figure 6.58: Sieve of *Lichens*. 
Figure 6.59: Sieve of Thallein.
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Figure 6.64: Version of the Sieve of Keqrops used in Tuurakemsu.
Figure 6.65: Version of the Sieve of Keqrops used in Kyania.
Figure 6.66: SWF.
Figure 6.67: CSWF.
Figure 6.68: SWF'.
Figure 6.69: CSWF'.
Figure 6.70: Sieve of *Krinoïdi*. 
Figure 6.71: Complement of the Sieve of *Krinoïdi*. 
Figure 6.72: Sieve of Kyania.
Figure 6.73: Sketches of Echange, Formula of ASK.
Figure 6.74: ASK.
Figure 6.75: Complement of ASK.
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Figure 6.77: Sieve of Epicycle as used in Tetora.
Figure 6.78: Complement of the Sieve of Epicycle as used in Tetora.
Figure 6.79: Sieve of *Paille in the Wind*. 
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Figure 7.1: Sieves in the Final Section of *Keqrops* and Segments of the Sieve of *Nekuia*, cont.
Figure 7.2: *Horos*, bars 47-50. Sieve Segments in the strings.
Figure 7.3: *Aia*, bars 68-70.
Figure 7.4: *Ata*, bars 68-70, Vertical Pentachords.
Figure 7.5: Sketches of Keqrops, Tetrachords of bars 126-137.

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<th>128</th>
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Figure 7.6: The Tetrachords of bars 126-137, as they are used in Keqrops.
Bars: | 126 | 127 | 128 |
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Bars: | 129 | 130 | 131 |
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Figure 7.7: Permutations of the Elements of Tetrachord γ in bars 126-131 of Keqrops.

Bars: | 0-12 | 13-14 | 14-18 | 18-21 |
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Figure 7.8: The Exchange of the Sieve and its Complement in the First Section of Akea.

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Figure 7.9: Akea, bars 21-29, the Time-points of the Accented Dyads in the Piano.
Figure 7.10: *Akea*, bar 36, Reiterated Chord.

Figure 7.11: *Akea*, T.12 Segment of the Original Sieve for the Piano Sieve-Clusters in bars 48-50.

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Figure 7.15: *Akea*, Pitch Content of bars 71-78.
Figure 7.16: *Akea*, Sieve-Clusters of bars 71-78, after Octave Reduction.

Figure 7.17: *Akea*, Pitch Content of the Sieve-Clusters of bars 71-78 (after Octave Reduction).
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Figure 7.19: À l’île de Gorée, Chords of the Chord Sequence in bars 30-60.
Figure 7.20: À l’île de Gorée, bars 34-53, Brass Chord Sequence.
Figure 7.21: À l’île de Gorée, bars 63-80, The Chords of the First Two Layers in the Chord Sequence.

Figure 7.22: À l’île de Gorée, Sieve of bars 99-101.
Figure 7.23: À l’île de Gorée, Harpsichord Sieve of bars 102-116.

Figure 7.24: À l’île de Gorée, Chords of the Sequences in bars 127-137.
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<td>6&lt;sup&gt;th&lt;/sup&gt;</td>
<td>a e e f b c</td>
</tr>
<tr>
<td></td>
<td>2 2 3 2 3</td>
</tr>
<tr>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
<td>a b b e c c</td>
</tr>
<tr>
<td></td>
<td>3 2 3 2 2</td>
</tr>
<tr>
<td>8&lt;sup&gt;th&lt;/sup&gt;</td>
<td>a c e d f c</td>
</tr>
<tr>
<td></td>
<td>2 3 2 3 2</td>
</tr>
<tr>
<td>9&lt;sup&gt;th&lt;/sup&gt;</td>
<td>a b f e c b</td>
</tr>
<tr>
<td></td>
<td>2 3 2 2 3</td>
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<tr>
<td>10&lt;sup&gt;th&lt;/sup&gt;</td>
<td>a e e f b c</td>
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<tr>
<td></td>
<td>2 2 3 2 3</td>
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<tr>
<td>11&lt;sup&gt;th&lt;/sup&gt;</td>
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<tr>
<td></td>
<td>2 3 3 2 2</td>
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<tr>
<td>12&lt;sup&gt;th&lt;/sup&gt;</td>
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Figure 7.25: À l’île de Gorée, bars 127-137, The Accented Tetrachords in the Woodwinds’ Chord Sequence.
<table>
<thead>
<tr>
<th>Period</th>
<th>Trichords</th>
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<tbody>
<tr>
<td>1st</td>
<td>d d e c e d b</td>
</tr>
<tr>
<td>2nd</td>
<td>b b f e f f a</td>
</tr>
<tr>
<td>3rd</td>
<td>f f e d b c b</td>
</tr>
<tr>
<td>4th</td>
<td>e d a f d d e</td>
</tr>
<tr>
<td>5th</td>
<td>f f c a b f f</td>
</tr>
<tr>
<td>6th</td>
<td>d d f f f f a</td>
</tr>
<tr>
<td>7th</td>
<td>c c f d f d c</td>
</tr>
<tr>
<td>8th</td>
<td>d d b a a d f</td>
</tr>
<tr>
<td>9th</td>
<td>c c f d f d c</td>
</tr>
<tr>
<td>10th</td>
<td>a a c b a d c</td>
</tr>
<tr>
<td>11th</td>
<td>d d c d b a d</td>
</tr>
<tr>
<td>12th</td>
<td>f f f</td>
</tr>
</tbody>
</table>

Figure 7.26: À l’île de Gorée, bars 127-137, The Trichords in the Strings’ Chord Sequence.
Figure 7.27: À l’île de Gorée, bars 127-137, The Trichords in the Brass’ Chord Sequence.

Figure 7.28: Tetora, pitch content and sieve of bars 27-30.
a) Chord Sequences and Transpositions:

| Bars 21-24 (T₀) | B | F | X | E | B | X | F | C | G | X | C | E | F | C | X | B | E | X | C | G | C | F | G | X | E | G | X | C | G |
|                 | 3 | 1 | 1 | 3 | 1 | 1 | 2 | 3 | 1 | 2 | 1 | 1 | 3 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | |

| Bars 40-42 (T₀) | F | C | δ | F | C | δ | E | F | C | δ | C | A | δ | E | E | F | μ | F | δ | C | F | δ | E | F | E | F | C | δ | η |
|                 | 3 | 1 | 1 | 3 | 1 | 1 | 2 | 3 | 1 | 2 | 1 | 1 | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 2 |

| Bars 48-50 (T₀) | δ | ζ | ζ | δ | ζ | δ | ζ | δ | ζ | δ | ζ | ζ | δ | ζ | ζ | δ | ζ | ζ | δ | ζ | ζ | δ | ζ | ζ | ζ | ζ | δ | ζ | ζ | ζ | ζ | ζ | ζ |
|                 | 2 | 1 | 1 | 2 | 1 | 2 | 3 | 1 | 2 | 1 | 1 | 3 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

b) Chord Sequences and Substitutions:

<table>
<thead>
<tr>
<th>40-42 → 48-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>F → δ</td>
</tr>
<tr>
<td>C → ζ</td>
</tr>
<tr>
<td>E → η</td>
</tr>
<tr>
<td>A → γ</td>
</tr>
</tbody>
</table>

Figure 7.29: *Tetora*, bars 21-24, 40-42, and 48-50, Chord Sequences.

Figure 7.30: *Tetora*, bars 21-24, Low Heptachord (X).
Figure 7.31: Tetora, Chords of bars 21-85.
a) Chord Sequences and Transpositions:

| Bars 59-62 | e  γ  e  γ  δ  e  γ | Bars 62-65 | δ  ς  δ  ς  δ  ς  β  δ  ς |
| Bars 62-65 (T₁) | 1 2 1 2 2 2 | Bars 65-88 (T₁) | a  ε  ς  a  ε  γ  a  ε |
| Bars 65-88 (T₁) | δ  e  ς  δ  ς  γ  γ  a  e  a  ς  γ  a  γ  a  ε  δ |
| Bars 69-71 (T₁) | γ  β  δ  ς  a  γ  β |
| Bars 71-73 (T₁) | β  a  ε  β  a  ς  δ  β  a |
| Bars 73-74 (T₁) | γ  γ  ε  γ  ς  γ  γ  γ |
| Bars 74-76 (T₁) | a  ε  ς  a  ε  δ  β  a  a  ς |
| Bars 83-85 (T₄) | α  ς  α  δ  β  a  α  ς |

b) Chord Sequences and Substitutions:

<table>
<thead>
<tr>
<th>59-62</th>
<th>62-5</th>
<th>65-8</th>
<th>69-71</th>
<th>71-3</th>
<th>73-4 (83-5)</th>
<th>74-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>e  → δ  → a  → γ  → β  → a  → γ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ  → ς  → ε  → β  → a  → ς  → γ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δ  → β  → γ  → a  → ς  → β  → γ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β  → ε  → ς  → δ  → ε  → δ  → γ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1  → 2  → 1  → 1  → 1  → 1  → 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2  → 3  → 2  → 1  → 2  → 3  → 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3  → 1  → 1  → 1  → 1  → 2  → 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.32: Tetora, bars 59-68, 69-76, and 83-85, Chord Sequences.

1 Retrograde of bars 73-4.
2 With an exception at the correspondence of the last chord of bar 72 with the second from the end of bar 83 (1 semiquaver instead of 2 or 3).
Figure 7.33: Tetora, chords of bars 115-137.
With an exception at the correspondence of the last chord of bar 119 with the fourth of bar 122 (3 semiquavers)

With some exception after the interruption of the normal evolution of the sequences (See the last six columns of part [a] of the table).

With one exception: in chord α′ C6 has been substituted for D#6, where C4 is middle C.

With some exception after the interruption of the normal evolution of the sequences (See the last six columns of part [a] of the table).

With an exception at the correspondence of the last chord of bar 119 with the fourth of bar 122 (3 semiquavers instead of 2).

Figure 7.34: Tetora, bars 115-128, Chord Sequences.

---

| Bars 115-116 (Tₐ) | δ' | ε' | e' | δ' | ε' | e' | δ' | ε' | e' | δ' | ε' | e' | δ' | ε' | e' |
|-------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|                   | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| Bars 116-119 (T₁) | ε' | e' | δ' | ε' | e' | δ' | ε' | e' | δ' | ε' | e' | δ' | ε' | e' | δ' |
|                   | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| Bars 119-122 (T₂) | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' |
|                   | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
|                   | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  |
| Bars 122-125 (T₃) | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' | γ' | α' |
|                   | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  |
|                   | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  |
| Bars 125-128 (T₄) | β' | δ' | β' | δ' | β' | δ' | β' | δ' | β' | δ' | β' | δ' | β' | δ' | β' |
|                   | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
|                   | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  |
|                   | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  |

---

<table>
<thead>
<tr>
<th>115-116</th>
<th>116-119</th>
<th>119-122</th>
<th>122-125</th>
<th>125-128</th>
</tr>
</thead>
<tbody>
<tr>
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<td>ε' → δ'</td>
<td>δ' → γ'</td>
<td>γ' → α'</td>
<td>α' → β'</td>
</tr>
</tbody>
</table>

| 15 | 15 | 15 | 15 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

---

3 With one exception: in chord α′ C6 has been substituted for D#6, where C4 is middle C.

4 With some exception after the interruption of the normal evolution of the sequences (See the last six columns of part [a] of the table).

5 With an exception at the correspondence of the last chord of bar 119 with the fourth of bar 122 (3 semiquavers instead of 2).
Bars 128-131:  
$\zeta' \gamma' \alpha' \delta' \beta' \beta' \gamma' \gamma' \alpha' \gamma' \gamma' \epsilon' \beta' \zeta' \beta' \alpha' \epsilon' \gamma' \delta' \zeta'$

Bars 131-134:  
$\zeta' \alpha' \delta' \alpha' \gamma' \gamma' \zeta' \delta' \epsilon' \gamma' \gamma' \zeta' \beta' \beta' \delta' \zeta' \beta' \beta' \alpha' \zeta' \gamma' \alpha' \beta' \epsilon' \gamma' \alpha' \beta' \delta' \alpha' \beta' \delta'$

Bars 134-137:  
$\beta' \zeta' \alpha' \epsilon' \delta' \epsilon' \delta' \alpha' \epsilon' \epsilon' \gamma' \beta' \gamma' \alpha' \beta' \epsilon' \gamma' \beta' \alpha' \epsilon' \delta' \alpha' \zeta' \beta' \alpha' \epsilon'$

Bars 134-137:  
$\epsilon' \alpha' \beta' \zeta' \alpha' \delta' \alpha' \epsilon' \epsilon' \alpha' \epsilon' \beta' \gamma' \epsilon' \beta' \alpha' \gamma' \epsilon' \epsilon' \beta' \beta' \delta' \alpha' \zeta' \beta' \alpha' \epsilon'$

Figure 7.35: *Tetora*, bars 128-138, Chord Sequences.

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<td>$T_4$</td>
<td>$T_5$</td>
<td>$T_6$</td>
<td>$T_7$</td>
<td>$T_8$</td>
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<td>$T_3$</td>
<td>$T_4$</td>
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</tr>
</tbody>
</table>

Figure 7.36: Rotations of the Triangle in *Tetora*. 