Iannis Xenakis and Sieve Theory

An Analysis of the Late Music (1984-1993)

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DECLARATION

I declare that the work presented in this thesis is my own.

Dimitrios Exarchos

Abstract

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This thesis is divided in three parts, the first two of which are theoretical and the third analytical. Part I is an investigation of Iannis Xenakis's general theory of composition, the theory of *outside-time musical structures*. This theory appears in many of Xenakis's writings, sometimes quite idiosyncratically. The aim of this part is to reveal the function of the non-temporal in Xenakis's musical structures, by means of a historical approach through his writings. This exploration serves to unveil certain aspects discussed more thoroughly through a deconstructive approach. The deconstructive is demonstrated in the classification of musical structures and aims partly at showing the nature of Time in Xenakis's theory.

Part II is preoccupied with Xenakis's Sieve Theory. In the earlier writings on Sieve Theory he presented a slightly different approach than in the later, where he also provided an analytical algorithm that he developed gradually from the mid 1980s until 1990. The rationale of this algorithm and the pitch-sieves of 1980-1993 guides Part III, which is preoccupied with a methodology of sieve analysis, its application, and an exploration of the employment of sieves in some of Xenakis's compositions of the 1980s. When possible, the analysis takes in consideration the pre-compositional sketches, available at the Archives Xenakis, Bibliothèque Nationale de France. The sketches reveal aspects of the application of Sieve Theory, not included in Xenakis's theoretical writings.

As with the application of other theories, Xenakis progressed to less formalised processes. However, this does not mean that Sieve Theory ceased to inform the process of scale-construction. As the conclusion of this dissertation indicates, he employed Sieve Theory in order to achieve structures that conform to his general aesthetic principles, that relate to various degrees of symmetry and periodicity.

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Abbreviations

Xenakis, Iannis

A/S 1985a: *Arts/Sciences: Alloys. The Thesis Defense of Iannis Xenakis*, trans by Sharon Kanach (Stuyvesant, New York: Pendragon Press).

FM 1992: *Formalized Music: Thought and Mathematics in Composition*, revised edn, compiled and edited by Sharon Kanach (Stuyvesant, New York: Pendragon Press).

MA 1976: *Musique. Architecture*, 2nd edn, revised and augmented (Tournai: Casterman).

K 1994: *Kéleütha (Ecrits)*, ed. by Alain Galliari, preface by Benoît Gibson (Paris: L'Arche).

The system for labelling pitches used in this dissertation is the one proposed by the Acoustical Society of America: pitches are denoted with an upper case letter, followed by a number indicating the octave in which they appear. The lowest C of the standard piano keyboard is denoted as C1 and the highest as C8 (so that C4 = 'middle' C).

Introduction

Xenakis's first reference to his theory of *outside-time musical structures* is found in *Musiques formelles* of 1963. With this theory he embarked on a project to show that what most composers consider to be the most important element of music is actually subordinate. Time in music, he said, is not everything (see FM 192). Certainly, Xenakis's theory was partly aimed at demonstrating, not only the position of time in music, but that the classical view had placed too much reliance on temporality. This is evident precisely in the fact that what Xenakis explored most was not the nature of time, but what is independent of it. Time-independent structures can be constructed in such a way that ordering is not important. When it is not necessary for an element to be preceded or followed by any particular other element, the structure is said to be 'outside time'. Thus, a melody is an *inside-time* structure, in the sense that it cannot be constructed (or conceived) without time-ordering its pitches. Note that melody is shown, at this stage, to belong to time without yet referring durations. What does not belong to time is the scale or mode a melody is based on. This is because a scale is a collection or a set of elements, where order is not significant (cf. the distinction between *set* and *sequence* by Squibbs 1996: 45-56).

At the beginning of the chapter 'Symbolic Music' in *Musiques formelles* Xenakis refers to a 'sudden amnesia' (FM 155). This is not unrelated to his outside-time structures. He suggests that we look at the basic thought-processes when listening to music. From these thought-processes he derives the function of time and indicates that durations too have an outside-time aspect. They are independent of a time-ordering in the sense that they form a *set* of values. His view is that any set with abelian group structure is outside-time. In mathematics an abelian group, named after mathematician Niels Henrik Abel (1802-1829), is one with a commutative (as well as associative) group operation. 'Commutative' means that the product of the group elements is independent of the order of the elements during the calculation. Xenakis's approach to temporal structures is such that durations are thought of as multiples of a unit. This is, among others, what 'amnesia' refers to: when sonic events occur they divide time into sections that are perceived as multiples of a unit. These 'quantities' are compared to each other and can be thought of in an order different from the one they occurred. In terms of mathematical operations, commutativity is one of the basic properties of addition and multiplication: when we add or multiply certain values the order we perform the operation is not significant. Comparing two time-intervals is no different. We can think of them in any order and compare their size; i.e. interval A is twice as large or half the size of B etc.

Xenakis continued to develop his theory of outside-time structures throughout most of his writings. But the direction of this development was not entirely clear; moreover, it does not seem that he meant to present a complete account of it. It appears in relation to his other, more 'concrete' compositional theories or along with more general comments on his view of the avant-garde and musical tradition. As it is not a case of one theory among others, we could also refer to it as a *metatheory*. Perhaps the most enigmatic characteristic of Xenakis's demonstration of this metatheory is the fact that he alternated between a tripartite distinction: a) outside-time, b) temporal, c) inside-time, and a dualistic one that omits the middle term. One could say that Xenakis talked essentially about two types of structures (or categories), outside- and inside-time, and that he occasionally included a third type to clarify the case of temporal structures; however true such an observation might be, it does not adequately explore Xenakis's thought and its consequences in relation to his general view of composition. The first chapter of this dissertation is preoccupied with tracing the metatheory, but not in a teleological way; i.e. it does not aim at reconstructing a theoretical schema that corresponds to the two or three types of structure. Rather, it is intended to unveil certain lines of thought and explore the nature of time for Xenakis and its relation to his two opposed categories.

The initial reference to outside-time structures is contextualised by his 'symbolic music' for solo piano, *Herma* (1961), where he employs set-theoretic operations on pitch-sets. Later, Xenakis extended his idea of outside-time structure to include his general attempt to axiomatise musical structures. This would be the foundation of a General Harmony which, like combinatorics eleven years before, was a means of overcoming the impasses of serialism (see K 39-43). Among others, this is the axis on which Xenakis based his metatheory in 1965, in the manuscript 'Harmoniques (Structures hors-temps)' (published as 'Vers une métamusique' in 1967 and included in FM 180-200). Xenakis advanced an outside-time conception of composition and showed that serial techniques are solely preoccupied with inside-time manipulations. On the other hand, he also indicated that outside-time structures could not possibly be *removed* from any musical language. In other words, harmony could not possibly be removed from any melody. Harmony here also includes the scale on which a melody is based. The French philosopher Jacques Derrida has demonstrated the relationship between scale (or harmony) and melody, as analogous to that of writing and speech (Derrida 1997: 214). In both cases there is a dual opposition: harmony/melody and writing/speech. For Xenakis, the serialists placed too much emphasis on the latter, i.e. the series, which in terms of time-ordering is equivalent to melody. Thus, in both cases one term is privileged over the other: inside- over outside-time for the serialists and speech over writing for classical metaphysics. What Xenakis did with his sharp comments on the outside-time aspect of the total chromatic is to show the possibility of a *usurpation* similar to the one Derrida indicated in relation to writing. The serialists, Xenakis thought, subordinated the scale, but did not manage to disengage from it: for Xenakis it would be impossible to get rid of outside-time structure. Chapter 2 explores the consequences of Xenakis's outside-time structures and comprises a critical, deconstructive approach in relation to his critique of serialism and the notions of symmetry and periodicity (as the expressions of outside- and inside-time structure respectively).

These notions of symmetry and periodicity are the fundamental criteria Xenakis was concerned with in his development and application of Sieve Theory. The central aim of the theory is the construction of outside-time structures. This is Xenakis's answer to the amnesiac attitude to outside-time structures. The structures he produced with Sieve Theory are mainly and ultimately pitch-scales; in their general and abstract form, sieves are thought of as points on a straight line. The first work in which Sieve Theory was applied is *Akrata* (1964-65, for brass ensemble) (see Harley 2004: 40 & Schaub 2005: 11). This marks the beginning of the early period of sieve-based composition, that includes some works of the 1960s; sieves were then used (as pitch scales) more frequently from *Jonchaies* (1977, for orchestra) onwards. As for rhythmic sieves, these were more frequent in the earlier period than later.[1] The present study is preoccupied with the pitch-sieves of the later period. These structures share the same general characteristics to such an extent, that one can refer to a single type of scale that underwent metabolae (transformations) until the early 1990s. The last work that makes use of such a type of pitch-sieve is *Paille in the Wind* (1992, for violoncello and piano). *Mosaïques* (1993, for orchestra) is based, as the title suggests, on extracts from previous works and is therefore the final work of the late period that uses sieves.[2]

The theory has been researched to a significant extent by Flint (1989: 39-49), Solomos (1996: 86-96), Squibbs (1996: 57-67), Jones (2001), Gibson (2001; 2003: 39-117), and Ariza (2005), among others. Sieve-theoretical expressions offer the possibility of examining a scale, comparing it with others, or transforming its structure. The two basic 'components' of a such a theoretical representation are Modular Arithmetic and Set Theory. It is a case of working with set-theoretical operations, but on modular sets. If the sieve-theoretical expression is our starting point, we can work on the formal level and produce sieves according to a variety of methods. But if the starting point is the sieve, producing the sieve-theoretical formula is not very straightforward. The basic problem of Sieve Theory is precisely the redundancy of formulae. As Gibson has shown, 'since multiple representations of a sieve are possible, they cease to be equivalent when they

[1] I should note here the reference Xenakis made of the rhythmic sieves in *Komboï* (1981, for harpsichord and percussion) (see Varga 1996: 171).

[2] Xenakis's late-period works have not been analysed as extensively as the early ones. In terms of pitch organisation and as a general characteristic, the works after 1993 do not show evidence of sieves; they rather tend to chromaticism (see Solomos 1996: 101). However, this research did not take into account all the works between 1993 and 1997.

undergo transformations' (2003: 72).[3] By this, he means that when one works with sieves on the level of their logical representation (formula), one applies certain transformational procedures, whose result is dependent precisely on the choice of the formula. It is therefore a methodological problem about the relationship between the theoretical means and the compositional outcome. From the analytical point of view, this obviously prevents comparison of different sieves: when one formula is derived from a sieve, it must be comparable to formulae derived from other sieves; and given that there is more than one formula for a single sieve, comparison of different sieves presupposes a method for the determination of their formulae.

This restriction was certainly clear to Xenakis. Bearing in mind that a simple formula is much more desirable than a complex one, the theoretical representation of irregular sieves is more problematic in this respect. But a unique formula for a single, irregular arithmetic progression would be too much to expect. Nonetheless, Xenakis continued using Sieve Theory relatively constantly. In the early phase (1960s) he relied much more on the calculation of a formula that would be the starting point of transformational systems. In his later sieves though (1980s), sieve formulae were no longer of the same type nor did they serve transformations. This is evident in a comment by Hoffmann in relation to the sieve of *Horos* (1986, for orchestra): 'This scale does not seem to be readily reducible to a closed sieve formula' (2002: 125). This 'analytical perplexity' is characteristically caused by Xenakis's computer program for the analysis of sieves (see FM 277-88).

[3] 'Si plusieurs représentations d'un crible sont possibles, elles ne s'équivalent pas lorsqu'elle se soumettent à des transformations'.

This is partly the reason why previous research does not comprise a complete analysis of the sieves of Xenakis's later music. His writings on the theory, along with its implementation in the music of the 1960s, demonstrate the possibilities it offers and especially the possibility of generating and transforming scales. Its application to the music of the 1980s is different: Xenakis applied much simpler transformations, such as cyclic transposition (which is relevant to, but does not necessitate Sieve Theory), or simple alterations straight on the actual scale. The question arises at this point, whether this means that Sieve Theory is redundant.

Chapters 3 and 4 deal with these questions of the redundancy of formulae and of Xenakis's implementation of the theory in the 1980s. In order to do so, a distinction between types of formulae is introduced. This distinction is based on two criteria that result in four different types. Firstly, the 'period' of the sieve (e.g. the octave in the major diatonic scale) can either be taken into account or not; and secondly, the formula can either be maximally simple or not. As I will show, Xenakis progressed to a 'simplified' conception of sieves where the period is not taken into account. These theoretical and methodological conclusions are not based only on the writings on the theory. The inclusion of the computer programs for the generation of sieves and sieve-formulae sheds light on the discussion. Xenakis did use Sieve Theory along with his analytical algorithm for his scales. Although it was different from the 1960s, the application of the theory in the 1980s offered a method of creating the symmetries and periodicities that Xenakis required.

In many cases during my research on sieve-construction, it was clear that Xenakis created scales and derived the formula afterwards. As mentioned above, this formula was not intended to serve as means of sieve transformations; rather, it revealed information about the structure of the sieve in terms of 'hidden symmetries' (FM 269-70). Of course, the information a formula reveals depends on its type. At this level, the aesthetic criteria that intervene in scale-construction (for example, the well-known paradigm of the Javanese *pelog* – see Varga, 1996: 144-5), also determine the type of formula. The 'internal symmetries' that Xenakis mentioned (FM 276), are revealed by the formula his analytical algorithm suggests. Chapter 5 comprises an analytical method that implements this algorithm and I propose a reading of the resulting formula, however inconvenient and imprecise it might seem at a first glance.[4] Its aim is to reveal the hidden symmetries of an irregular sieve and deduce from this a certain degree of symmetry or asymmetry. In Chapter 6 I present an analysis of the most frequent sieves of the later period. I have found that Xenakis developed his analytical algorithm over a period of at least four years (up to its publication in 1990). Throughout this period (in fact throughout the 1980s) the aesthetic criteria of sieve construction and analysis remained the same. This facilitates comparison, as a difference in degree (of symmetry) can be meaningful only when comparing objects (sieves) of the same type. This is the reason that only sieves that retain

[4] Jones (2001) proposed a 'concise' formula (one with a small number of modules) which does not account for all the points of the sieve (an indication of the percentage of the points accounted for is provided along with the formula). Cases where two distinct sieves are expressed by the same formula are overcome by attaching the interval vector of the sets, combined with the 'product of primes' to ensure unique representation. Indeed, the result is a unique representation of the sieve, but it does not reflect the sieve's structure. Unfortunately, the font used by *Perspectives of New Music* for the code is not useful for the reader: upper case letter I and number 1 (one) are indistinguishable. I am thankful to Dr Evan Jones for providing me the code of his program.

certain general characteristics are included in this study. These characteristics have to do with the size of the intervals, the number of pitches, or the range of the sieve. For example, *Embellie* (for solo viola), although it was composed in 1981, uses a sieve based on quarter-tone intervals and exhibits a range different from the average range of the sieves in this period; in this sense, it belongs to the earlier period of Sieve Theory. For this reason, it is not analysed here. In general, occasional quarter-tone passages have not been taken into account here, following Xenakis's assertion that intervals are also to be taken in their acoustical aspect (see Harley 2002: 15-16).

The later period of sieve-based composition actually starts later than the first use of the characteristic type of sieves. This type of sieves is based on an irregular, non-repetitive succession of intervals between a semitone and a major 3rd. Although Xenakis used such a sieve in *Aïs* of 1980, Solomos (1996: 86-90) designates the period of the sieves between 1984 and 1993. The reason for doing so is related to the general style of composition that works of this period exhibit: 'extremely overloaded and made up of a succession of monolithic sections' (Solomos 2002: 14). Xenakis gradually abandoned glissandi and quarter-tones. Another characteristic is the idea of layers: 'sections of uniformly identifiable material tend to be shorter, to contain more interruptions or secondary layers of other material' (Harley 2001: 45). Furthermore, he proceeded to new, less formalised compositional techniques, which were also used for the inside-time employment of sieves. The final chapter of this thesis is devoted to the inside-time structures: in other words, to the analysis of some works of this later period in terms of how Xenakis used sieves in his music. This analysis is inevitably extended to other inside-time structures that have the form of 'points on a straight line', such as rhythmic structures that might not have been constructed with the help of Sieve Theory. In his inside-time treatment, Xenakis used other techniques that are not analysed here. If Sieve Theory produces outside-time structures, the techniques of group transformations (see FM 201-41; Vriend, 1981; Gibson, 2002: 48ff.; 2003: 152-4; and Schaub, 2005) or cellular automata (see Hoffmann 2002: 124-126; Gibson 2003: 166-8; Harley 2004: 176-180; Solomos, 2005b) are aimed at arranging these structures inside time.

For my research I visited the Archives Xenakis in the Bibliothèque Nationale de France, in Paris, on two occasions: April and November 2006. I managed to study Xenakis's pre-compositional sketches for the following works: *Jonchaies*, *Palimpsest*, *Aïs*, *Shaar*, *Idmen A* and *B*, *Horos*, *Akea*,[5] *Keqrops*, *Jalons*, *XAS*, *Ata*, *Echange*, *Epicycle*, *Kyania*, and *Tetora*. Works that are included in this dissertation and of which there are no sketches available in the Archives Xenakis, include: *Komboï*, *Thalleïn*, *À l'île de Gorée*, *Tracées*, and *Knephas*. I should note that I did not have the chance to look for sketches for each work I include in my research. Dr Ronald Squibbs kindly provided me a copy of the page of the sketches with the sieve of *Mists*. It should also be noted that in my research on these pre-compositional sketches, I focussed only on the sieves and relevant pre-compositional processes. During this research, valuable information has been found, that concerns compositional techniques that are not included here. Although this is an exhaustive study of the sieves used in works between 1980-1993,[6] it does not claim to be a complete account of each pre-compositional sketch mentioned.

[5] The sketches of *Akea* are – probably mistakenly – classified in the dossier with the sketches of *Ata*.

[6] Unfortunately, I could not locate and get hold of the score of *Oophaa* (1989, for harpsichord and percussion).

PART I

1 Outside-Time Structures

In this chapter I will attempt a historical approach through Xenakis's writings that refer to outside-time structures. This however, is not aimed at suggesting a certain teleological evolution in his theoretical thinking; it is rather an exploration of his concept of musical structures in relation to time that serves to unveil certain aspects discussed more thoroughly in Chapter 2, which forms a deconstructive, critical approach.

1.1 Literature

Xenakis started developing his theory of outside-time musical structures in the mid 1960s. The earliest reference is found in his first monograph, *Musiques formelles* of 1963. Its concluding chapter is titled 'Musique symbolique' and introduces Xenakis's application of Set Theory and an analysis of *Herma* (1961, for solo piano). A seed for this chapter is traced back to 1962 in a text titled 'Trois pôles de condensation' (which does not include the analysis of *Herma*). 'Musique symbolique' is followed by 'La voie de la recherche et de la question' in 1965 and 'Towards a Philosophy of Music' in 1966. An extensive demonstration of his theory, with examples of non-Western music cultures, is found in 'Vers une métamusique' of 1967, whose manuscript dates back in December of 1965 and is titled 'Harmoniques (Structures hors-temps)'; the latter was originally a symposium paper (see Solomos 2001: 236 & Turner 2005).

Excluding references in minor writings, Xenakis showed a renewed interest to the notion of time in music in the early 1980s. In 1981 he published an article called ‘Le temps en musique’, which was extensively enlarged and published as ‘Sur le temps’ in 1988. It then appeared with additional material as chapter X in the revised edition of *Formalized Music* in 1992, titled ‘Concerning Time, Space and Music’. The evolution of Xenakis’s thought through these writings can be divided in two periods: the first is the formation of his theory during the 1960s and the second reflects a more thorough investigation of the nature of time in music as found in his writings and interviews of the 1980s.[7] The elaboration of the theory appeared sporadically in several writings, such as articles, books, interviews. For this reason it was never presented in its entirety and there is no single writing that is wholly devoted to it. His theory was occasionally approached quite idiosyncratically and frequently under a different light; therefore a straight examination of the text wherein it is elaborated is necessary.

1.2 Overview

The theory of outside-time musical structures is not a theory among others. Xenakis’s approach to composition is characterised by the title of his first major publication, *Musiques formelles*, which reflects the title of the more recent publication, *Formalized Music*. His choice to use terms such as ‘formalisation’ or ‘axiomatisation’ is indicative of his approach to composing with tools borrowed from scientific areas and developed

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[7] In my tracing of the theory through Xenakis’s writings I will follow a chronological order according to the first publication of the articles, but when referencing, I will use the latest edition of each one, allowing for comparisons with earlier ones as appropriate.

according to his philosophy of music and/or practical compositional matters. On another level, all his theoretical tools (Stochastics, Sieve Theory, etc) fall into the scope of his general view on composition that is partly concerned with unveiling the nature of time in music. This is a theory that describes musical structures, including his specialised theories, music perception (from a psycho-physiological standpoint) and analysis, and shows a general underlying abstract thinking. Therefore, it is a theory in an indirect sense, a *meta*theory of composition.

The metatheory of outside-time structures is a matter of a general response to the question of the nature of time in music: 'what remains of music once one removes time'? (MA 211).[8] However, this question is only a starting point, and what remains seems to be one category among others. The theory, in its typical form, outlines three categories of musical structures: a) *outside-time*, b) *temporal*, and c) *inside-time*.[9] The first category is attempted to reply to the above question; while the inside-time structure is the actual composition, the outside-time category refers to structures that remain independent of time. As regards to the temporal category, Xenakis frequently made clear that this is a much simpler category and that time (in music) is a 'blank blackboard' where structures or architectures are inscribed into. In this chapter I will trace the evolution of this classification of musical structures through Xenakis's writings.

[8] 'Que reste-t-il de la musique une fois qu'on a enlevé le temps ?'

[9] Following the practice Squibbs (1996) and Flint (1989), I will use the term *inside-time*, instead of Xenakis's *in-time*, as a more obvious antonym to *outside-time*.

1.3 Symbolic Logic ('Symbolic Music' – 1963)

In the earliest of his writings on the matter Xenakis related the outside-time structure of music with his approach and application of Set Theory in *Herma*. The subtitle of the work is 'Symbolic Music for Piano' and it is founded on 'symbolic logic'. For Xenakis a sonic event is 'a kind of statement, inscription, or sonic symbol' (FM 156). These symbols stand for elements that undergo manipulation with the aid of logical functions or operations (using Boolean algebra). At this stage outside-time structures are thought of as logical structures or as logical operations that are independent of time. The first appearance of the schema of this classification is the following:

> [A] musical composition could be possibly viewed under the light of fundamental operations and relations, *independent* of time, which we will call *logical structure or algebra outside-time*.
>
> Afterwards, a musical composition examined from a temporal viewpoint, shows that sonic events create, on the axis of time, durations that form a set equipped with an abelian group structure. This set is structured with the aid of a *temporal algebra* independent of the outside-time algebra.
>
> Finally, a musical composition could be examined from the point of view of the correspondence between its *outside-time algebra* and its *temporal algebra*. Thus we have the third fundamental structure, the *inside-time algebraic structure* (MA 36-7).[10]

The above distinction is found in 'Trois pôles de condensation' of 1962 – the predecessor of 'Symbolic Music'. This is the only occasion where Xenakis phrases his theory using the term 'logical algebra'. What is important in this phrasing is that the logical functions

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[10] '[U]ne composition musicale peut être vue d'abord sous l'angle d'opérations et relations fondamentales, *indépendantes* du temps, que nous appellerons *structure logique ou algébrique hors-temps* :

Ensuite une composition musicale examinée du point de vue temporel montre que les événements sonores créent, sur l'axe du temps, des durées qui forment un ensemble muni de structure de groupe abélien. Cet ensemble est structuré à l'aide d'une *algèbre temporelle* indépendante de l'algèbre hors-temps.

Enfin, une composition musicale peut être examinée du point de vue de la correspondance entre son *algèbre hors-temps* et son *algèbre temporelle*. Nous obtenons la troisième structure fondamentale, la *structure algébrique en-temps*'.

are themselves shown to be outside-time; it is not merely (or not yet) an attempt to describe which types of musical entities are independent of time. Logic here is not a general 'reasoning' but refers to the abstraction Xenakis had always aimed at; abstract relations between elements render a structure that is definitely not about becomingness. Saying this of course, is itself an abstraction and what remains is to untangle the elements of an entity and illustrate which of its aspects might be independent of time.

'Symbolic Music' (1963) is part of Xenakis's first major publication concerning the matter and is more specific than before. However, the sketching out of the theory will remain similar as regards to the classification. The temporal algebra remains situated between the outside and the inside. Xenakis makes clear that this category (temporal) is much simpler than the outside-time one. It serves only as a means of rendering the music perceptible. More specifically, the temporal category is occupied by time as such; however, time itself is not viewed simplistically. This is the period just after the completion of *Herma* where he first employed logical functions, which later led him to a more extensive application of these operations and the development of his Sieve Theory. It could be said that, following Stochastics and Probability Theory, *Herma* and 'Symbolic Music' mark the beginning of a new period in the evolution of Xenakis's theoretical thinking. At the beginning of that stage Xenakis started to introduce considerations that undermine the classical view of the importance of time in music.

Whereas in his previous text he talked on a more abstract level, i.e. the possible ways of looking at a composition according to his classification of musical structures, in 'Symbolic Music' he is more concerned with the actual perception of time in music. He demonstrates his views by introducing Piaget's research on the child's perception of time.

> Let there be three events a, b, c emitted successively.
>
> *First stage*: Three events are distinguished, and that is all.
>
> *Second stage*: A "temporal succession" is distinguished, i.e., a correspondence between events and moments. There results from this
>
> $$a \text{ before } b \neq b \text{ before } a \qquad \text{(non-commutativity).}$$
>
> *Third stage*: Three sonic events are distinguished which divide time into two sections within the events. These two sections may be compared and then expressed in multiples of a unit. Time becomes metric and the sections constitute generic elements of set T. They thus enjoy commutativity.
>
> According to Piaget, the concept of time among children passes through these three phases.
>
> *Fourth stage*: Three events are distinguished; the time intervals are distinguished; and independence between the sonic events and the time intervals is recognized. An *algebra outside-time* is thus admitted for sonic events, and a secondary *temporal algebra* exists for temporal intervals; the two algebras are otherwise identical. (It is useless to repeat the arguments in order to show that the temporal intervals between the events constitute a set T, which is furnished with an Abelian additive group structure.) Finally, one-to-one correspondences are admitted between algebraic functions outside-time and temporal algebraic functions. They may constitute an algebra in-time (FM 160).

This structure which time is furnished with is given by durations[11] or time intervals that are marked by the sonic events (sections of time). Since durations may be compared with each other and expressed according to a unit, algebraic functions can be applied on these durations as well. Therefore, the set of durations is a commutative group, in which the order of appearance is not significant.[12] This fact renders temporal intervals themselves outside of time. With durations of course, Xenakis does not imply pure time-flow but

[11] It should be noted that the idea of duration is here used in its elementary sense of 'time-value'; and not in the sense philosopher Herni Bergson used it.

[12] Commutativity and associativity are two properties that belong both to addition and multiplication. In terms of temporal intervals, this means that intervals can be expressed as multiples of unit, no matter the order of their appearance.

metric time. The *discreteness* of metric time allows for the temporal intervals to be handled, analysed, or perceived as outside-time entities. Before relating the idea of discreteness and outside-time structures, the first category is shown to include the three more obvious properties of sound: pitch, intensity, and duration.

> [M]ost musical analysis and construction may be based on: 1. the study of an entity, the sonic event, which, according to our temporary assumption groups three characteristics, pitch, intensity, and duration, and which possesses a *structure outside-time*; 2. the study of another simpler entity, time, which possesses a *temporal structure*; and 3. the correspondence between the structure outside-time and the temporal structure; the *structure in-time* (FM 160-1).

On the one hand, metric time is shown to be subordinate to outside-time structures, but on the other hand, temporal intervals are privileged and assigned to the first category. What places the set of durations or the temporal structure outside of time is essentially the presence of commutativity. In this sense, the outside-time and the temporal are both *ordered structures*.[13] The sonic events on the one hand and durations on the other belong to two different categories that share an almost identical algebra; in the first case this algebra refers to the structure of the sonic events themselves, and in the second to the time-intervals that are designated by these events. Thus temporal intervals as such are part of a secondary structure, as they are issued from the sonic entities. In both cases,

[13] Xenakis defined ordered structures as follows:

> [‘Totally ordered structure’ means that] given three elements of one set, you are able to put one of them in between the other two. […] Whenever you can do this with all the elements of the set, then this set, you can say, is an ordered set. It has a totally ordered structure because you can arrange all the elements into a room full of the other elements. You can say that the set is higher in pitch, or later in time, or use some comparative adjective: bigger, larger, smaller (Zaplitny 1975: 97).

See also Perrot 1969: 62 & Xenakis 1996: 144.

structure is defined as the relations and operations *between* the elements (sonic events or temporal distances).

> Besides these logical relations and operations outside-time, we have seen that we may obtain classes (*T* classes) issuing from the sonic symbolization that defines the distances or intervals on the axis of time. The role of time is again defined in a new way. It serves primarily as a crucible, mold, or space in which are inscribed the classes whose relations one must *decipher*. Time is in some ways equivalent to the area of a sheet of paper or a blackboard. It is only in a secondary sense that it may be considered as carrying generic elements (temporal distances) and relations or operations between these elements (temporal algebra) (FM 173).

There are two remarks here that relate to the nature and position of time in Xenakis's theory. On the one hand, there is a temporal structure, which time is furnished with, and it is found in the set of temporal intervals as generic elements and the relations between these elements. On the other, time itself functions as a space of inscription or as a blackboard where sonic events are inscribed into as symbols that form part of the outside-time structure; this structure is found in the relations and operations between these sonic symbols. This second remark will be discussed in Chapter 2.

In this way, time is shown to be something more than just a set of elements with a commutative group structure. It must be clarified that what Xenakis subordinates at this stage is not time as such, but precisely this set of elements, or the temporal structure that time *possesses* (and of course this structure is not everything about time as such). Therefore, it is the temporal structure that is in proximity with the outside-time one, in the sense that the two share a common algebra. Time as such remains a medium that renders structures perceptible.

1.4 Two Natures ('La voie de la recherche et de la question' – 1965)

The positioning of the temporal as a medium between outside and inside time structures serves to distinguish the two opposed poles in Xenakis's formation. In the previous two stages of his theory he described a) the logical operations and b) the sonic events and their characteristics as being outside time. In 1965 he proceeds to a more simplified distinction: the mediating temporal category is now absent, and Xenakis attempts a clarification that is more than an assumption. The key term in this clarification is ordering, or arrangement in time.

> We have to distinguish between two natures: inside-time and outside-time. That which can be thought of without changing from the before and the after is outside-time. Traditional modes are partially outside-time, the logical relations and operations applied on classes of sounds, intervals, characters… are also outside-time. Those whose discourse contains the before or the after, are inside-time. The serial order is inside-time, a traditional melody too. All music, in its outside-time nature, can be rendered instantaneously, flat. Its inside-time nature is the relation of its outside-time nature with time. As sonorous reality, there is no pure outside-time music: there is pure inside-time music, it is rhythm in its pure form (K 68).[14]

With the 'before' and the 'after' Xenakis obviously refers to the possibility of permutations of the (commutative) elements of an outside-time structure. Although he does not mention it clearly, this dual opposition can be exemplified in the relation between a scale and a melody based on it. This is because the arrangement of the degrees of a scale is not temporal but hierarchical. Hierarchy here must be thought in a different sense than the tonal hierarchy of the scale degrees. Hierarchy is rather related to the

[14] 'Il faut distinguer deux natures : en-temps et hors-temps. Ce qui se laisse penser sans changer par l'avant ou l'après est hors-temps. Les modes traditionnelles sont partiellement hors-temps, les relations ou les opérations logiques infligées à des classes de sons, d'intervalles, de caractères… sont aussi hors-temps. Dès que les discours contient l'avant ou l'après, on est en-temps. L'ordre sériel est en-temps, une mélodie traditionnelle aussi. Toute musique, dans sa nature hors-temps, peut être livrée instantanément, plaquée. Sa nature en-temps est la relation de sa nature hors-temps avec le temps. En tant que réalité sonore, il n'y pas de musique hors-temps pure ; il existe de la musique en-temps pure, c'est le rythme à l'état pur'.

axiomatics of the set of natural numbers. Therefore, on the outside-time side we have the notes of a scale or a mode that appear from the lower to the higher. (Intervals are outside-time in the sense that they can be compared in terms of their size.) On the other side, we have melody or the series, as an ordering of these elements. In the same way that a melody is based on a scale or mode, the series is based on the total chromatic and is a reordering of its elements. This is the first time that Xenakis refers to the inside-time nature of the series and it is a new starting point of bringing back his critique of serialism – this time under the light of his general compositional theory and not Stochastics. This, however, will lead to a much more complicated discussion and it will be developed in his following writings.

Although the emphasis is now given on clarifying the dual opposition of inside and outside, time is still included in Xenakis's discourse, but with a somewhat different function. Here time is clearly a catalyst, necessary for bringing music into life – in terms of perception, that is. The inside-time is the *relation* of the outside-time with time. This reveals another aspect of the position of time in the classification. There is no temporal category here and Xenakis does not mention the set of time intervals as furnished with commutativity – namely, the temporal structure. (It goes without saying that the commutativity of time intervals implies that the before and after do not change this structure.) As he mentioned previously, the temporal and the outside-time algebras are identical; therefore, in this dualistic distinction the temporal structure would also be outside time.

We see that although the second category of the theory collapses to the first, the notion of time is still included in the classification, and this is in relation to the third category. Recall that in the preceding demonstration the third category derives from the correspondence of the outside-time with the *temporal* structure, whereas now from the relation of the outside-time nature with *time*. This reveals that for Xenakis there seem to be two different lines of thought when he places time in relation to the other two categories; and this is shown by the fact that the ‘middle’ category is related to the other two in two different ways. On the one hand, time is (in a secondary sense) included in the outside-time category as their corresponding algebras are identical; on the other, it is shown to be ‘rhythm in its pure form’.

Pure inside-time music can be conceived only in the total absence of outside-time structure. Of course, for Xenakis there is no music that totally lacks outside-time structures; in the case of a serial composition for example, the outside-time structure that it is based on is the total chromatic, which indeed is outside-time, albeit too neutral. The movement of thought in the two articles can be seen in the gestures that Xenakis makes in relation to the middle category. From an entity that is *simpler* than the sonic event itself, to *pure* inside-time music; or from a view that has time as *metric* to another that has time as *rhythm* in a much more general sense than metre. This movement does not imply that he abandoned the older view in favour of the new one. It is a movement between two lines of thought that are not mutually exclusive (although at the same time still independent of each other). However, his metatheory is not aimed at understanding the nature of time as such, nor its function in music; Xenakis had from the outset been concerned with what remains after time has been *removed*. Distinguishing between two different aspects of the role of time in his schema, serves at demonstrating the natures of the two extreme poles. As a ‘temporary assumption’ then, time participates in both the outside and the inside time categories. By assigning the temporal in the middle category Xenakis made clear that, contrary to the classical view, time includes an outside-time aspect; and by identifying pure inside-time music with pure rhythm, he indicated time as being disentangled from outside-time structures.

1.5 Tomographies Over Time ('Towards a Metamusic' – 1965)

Immediately following the publication of 'La Voie' Xenakis wrote the manuscript for 'Towards a Metamusic', which was however published two years afterwards (1967). His discourse brings back the notions of categories (instead of natures) and the classification includes again the temporal category. At this point Xenakis refers to the idea of the scale, which is considered central in his theory (as well as in Sieve Theory).

> I propose to make a distinction in musical architectures or categories between *outside-time*, *in-time*, and *temporal*. A given pitch scale, for example, is an outside-time architecture, for no horizontal or vertical combination of its elements can alter it. The event in itself, that is, its actual occurrence, belongs to the temporal category. Finally, a melody or a chord on a given scale is produced by relating the outside-time category to the temporal category. Both are realizations in-time of outside-time constructions (FM 183).

In this article there is an extended demonstration of ancient Greek and Byzantine scales, which serve Xenakis to enforce his arguments and the presentation of his ideas.

The discussions from both two previous articles are in a way brought in here too, although not with a straightforward terminology. As I have already suggested, the two approaches regarding the temporal category are not mutually exclusive. We can here remark that the temporal category is, unlike before, not shown to be equipped with a secondary ordered structure. The temporal is mentioned only in relation to the instant and the realisation of the sonic event. Although 'Metamusic' brings back the tripartite classification of 'Symbolic Music', the temporal category seems to be approached from the same viewpoint as in 'La voie'. In other words, this formulation is not concerned so much with what kind of structure time possesses, but with what belongs to the temporal category in a less abstract way of thinking.

This change of viewpoint is also apparent in the way that the sonic event itself is treated. Whereas in 1963 the sonic event is shown to possess an outside-time structure, it is now shown to belong to the temporal category, as far as its actual occurrence is concerned. Therefore, the sonic event is not 'a kind of symbol' as previously stated but is here related to immediate reality. This is a matter of a less abstract approach to both the sonic event and the temporal category. This less abstract approach is formulated more successfully later, in 1976, in *Arts/Sciences: Alloys* and the discussion of the reversibility of time (Section 1.8). When a composition is viewed under the angle of outside-time relations and operations, then both the sonic event and time are shown to possess an outside-time or ordering structure; when it is viewed from the temporal angle, the event itself belongs to the temporal category (as an instantaneous reality) and time is shown to be pure instead of metric. In the following part of 'Metamusic' Xenakis provides the point of view that he had been concerned with from the outset: the sonic event (or architecture) is outside-time and time as such belongs to the temporal category, where the latter is considered to be pure.

> In order to understand the universal past and present, as well as prepare the future, it is necessary to distinguish structures, architectures, and sound organisms from their temporal manifestations. It is therefore necessary to take "snapshots", to make a series of veritable *tomographies* over time, to compare them and bring to

> light their relations and architectures, and vice versa. In addition, thanks to the metrical nature of time, one can furnish it too with and outside-time structure, leaving its true, unadorned nature, that of immediate reality, of instantaneous becoming, in the final analysis, to the temporal category alone (FM 192; italics added).

Although Xenakis presents his classification from different viewpoints at different times, it remains clear that he insists on the importance of the outside-time structure of music (or algebra, architecture, nature). Inside-time structures always remain as the second term of the dualistic approach that he occasionally tends to suggest. More importantly, these two terms offer Xenakis the possibility to approach the temporal category, or time, from two different points of view. The approach he might take each time, affects also the way that the sonic event is interpreted. There are therefore two ways of thinking, which are based upon two opposed tendencies (outside/inside) that rarely seem to be stabilised in a formulation, although always involved in it. This dichotomy is found again in the subsequent article, which presents a formulation similar to that of 'La voie'.

1.6 'Towards a Philosophy of Music' (1966)

In the previous formulation of the dichotomy, time, or the temporal category, had not been excluded. It was considered only in relation to its pure nature, that of instantaneous becoming; this observation was also maintained in the tripartite classification of 'Metamusic'. In 'Towards a Philosophy of Music' the formulation is of a dualistic nature, but more refined as regards to the middle term. Time is referred to as both possessing an ordered structure and related to instantaneous creation. Unlike 'Metamusic' the reference to both natures of time is concisely demonstrated in the classification:

> It is necessary to divide musical construction into two parts: 1. that which pertains to time, a mapping of entities or structures onto the ordered structure of time; and 2. that which is independent of temporal becomingness. There are, therefore, two categories: *in-time* and *outside-time*. Included in the category outside-time are the durations and constructions (relations and operations) that refer to elements (points, distances, functions) that belong to and that can be expressed on the time axis. The temporal is then reserved to the instantaneous creation (FM 207).

It is clear that time possesses an ordered structure, which is outside of time. More specifically what belongs to that category is the durations, or time-intervals, as a set of generic elements that enjoys commutativity. In other words, metric time. The 'temporal' has now the place that time had in 'La voie', that is pure time of immediate reality.

1.7 Ontological/Dialectical ('Une note' – 1968)

In 1968 Xenakis demonstrated a slightly differentiated classification in 'A note' in *La Revue Musicale* (published in the following year). Unlike all his previous references, where he alternated between a dichotomy and a tripartite classification with alternating viewpoints, here he talks about two categories but with a 'triple correction'. The two categories have the form of the dichotomy outside-time/temporal and they represent the ontological/dialectical dichotomy that the philosophies of Parmenides and Heraclitus represent for Xenakis. These two categories intermingle and their mapping is the 'realisation' or what he termed formerly the inside-time (although this term in not used here):

> There is a mental crystallisation around two categories: ontological, dialectical; Parmenides, Heraclitus. From there comes my typification of music, outside-time and temporal that lights so intensely. But with a triple correction:
>
> a) in the outside-time, time is included,

b) the temporal is reduced to the ordering,
c) the ‘realisation’, the ‘execution’, that is the actualisation, is a play that makes a) and b) pass into the instantaneous, the present which, being evanescent, does not exist.

> Being conscious, we have to destroy these liminal structures of time, space, logic… So with a new mentality, with a past, future and present interpenetrating, temporal but also spatial and logical ubiquity. That’s how the immortality is. The omnipresent too… without flares, without medicine. With the mutation of the categorising structures, thanks to the arts and sciences, in particular to music, obliged as she has been recently to dive into these liminal regions (Xenakis 1969: 51).[15]

The outside-time is privileged over the temporal, which is in turn reduced to ordering and finally the instantaneous refers to the present which does not exist. This is an obvious remark about metric time and what Xenakis considers to be included in the outside-time is precisely this metric time as a set of elements that has an ordered structure. Pure rhythm or pure time have no place in this formulation and certainly the instantaneous or the present is not shown to be related directly to this purity. He makes a gesture of overturning an old way of thinking and suggests a new one where tenses ‘interpenetrate’; this can be thought only when time intervals are taken abstractly, as multiples of a unit that are commutative. They then form entities outside of time (immortal, omnipresent). Time, as it is included in the outside-time, is then shown only in

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[15] ‘Il y a cristallisation mentale autour de deux catégories : ontologique, dialectique ; Parménide, Héraclite. D’où ma typification de la musique, hors-temps et temporelle qui s’éclaire ainsi intensément. Mais avec une correction triple :

a) dans le hors-temps est inclus le temps,
b) la temporelle est réduite à l’ordonnance,
c) la « réalisation », l’ « exécution », c’est-à-dire l’actualisation, est un jeu qui fait passer a) et b) dans l’instantané, le présent, qui étant évanescent, n’existe pas.

Il faut, étant conscients, détruire ces structure liminaires du temps, de l’espace, de la logique… Mental donc neuf, passé futur présent s’interpénétrant, ubiquités temporelle mais aussi spatiale et logique. Alors l’immortalité est. Le partout présent, aussi… sans fusées, sans médecine. Par la mutation des structures catégorisantes, grâce aux sciences et aux arts, en particulier à la musique, obligée qu’elle a été de se plonger dans ces régions liminaires récemment’.

its metric sense and not in its pure, which seems to be for Xenakis something more than the evanescent present. No matter how Xenakis's change of viewpoints influences his demonstration of the temporal category or of time, it remains as a constant in his theory that the outside-time, the ontological in this case, is the privileged term in a discourse of polarity. In his subsequent publications he is concerned with classifying less than in his former ones; he is interested primarily in the way memory functions in music perception and the consequences these observations might have in composition.

1.8 *Arts/Sciences: Alloys* (1976)

In 1976 Xenakis was awarded a 'Doctorat d'État' and his thesis defence was published in 1979. All his writings mentioned so far (at least the ones included in his books published by then) were submitted for the award and taken into account in the discussion between Xenakis and the jury. It is therefore a temporary concluding point, before his final article on Time. In his thesis defence his theory is mentioned under a discussion on the possibility of the reversibility of time in his music. The reference to outside-time structures, then, is made only in order for Xenakis to clarify that his view does not necessarily imply a reversible time. This clarification is, importantly enough, a way for him to distinguish between the two natures of time, which also reflect the overall polarity of outside/inside. Reversibility is for him simply one among the several outside-time permutations that temporal intervals can undergo. It is clearly a matter of distinguishing between metric time and pure temporal flow.

> [W]hen I talk about time intervals, they are commutative. This is to say that I can take time intervals now or later and commutate them with other time intervals.

> But the individual instants which make up these time intervals are not reversible, they are absolute, meaning that they belong to time, which means that there is something which escapes us entirely since time runs on (A/S 69).

The idea of reversibility is for Xenakis related to the non-temporal; what is temporal is by definition irreversible. In that case, what escapes us is related to real time as opposed to metric. The two lines of thought are clarified further on:

> There are some orders which can be outside of time. Now, if I apply this idea to time, I can still obtain these orders, but not in real time, meaning in the temporal flow, because this flow is never reversible. I can obtain them in a fictitious time which is based on memory (A/S 71).

Memory serves here as a means of thinking about time abstractly and enables man to construct a metric structure in order to perceive time and the composer in order to work with durations and time intervals:[16] 'There is the temporal flow, which is an immediate given, and then there is metrics, which is a construction man makes upon time' (A/S 97). The time instants and the effect they have on memory is, for Xenakis, an important remark, as it is a starting point for his elaboration of the outside-time aspect of time. This will substitute the paradigm borrowed from Piaget. I will explore this in the following chapter, in relation to the idea of the *trace*.

[16] Note that time-intervals and durations are not always identical. The former are the temporal distances marked by sonic events (i.e. the distance between two time-points) and the latter refer to the duration of these sonic events (i.e. the duration of a pitch).

2 Outside-Time Structures as Writing

As early as the first statement of his theory Xenakis referred to the temporal as related to the category of outside-time structures. I will use the relationship between time and outside-time structures in order to unveil the character of the latter as writing. For Xenakis time is, as I have already pointed out, a ‘space of inscription’. There are several references that have time as a white ‘sheet of paper’ or as a ‘blank blackboard’. This metaphor should be studied more thoroughly, as it implies a gesture that overturns the classical idea that time in music is everything. What is more interesting is that the temporal has been shown from two opposed angles that place it in both poles of the dichotomy of outside/inside. It can be shown that these two aspects of time also function in a way that disturbs this dichotomy, which is not different from the one of writing/speech. For this reason, Jacques Derrida’s exploration of writing is useful here; primarily because Derrida equates the relationship of the scale (for Xenakis the primary outside-time paradigm) and the origins of music with the one of writing and speech: ‘The chromatic, the *scale* [gamme], is to the origin of art what writing is to speech. (And one will reflect on the fact that *gamma* is also the name of Greek letter introduced to the system of literal musical notation)’ (Derrida 1997: 214).

2.1 The Third Term

It is clear that Xenakis’s formulation as a binary opposition ‘with a triple correction’, involves a ‘third’ term in the way that Derrida has shown (see Derrida 2001: 5). This third term participates in both sides of the polarity. (Xenakis had always made certain to stress that time participates in the outside-time, as something not generally taken for granted; but he also did that in order to demonstrate that the first category bears much more significance than the second.) Participating in both categories, the temporal is a mediator between the two. This is a consequence of the heterogeneous nature time has for Xenakis: metric time and temporal flux, a manmade construction and an immediate given. Heterogeneity does not allow time to be a stable part of the schema, and this is why it is occasionally excluded from Xenakis's writings, or phrased differently, or viewed from different angles. Time is the element that resists systematisation and therefore, more than just being a mediator, it escapes integration into the system.

The temporal belongs neither to the outside- nor to the inside-time; but on one hand it possesses a structure that belongs to the former and on the other hand its irreversibility places it with the latter. Derrida talks of the 'third' in a way that brings light to this discussion: 'It is at the same time, the place where the system constitutes itself, and where this constitution is threatened by the heterogeneous' (2001: 5). The temporal, as the middle or the third term, obscures the limits of what is outside and what inside. What is obvious from Xenakis's writings, is that ordered structures (including temporal ones) are outside time while time as such remains pure; inside time then are the outside-time structures when affected by the catalytic action of time. But more than a catalyst, time is the function that renders the outside-time perceptible, in other words inside-time. I will show how this disruption takes place after I demonstrate the way Xenakis developed his critique of serialism.

2.2 The Critique of Serialism

The idea of the scale is central in any discussion on the matter, and it is always the primary example of Xenakis's demonstration of the theory. A scale is a well-ordered set, an object outside time. Having this observation as a starting point, we can re-formulate Xenakis's criterion for his evaluation of serialism's compositional practice. Xenakis points out a progressive degradation of outside-time structures: 'This degradation of the outside-time structures of music since late medieval times is perhaps the most characteristic fact about the evolution of Western European music' (FM 193). Xenakis's first theoretical endeavour was his famous manifesto against serialism, 'La crise de la musique sérielle' of 1955. There, he identifies a crisis and a degradation of polyphonic linear thought as situated at the basis of this compositional technique (see K 39).

2.2.1 General Harmony

This critique is also included in his theory of outside-time structures. His critical stance remains, ten years after 'La crise', in 'La voie'. The starting point of his argument is precisely the placing of the tempered chromatic scale outside of time. The outside-time character of the chromatic is a privilege that the serialists (among others) failed to observe:

> [The tempered chromatic scale] is for music what the invention of natural numbers is for mathematics and it permits the most fertile generalisation and abstraction. Without being conscious of its universal theoretical value, Bach with his *Well-Tempered Clavier* was already showing the *neutrality* of this scale, since it served as a support for modulations of tonal and polyphonic constructions. But only after two centuries, through a deviating course, music in its totality and its flesh breaks decisively from tonal functions. It then confronts the void of the neutrality of the tempered chromatic scale and, with Schönberg for example,

> regresses and falls back to more archaic positions. It does not yet acquire the scientific awareness of the *totally ordered structure* that this privileged scale comprises. Today, we can affirm with the twenty-five centuries of musical evolution, that we arrive at a universal formulation concerning the perception of pitch, which is the following:
>
> *The totality of melodic intervals is equipped with a group structure with addition as the law of composition* (K 69).[17]

The tempered chromatic is then a landmark in the history of music that went unnoticed. Of course this does not mean that outside-time structures did not exist before or that they were necessarily poorer. On the contrary, the chromatic is a neutral structure, much poorer itself than, say, the diatonic scale or Byzantine and ancient Greek modes, which have a differentiated and much more sophisticated structure. By corresponding the chromatic with the set of natural numbers Xenakis did not merely show that a new structure as such was discovered; what actually happened, for him, is an opening up of possibilities for constructing new structures, e.g. scales, with mathematical tools, such as Set Theory. Under the scope of such possibilities Xenakis conceived (at around the same time) his Sieve Theory, which was eventually developed exclusively towards the construction of pitch scales. He acknowledges of course that it was in France that the outside-time category was reintroduced, both in the domain of pitch and of rhythm; this was done by Debussy with the invention of the whole-tone scale and Messiaen with his

[17] '[La gamme chromatique tempérée] correspond en musique à l'invention des nombres naturels des mathématiques et c'est elle qui permet la généralisation et l'abstraction les plus fécondes. Sans être conscient de sa valeur théorique universelle, J.-S. Bach avec son *Clavier bien tempéré* montrait déjà la *neutralité* de cette gamme, puisqu'elle servait de support aux modulations des constructions tonales et polyphoniques. Mais ce n'est que deux siècles plus tard, par un cheminement dévié, que la musique dans l'ensemble et dans sa chair rompt, définitivement, avec les fonctions tonales. Elle se trouve alors devant le vide de la neutralité de la gamme chromatique tempérée et, en la personne de A. Schönberg par exemple, elle recule et se replie sur des positions plus archaïques. Elle ne prends pas encore une conscience épistémologique de la *structure d'ordre total* que cette gamme privilégiée renferme. Aujourd'hui, on peut affirmer qu'avec les vingt-cinq siècles d'évolution musicale, on aboutit à une formulation universelle en ce qui concerne la perception des hauteurs, qui est la suivante :

L'ensemble des intervalles mélodiques est muni d'une structure de groupe avec comme loi de composition l'addition'.

modes of limited transpositions and non-retrogradable rhythms (which I will discuss later). However, Messiaen did not advance this thought into a general necessity and abandoned it, 'yielding to the pressure of serial music' (FM 208).

In 'Towards a Metamusic' Xenakis reminds us of the suggestion he made in 1955: the introduction of probabilities and a 'massive' conception of sound that would include serialism's linear thought merely as a particular case. He then goes on to pose the question whether this suggestion itself implied a general harmony, only in order to reply: 'no, not yet' (FM 182). The introduction of probabilities does include serial manipulation as a mere case, but it does not constitute a general approach to composition – it does not stand as a general harmony. More specifically, this general harmony is provided by his outside-time musical structures. This harmony is seen not in a traditional – limiting for Xenakis – sense of the homophonic or contrapuntal language. A truly general harmony must be able to include, potentially, all types of musical structures of the past and present, all styles and personal languages. Precisely this idea of a personal language is shown by Xenakis to rest in the outside-time category (FM 192).

2.2.2 Magma

In serial music, he says, there is an exaggerated emphasis on temporal structures, as it is based on a temporal succession, or a time-ordering, of all pitches of the chromatic scale (succession of elements inside time). In other words, it is impossible to discern between structures (architectures, sound-organisms, etc) and their temporal manifestations. Serial music remains for Xenakis 'a somewhat confused magma of temporal and outside-time structures, for no one has yet thought of unravelling them' (FM 193). What needs to be unravelled then is essentially the outside-time structure from its temporal manifestation (instantaneous becomingness). In the case of the pitch organisation of a serial composition, these two elements are the chromatic scale, which is placed outside time, and the series, which is inside time. What Xenakis means when he says ‘temporal’ here is not metric time. It should be remembered that metric time refers to the ordered set of durations, of temporal intervals (in a temporal structure), which is a commutative group and which is outside time. What he talks about here is the element of pitch, without taking into account any durations pitches might be associated with. Therefore, ‘temporal’ stands here for the inside-time ordering of the twelve pitch-classes; thus the ‘magma’ Xenakis refers to consists of the outside- and the inside-time categories.

As I have mentioned earlier, for Xenakis, apart from ‘pure rhythm’, there is no pure inside-time music. Outside-time structures do exist (for example, the total chromatic in the case of a serial composition) and are just *perceived* in time: ‘Polyphony has driven this category [of outside-time structures] back into the subconscious of musicians of the European occident, but has not completely removed it; that would have been impossible’ (FM 208). The magma that serial music is then, should be a natural and an expected one. What he actually points at, is the neglecting of the outside-time that is responsible for the degradation of music. It is a matter of a *confused* magma where the two categories are in a disproportioned, unbalanced relationship; Xenakis’s suggestion then should be seen not as disentangling the two categories, but that the outside-time category should be given more attention, as it is always already there. It is therefore not a matter of reintroducing it, but taking into account its existence, noticing the possibilities it offers in composition and the effects it has in musical perception. Its consequences, it seems for Xenakis, are at work no matter whether we acknowledge it or not.

2.3 The Temporal as Outside-Time

The temporal element in the series is then the ordering of the pitch-classes, as an inside-time structure. This clarification is important to be made in order to understand how the polarity of outside/inside functions for Xenakis in serial music. For this purpose, I will compare the idea of the scale (outside-time) and that of the rhythmic sequence (temporal). In his final article on Time ('Concerning Time, Space and Music' – 1981) Xenakis focuses on the temporal, or the middle category, and its relation to the outside-time. He demonstrates the outside-time aspect of time, leaving the temporal flux (which would place structures inside time) as the other element where music participates.

> 1. We perceive temporal events.
> 2. Thanks to separability, these events can be assimilated to *landmark points* in the flux of time, points which are instantaneously hauled up outside of time because of their *trace* in our memory.
> 3. The comparison of the *landmark points* allows us to assign to them distances, intervals, durations. A distance, translated spatially, can be considered as the displacement, the step, the jump from one point to another, a nontemporal jump, a spatial distance.
> 4. It is possible to repeat, to link together these steps in a chain.
> 5. There are two possible orientations in iteration, one by accumulation of steps, the other by a de-accumulation (FM 264-5; the author italicises only 'landmark points').

This formulation concerns temporal structures when placed outside of time. His final publication then is concerned only with the middle category in its outside-time aspect. The other aspect, that of temporal flow is left unmentioned here.

Messiaen's non-retrogradable rhythms are shown by Xenakis to belong to the outside-time category. More precisely, it is a case of a temporal structure that is placed outside time. There are two elements involved in such structures: the time-instants and the temporal intervals between them (no matter whether durations are associated with the whole or a part of any such temporal interval). If we correspond the time-instants and the temporal intervals with the pitches and pitch-intervals of a scale, it can be shown that, as Xenakis often said, the temporal structure (the rhythmic sequence) is simpler that the outside-time structure (the scale). In the case of the scale there are two possible ways of hierarchical arrangement: the very idea of the scale suggests that pitches are placed from the lower to the higher; but the intervals themselves (such as the semitone and the tone in the diatonic) might also be compared in terms of their relative sizes, perceived and expressed as multiples of a unit, and in a way arranged from the smaller to the larger (or commutated). Neither of these two ways of arranging include the 'before' and the 'after'. In the case of a rhythmic sequence though, there is only one way of doing so: as I have quoted Xenakis saying (see section 1.8), while you can compare the sizes of temporal intervals, commutate them, or arrange them from the smaller to the larger, time instants are not commutative, not reversible; they belong to time, to pure temporal flow. This is due to the heterogeneity of time: a rhythmic sequence has a part that is outside-time (temporal intervals) and another that is inside-time (time-instants). Therefore, a reordering of the pitches of a scale (and not of the intervals involved in it) is inside time; in a rhythmic sequence this would be inconceivable, as time instants are fixed to the flow of time.

2.4 Outside-Time as Supplement

The metaphor for writing Xenakis frequently used is not aimed at suggesting that music functions as language does. In the *Conversations* with Varga he clearly says it: 'music is not a language: it does not have the task of expressing something through sounds and symbols. Music stands by itself, there's nothing beyond it' (Varga 1996: 82). Nonetheless, the idea of symbols might suggest a similarity with language, that in conjunction with the idea of writing can lead to an analogy between the dichotomy outside/inside and writing/speech; furthermore, it can be explored in relation to the deconstructive as unveiled by Derrida and his reading of the passages relevant to music in Rousseau's *The Essay on the Origin of Languages*. In *Of Grammatology*, published in the same year as 'Towards a Metamusic', Derrida argues for an analogy of the histories of language and music (that is, the histories of the two as read in Rousseau). There he focuses on the degradation of music that Rousseau considered to have taken place. According to this idea, there is an originary separation between speech and music. For Rousseau it is obvious that song is the origin of music and that itself derives from speech. For him, music and song grew apart; it is a case of a degradation caused by the forgetting of the origin of music.

Although I am not attempting an interpretation of Rousseau's views on the matter, there is an interesting analogy between his treatment of the opposition melody/harmony and Xenakis's view of harmony as being outside of time and melody inside. The two obviously privilege the opposite side of a dichotomy which seems to be the same for both: melody versus harmony. Harmony, independent of any other qualities such as time and rhythm, stands on its own; also, it is for both a 'rational science'. But for Rousseau harmony is the cause for music's degradation, which should have always been united with speech, with the inflections and accent of the spoken language. The comparison here is useful only for two purposes: first, to show the intention on Xenakis's part to demolish the classical view that has time as the essence of music, and secondly in order to see how the idea of supplementarity, shown in relation to Rousseau's view, affects Xenakis's.

Derrida has shown that, for Rousseau, music grew as a supplement to the unity of speech and song; supplement alludes here to the idea of writing in relation to speech. 'The growth of music, the desolating separation of song and speech, has the form of writing as "dangerous supplement": calculation and grammaticality, *loss of energy and substitution*. The history of music is parallel to the history of the language, its evil is in essence graphic' (Derrida 1997: 214). The 'graphic' element of music is described as grammaticality or as the 'rational science of intervals' that is alien to and the supplement of the natural song as presence. As with writing, the 'science of intervals' is located outside the full presence of the song, which is considered by Rousseau united with the inflections of the voice in speech. It would be superfluous to indicate here the obvious analogy between song being present to itself and inside time, and harmony (calculation of intervals) being outside this presence and outside time.

2.5 Symmetry

2.5.1 Series

Xenakis's scientific approach stands at the antipode of what Derrida is preoccupied with in reference to Rousseau's degradation of music. It is, for him, precisely too much emphasis on melody that has caused the degradation of music. It is interesting here to see how the deconstructive works when privileging harmony over melody; or in the case of serialism, the (chromatic) scale over the series. No matter how Xenakis might have phrased his critique over the years, it can be shown that the 'magma' he pointed at is essentially issued by the series itself; and this is due to the structural difference between a given melody and the series. Although according to Xenakis's theory both the series and melody are inside-time structures, the two are not identical. For Xenakis, the degradation he referred to, did not escape the attention of the Viennese school:

> [A]tonalism, prepared by the theory and music of the romantics [*sic*] at the end of the nineteenth and the beginning of the twentieth centuries, practically abandoned all outside-time structure. This was endorsed by the dogmatic suppression of the Viennese school, who accepted only the ultimate total time ordering of the tempered chromatic scale. Of the four forms of the series, only the inversion of the intervals is related to an outside-time structure. Naturally the loss was felt, consciously or not, and symmetric relations between intervals were *grafted* onto the chromatic total in the choice of the notes of the series, but these always remained in the in-time category. Since then the situation has barely changed in the music of the post-Webernians. This degradation of the outside-time structures of music since late medieval times is perhaps the most characteristic fact about the evolution of Western European music, and it has led to an unparalleled excrescence of temporal and in-time structures. In this lies its originality and its contribution to the universal culture. But herein also lies its impoverishment, its loss of vitality, and also an apparent risk of reaching and impasse (FM 193-4; italics added).

This is an obvious reference to Webern, who revealed serialism's potential, mainly discovering symmetrical relations between different forms of the tone-row. Symmetry is par excellence a geometrical phenomenon and therefore belongs to the outside-time category. And for Xenakis, this is the outside-time element that was *grafted* onto the process of constructing an inside-time structure, the series, in order for the serial technique to recover from its degradation. But this *possibility* for symmetry was already included in dodecaphonism's potential. The tone-row cannot be reduced to a mere succession of elements; its four forms might stand in such a relation to each other that can reveal correspondences and symmetries much more profound than vertical or horizontal reflections. The impasse was dealt with by revealing certain aspects of the interior of the series that can inform the structural principle of the composition, and not by imposing symmetrical forms from outside. There are here the two characteristics of the supplement: symmetry substitutes the mere inside-time ordering of the pitch-classes and at the same time adds itself as a structural principle. Xenakis's theory fails to see symmetry, an outside-time characteristic, as deriving from an inside-time structure; it is a case of a much more profound magma, where distinguishing the two categories is never straightforward. We can now see this way of thinking according to the logic of the supplement, which 'would have it that the outside be inside, that the other and the lack come to add themselves as a plus that replaces a minus, that what adds itself to something takes the place of a default in the thing, that the default, as the outside of the inside, should be already within the inside' (Derrida 1997: 215).[18]

Xenakis's privileging of outside-time structures is a method intended to establish the foundations of a general harmony. On the other hand, Derrida's method (and not strategy in a teleological sense) is to distance oneself from the binary opposition and allow for any deconstructive functions, without privileging one or the other side. Derrida has demonstrated the deconstructive in the function of harmony in relation to melody.

[18] It is not my intention to show Xenakis's approach as similar to Derrida's, as there is no evidence that he was aware of the latter's work (Derrida was working on these themes at around the same time). What is important here though is the logic of supplementarity and of incompleteness of an account on the matter (which is also reflected in that Xenakis had not written any complete treatise on outside-time structures).

According to the logic of the supplement 'there is already harmony within melody' (Derrida 1997: 212). Writing, as the supplement of speech, allows for *spacing* (the becoming-space of time and the becoming-time of space) (see Derrida 1997: 68) and elements are put in distance from each other (intervals). In the same way that punctuation is an act of inscribing into space, the function of harmony is parallel to that of writing as spacing. Outside-time structures, or the rational science of intervals (such as the intervals in a pitch scale) follow the same logic. Interval here is the nonpresent, the unperceived.

2.5.2 Non-Retrogradable Rhythms

The idea of symmetry, as the outside-time element in a structure, is also found in the case of non-retrogradable rhythms. Their placing outside of time by Xenakis raises the question of whether there are rhythmic sequences that, as the scales, are wholly or partially outside of time. As I have shown when comparing it with the scale, a rhythmic sequence possesses a temporal structure (the set of temporal intervals placed outside of time) and an inside-time aspect (the time-instants). Generalising therefore, all rhythmic sequences posses an outside-time aspect. In *Arts/Sciences*, in the discussion on the reversibility of time, Xenakis made his view clear: real time is not reversible. There, he also uses symmetry as an example for a state of order (as opposed to states of disorder in his stochastic music). Furthermore, for Xenakis symmetry does not entail reversibility of time, 'because there can be order in non-temporal things' (A/S 70). In non-retrogradable rhythms you cannot reverse the inside-time order of the successive intervals. But reversing is only one among the several reorderings or permutations that can be applied inside time; symmetry merely limits the inside-time operations that one could apply to such a structure. In this sense, the set of temporal intervals does not need symmetry in order to be thought of as an outside-time structure. It remains then that symmetry must be related to the other element of a rhythmic sequence: time instants themselves. Symmetry stands as an additional outside-time element, apart from the set of temporal intervals; non-retrogradable rhythms are therefore shown to be outside time by relating symmetry, which is a geometrical, non-temporal phenomenon, to their inside-time aspect. And as this symmetry does not imply reversibility of time (which is in any case impossible) the time instants are both outside time (as part of a symmetrical construction) and inside time by definition. As with the case of the series, symmetry is found to operate as a supplement; the idea of supplementarity is precisely found in this broaching of an inside-time structure (or the inside-time aspect of a temporal structure) with a non-temporal element.

2.6 Spacing

In the opening of 'Symbolic Music' Xenakis talked about a certain *amnesia* (see FM 155). That is, the forgetting of the origin of musical structures, such as the scales, modes and rhythms that personal languages and styles are built on. Although this is not an explicit metaphor for writing, it can be seen as an attempt of abstracting the originary elements of musical structures. Let us take once more the example of the scale, disregarding the cadential and hierarchical relationships between its elements (in other words, its origins). The elements of a scale neither refer to something other than themselves, nor are they present to themselves; they are defined *in relation* to each other. If the notion of the trace is relevant to temporal structures, it must also be relevant to outside-time ones. The interval constitutes both the difference between the elements and the deferral of the elements' definition. The difference between the elements is seen as a simple consequence of the (abstract) hierarchy that governs a totally ordered structure: elements are arranged from the lower to the higher, or from the smaller to the larger; in other words, there is a spatial distance between them. At the same time, each one element, as a 'landmark point' in our memory, is not defined until it is compared with the others; in other words, the assigning of spatial distances is deferred until we relate the *trace* of each one element with the others.

Pitch intervals can be seen as parallel to temporal intervals. Both a scale and a rhythmic sequence can be thought of as points on a straight line (the straight line of natural numbers in the former case and time in the latter). Thus the comparison between the two can be seen even more abstractly: points refer either to pitches or to time-instants and the intervals between two successive points refer either to pitch-intervals or to temporal intervals. The inside-time placing of the scale is carried out by the time-ordering of the points (as in a melody or a series), whereas the inside-time aspect of a rhythmic sequence stems from the fact that its points are fixed to the temporal flow. In the case of the rhythmic sequence therefore, the points are always inside time and the intervals outside of time ('nontemporal jumps'). But what has been said about elements or points can also be said about intervals. If intervals are perceived as multiples of unit, the idea of the trace defers the assigning of this unit to any interval until it is compared with another. The scale, as both a set of discrete elements and a succession of intervals, is itself conditioned by the function of writing as spacing: the becoming-space of time and the becoming-time of space. The non-distinction of the becoming-space and the becoming-time implies the impossibility of this other distinction between the (outside-time) set of points and the (inside-time) succession of intervals. It is a case of *différance*: 'of discontinuity and of discreteness, of the diversion and the reserve of what does not appear' (Derrida 1997: 69).

The representation as points on a straight line is essential in Xenakis's definition of a sieve (see FM 268); this is his own 'solution to the problems of outside-time structures' (preface to *Jonchaies*). His development of Sieve Theory is driven by the 'question of symmetries (spatial identities) and periodicities (identities in time)' (FM 268). The most elementary sieve is a single periodic interval, called a *module* (I will explain this further in Chapters 3 & 4). This is also the case of the chromatic scale: a periodic interval of a semitone. As I will show in Chapters 5 & 6, Xenakis conceived his sieves of the later period as multiplicities of such elementary sieves, but with different interval each; i.e. modules of different size. He referred to these modules both as symmetries and as periodicities. Apart from just a way of referring to the constituent elements of the sieve, this reflects Xenakis's approach to sieve-construction of the late period. Outside-time structures (sieves) are constructed and conceived as multiplicities of inside-time identities (periodicities). In the case of the series, an outside-time characteristic (symmetry) is achieved by the inside-time ordering of the twelve pitch-classes; similarly, in the case of his sieves an inside-time property (periodicity) is at work in an outside-time structure.

PART II

3 Sieve Theory

3.1 The Sieve of Eratosthenes

Xenakis developed Sieve Theory during his stay in Berlin, having received a Ford Foundation grant to live and work in West Germany, in 1963. The theory mainly concerns the creation of scales, arrived at through the combination of residue class sets. The primordial sieve in mathematics is known as the Sieve of Eratosthenes. The importance of this technique to Xenakis is fundamental; it has provided him with a method for ‘filtering’ elements in order to create and manipulate structures. Furthermore, Xenakis’s and Eratosthenes’s methods share a common origin in the foundations of arithmetic; I will show that the two are directed to the foundational role of prime numbers. The Sieve of Eratosthenes is a method for determining the prime numbers up to a given integer n. It is based on the following simple procedure: we write down in a matrix, in ascending order, all the integers from 2 to n. We leave the first element (2) and erase all its multiples, we leave the next number that has not been erased (3) and erase all its multiples, and so on. We proceed until we reach prime number p, where $p \leq \sqrt{n}$. The remaining integers are the prime numbers between 2 and n. In Figure 3.1 n = 50. The table consists of four parts (each for one stage of the process) and shows the cross-outs for each element: in the top left part of the table we have erased all the multiples of 2 (every second number), in the top right part all the multiples of 3 (every third number), and so on for 5 and finally 7, which is the greatest prime $\leq \sqrt{50}$.

More specifically, the first stage of the procedure (top left) starts at 2 and proceeds by steps of 2, the second stage (top right) starts at 3 and proceeds by steps of 3, the third (bottom left) at 5 and proceeds by 5, and the fourth (bottom right) at 7 and proceeds by 7. Some steps of one stage coincide with steps of another stage; this is shown as double or triple cross-outs. Afterwards, only the starting points of these four stages are allowed through the sieve (i.e. 2, 3, 5, 7); these elements are then added to the set of numbers greater than 7 that have not been erased, and the resulting set is the set of primes up to 50. These are the numbers of the bottom right part of the table that have not been erased:

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$$

What Xenakis drew from this is not merely the idea of filtering – passing the elements of a set through a sieve – but also the process of using starting points and steps of a specific distance. However, Xenakis's application of Sieve Theory is not intended to determine primes: his sieves allow both the starting points and all the following steps. Each of the four stages in the Sieve of Eratosthenes is for Xenakis an infinite set of numbers, that might coincide with each other in a more or less complex way. The degree of complexity is a matter of compositional decision and aesthetics. This was done in a period when he would attempt to take further his investigation towards formalisation; this time though not with stochastics and probabilities, but with the aid of the deterministic laws that govern Number and Set Theory. However, both cases were for him a matter of generating outside-time structures of music.[19]

A sieve, then, refers to a selection of points on a straight line; this is the abstract image of sieves: ‘Every well-ordered set can be represented as points on a line, if it is given a reference point for the origin and a length *u* for the unit distance, and this is a sieve’ (FM 268).[20] The theory was used in order to construct symmetries at a desired degree of complexity. This was achieved by the combination of two or more *modules*. A module is notated by an ordered pair (*m*, *r*) that indicates a *modulus* (period) and a *residue* (an integer between zero and *m*-1) within that modulus.[21] For example, for $m = 3$ and $r = 1$ we have the following module:

$$(3, 1) = \{1\ 4\ 7\ 10\ 13\ \ldots\}.$$

Elements that lie in distance equal to the value of the modulus are said to be *congruent modulo m*. In other words, elements that yield the same residue (*r*) when divided by the same number (*m*) belong to the same congruence class. In the example, elements 4, 7, and 10 are congruent modulo 3:

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[19] The first published material on sieve theory was in Xenakis’s article ‘La voie de la recherche et de la question’, *Preuves* 177 (1965), later included in MA. The first extended reference was made in ‘Vers un métamusique’, *La Nef* 29 (1967). This was followed by the ultimate publication of ‘Sieves’, *Perspectives of New Music* 28/1 (1990). The two latter articles appeared later as chapters VII and XI of FM.

[20] Xenakis referred to unit distance (e.g. the semitone in the major diatonic scale) also as Unit of Elementary Displacement (ELD).

[21] The terminology I use in this dissertation is based on Xenakis (FM) and Squibbs (1996).

$$4 \equiv \ 7(\text{mod}3)$$

$$7 \equiv 10(\text{mod}3).$$

3.2 Logical Operations

By applying the set-theoretical operations of union (+), intersection (·), and complementation (–), or a combination of them, one can construct more complex sieves.

3.2.1 Union

The union of two modules is the binary operation that includes all the elements that originally belong to both modules.[22] For example, the union of modules (3, 0) and (4, 0) is

$$(3, 0) + (4, 0) = \{0\ 3\ 4\ 6\ 8\ 9\ 12\ 15\ 16\ 18\ 20\ 21\ 24\ldots\}.$$

The period of this sieve is equal to the Lowest Common Multiple (LCM) of 3 and 4, that is, 12.[23] The *intervallic structure* of a set is a listing of all its successive intervals:

3 1 2 2 1 3 3 1 2 2 1 3.

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[22] We can only form a union of distinct sets. An example given by Xenakis (FM 270) involves the union of residue classes (2, 0) and (6, 0); but (6, 0) is a subset of (2, 0) and therefore the union is redundant.

[23] Finding the period of a sieve in this case is quite straight-forward, because 3 and 4 are coprime (their Greatest Common Divisor is 1). If this was not the case the procedure would have to involve Euclid's algorithm (see FM 270).

Since the set repeats itself after element 12, or in other words its intervallic structure repeats after 12 units (e.g. semitones), it suffices to represent it by only one occurrence of its period:

$$(3, 0) + (4, 0) = \{0\ 3\ 4\ 6\ 8\ 9\}.$$

The period, 12, would appear after element 9; but after modular reduction, it is equal to element 0. Thus $12 \equiv 0(\text{mod}12)$ and the following elements would be $15 \equiv 3(\text{mod}12)$, $16 \equiv 4(\text{mod}12)$, and so on.

3.2.2 Intersection

Intersection refers to the coincidences, or to the common elements of two modules. In the case of the major diatonic scale we can choose to represent the sieve either using its period (12 semitones) or using moduli 3 and 4; the operation of union is used in the former case, and a combination of union and intersection in the latter.

$$(12, 0) + (12, 2) + (12, 4) + (12, 5) + (12, 7) + (12, 9) + (12, 11) =$$

$$(4, 0)\cdot(3, 0) + (4, 2)\cdot(3, 2) + (4, 0)\cdot(3, 1) + (4, 1)\cdot(3, 2) + (4, 3)\cdot(3, 1) + (4, 1)\cdot(3, 0) + (4, 3)\cdot(3, 2)$$

Each intersection in the latter formula corresponds to a module in the former. Thus, (12, 0) = (4, 0)·(3, 0), (12, 2) = (4, 2)·(3,2), and so on. Within the scope of a single occurrence of a period, an intersection corresponds to a unique point. 3 and 4 are coprime and therefore their product equals the period of 12 semitones. We can now choose to regroup these elements around the modulus of 4. In order to do this we merely use the distributive property:

$$(4, 0) \cdot [(3, 0) + (3, 1)] + (4, 1) \cdot [(3, 0) + (3, 2)] + (4, 3) \cdot [(3, 1) + (3, 2)] + (4, 2)\cdot(3, 2).$$

This alternative formula for the same sieve is aimed at facilitating comparison with other sieves that might share modulus 4, or maybe with other versions of the same sieve. As I will demonstrate later on, the use of each logical operation also depends on the type of sieve the formula represents.

3.2.3 Complementation

Complementation is the only unary of the three operations and refers to all the elements that are not members of the original module. Whereas the two binary operations might reveal some aspects of the type of sieve they represent, complementation is used only in order to simplify the notation.[24] For example, the union of intersections (4, 0)·(3, 0) and (4, 0)·(3, 1) can be rewritten as follows:

[24] Xenakis used complementation in his former writings only. In (1990) he only used the two binary operations of union and intersection.

$$(4, 0)\cdot(3, 0) + (4, 0)\cdot(3, 1) = (4, 0) \cdot [(3, 0) + (3, 1)]$$

All the possible modules based on modulus 3 are (3, 0), (3, 1), and (3, 2). The only module that is missing from within the square brackets is module (3, 2). Thus we can finally write:

$$(4, 0)\cdot(3, 0) + (4, 0)\cdot(3, 1) = (4, 0)\cdot(\overline{3,2}).$$

3.3 Transcription

In FM (271-3) Xenakis presents a way of transcribing a formula into the actual sieve. The problem of transcribing a sieve is reduced to identifying the residues of the intersections. If $(m_1, r_1)\cdot(m_2, r_2) = (m_3, r_3)$ then m_3 is LCM of m_1 and m_2; r_3 is found through algorithmic calculations involving basic theorems of arithmetic. However, there is a much more immediate and simpler method proposed by Squibbs and elaborated by Gibson. As this method is applicable manually it is restricted to relatively smaller sizes of moduli (but is easily applied to moduli up to the audible range counted in semitones). It involves the construction of a matrix that represents two moduli that might be combined.[25] Xenakis himself had indicated that in a series of multiple intersections we would have to gradually calculate the intersection in pairs of modules (see FM 271). Despite the limitation that this method exhibits, it is a valuable tool in other ways as well:

[25] It is important to mention here that in order to construct such a matrix it is necessary that the moduli are coprime. I will discuss this in detail below.

it can assist in locating different transpositions of the sieve and provide a synoptic view of the type of sieve they express.

In order to represent a combination of two modules we construct a matrix whose dimensions correspond to the two moduli. Then we enter 0 at the top left cell and proceed diagonally, entering the consecutive natural numbers wrapping around back to the left or to the top of the matrix when the right or bottom edges have been reached. Each column corresponds to the residues of the one module and each row to the residues of the other. Figure 3.2 shows the matrix for moduli 4 and 3. The residues of modulus 4 are assigned to the four columns and the residues of modulus 3 to the three rows. Thus within the matrix we have all the residues of modulus 12, from 0 to 11. The columns and rows represent the seven possible modules based on moduli 3 and 4: (3, 0), (3, 1), and (3, 2) for the rows and (4, 0), (4, 1), (4, 2), and (4, 3) for the columns. The residues do not appear in order: for example, the elements of module (3, 1) appear in the second row as 4, 1, 10, 7. In more practical terms they represent the points covered when we start at 1 and proceed by steps of 3.

3.3.1 Transcription of unions

It should be obvious by now that when we want to transcribe a formula of straightforward unions of modules we simply need to locate the columns and rows that correspond to the them. The union of modules (3, 0) and (4, 0) is shown in the matrix of Figure 3.3. The first column and first row contain all the elements of the sieve. Accordingly, we can transcribe the union of more than two modules based on moduli 3 and 4. For example, row 1 and columns 0 and 2 contain all the elements of sieve (3, 1) + (4, 0) + (4, 2).

Finally we arrange the elements in the right order and given a point of departure, for example 0 = middle C, we transcribe the sieve to musical notation.

3.3.2 Transcription of intersections

In order to locate the coincidence (intersection) of modules (4, r_i) and (3, r_j) we need to find the cell where the two corresponding modules meet. Thus, (4, 3)·(3, 1) = (12, 7). Element 7 is found in the intersection of the fourth column (column 3) and the second row (row 1) of the matrix. In other words, with intersection we need to determine the residue of the new module (its modulus is the LCM of the two original moduli). This method of transcription is carried out for each of the intersections in a formula, until we reach a collection of points selected from the matrix. As an intersection of two modules corresponds to a unique point within the scope of one period, it might be helpful to transcribe any given formula in the form of a series of pairs of intersections (simply using the distributive property). In the case of the major diatonic we can use the following form (as shown in Section 3.2.2):

(4, 0)·(3, 0) + (4, 2)·(3, 2) + (4, 0)·(3, 1) + (4, 1)·(3, 2) + (4, 3)·(3, 1) + (4, 1)·(3, 0) + (4, 3)·(3, 2).

The points this formula produces are shown in the matrix of Figure 3.4. We then correspond each pair with a point in the matrix and proceed to musical notation. In case there are more than two moduli involved we have to construct sub-matrices and calculate the intersections in pairs of modules, following the procedure Squibbs has indicated

(1996: 306). If, for example, we desire to find the intersection of (3, 1), (4, 2), and (5, 4) we use a submatrix for moduli 3 and 4, and a larger matrix for 12 and 5, shown in Figure 3.5. The intersection of the first two modules is element 10 (see matrix of Figure 3.2). Therefore,

$$(3, 1)\cdot(4, 2) = (12, 10)$$

We then find the intersection of (12, 10) and (5, 4) in the larger matrix (Figure 3.5):

$$(12, 10)\cdot(5, 4) = (60, 34)$$

and therefore

$$(3, 1)\cdot(4, 2)\cdot(5, 4) = (60, 34).$$

3.4 Types of Sieves

One way to categorise sieves is according to their symmetry and periodicity. Xenakis refers to the notions of symmetry and periodicity as two distinct levels of identity: in the opening of his article on sieves he talks about ‘spatial identities’ and ‘identities in time’, correspondingly; he then refers to these levels as being internal and external to the sieve (FM 268). A sieve’s symmetry is the one evident in its intervallic structure, and its periodicity is evident in its periodic nature. The theory offers the possibility of decomposing a sieve; as I will show later on, this decomposition aims at a deeper level of symmetry, one that lies between absolute symmetry and absolute asymmetry. The categorisation of Sieves presented here, follows a similar approach to the one offered by Gibson (2003: 56). However, his starting point is different: his categorisation is carried out under the prism of an analytical methodology, more efficient and practical than the one proposed by Xenakis, while the one I present here is concerned with exhausting all the forms a sieve can have on the theoretical level (I will develop an analytical methodology in Chapter 5). My suggestion for a theoretical classification of sieves is aimed at (a) in describing the types of sieves Xenakis was concerned with, and (b) illustrating the relationship between Sieve and Number Theory.

There are four types of sieves, two for each of the two criteria of symmetry and periodicity:

(a) Symmetry refers to the intervallic structure of the sieve. Thus, a sieve can have either a symmetric (palindromic) or an asymmetric intervallic structure.

(b) Periodicity refers to the period of the sieve: this can be either a prime or a composite number.

3.4.1 Symmetry

According to symmetry, we call a sieve either symmetric or asymmetric. Symmetric sieves are not generally in line with Xenakis's compositional aesthetics. However, it is important to describe them in detail, as this will enable a demonstration of the nature of asymmetric sieves discussed below.

3.4.1.1 Symmetric Sieves

We call symmetric any sieve that exhibits symmetry in its intervallic structure. Symmetric sieves might be expressed merely as a union of two different modules. This is the case with the union of (3, 0) and (4, 0) already shown. The sieve's intervallic structure is palindromic: 3 1 2 2 1 3. This symmetry is found in the notional axis in the centre of the intervallic structure.

We also call symmetric the sieves that have a palindromic intervallic structure under cyclic transposition (discussed in the following section).[26] Cyclic transposition can be achieved by placing the final interval(s) of the intervallic structure before the initial one(s), or vice versa. If the result is a shift of the structure to the right, it is a case of an upward or positive transposition. (But in the scope of a period, a negative transposition could be also expressed as a positive one: the value of the period can be added to a negative index and result to an equivalent positive one.) The above structure would thus be 3 3 1 2 2 1, which is shifted three semitones to the right. If we repeat this four more times (or if we place the initial interval at the end) we have 1 2 2 1 3 3. The structure has now shifted nine semitones to the right. The two structures are now not symmetric as such. However, this does not imply that a simple cyclic transposition has changed the type of the sieve. These two sieves are equivalent as versions of one and the same sieve: they are symmetric under cyclic transposition.

[26] See preface to the score of *Mists*.

3.4.1.1.1 Symmetric Sieves with Even Number of Intervals

A palindromic structure is sufficient to call a sieve symmetric; and any sieve can have either an even or an odd number of intervals. Whereas this distinction does not affect the analysis of asymmetric sieves (discussed below) it is relevant to symmetric ones in terms of their intervallic structure as found in cyclically transposed versions of the original. As implied by the observation on the notional axis of symmetry, the above sieve has an even number of elements and intervals (6). Consequently, from all its cyclic transpositions two of them are symmetric. These two transpositions are situated at half a period's distance from each other. In the example these will be the transpositions 3 1 2 2 1 3 and 2 1 3 3 1 2, at 0 and 6 semitones. These two intervallic structures belong to sieves (3, 0) + (4, 0) and (3, 0) + (4, 2). Although any of the two could be taken as a starting point with regard to transpositions, the first of the two is preferred.

An efficient way of telling whether a sieve is even-symmetric is offered by its matrix. In the case of a simple union of modules all the transpositions appear as a simple intersection of complete rows and columns. In particular, in simple unions of only two modules, the number of intervals is $m_1 + m_2 - 1$.[27] I will use the two examples of transpositions of three and nine semitones in order to demonstrate their appearance in the matrix. The cyclic positive transposition of three semitones of (3, 0) + (4, 0) yields the following sieve: {0 3 6 7 9 11} and is shown in Figure 3.6. The transposition of nine semitones to the right of the same sieve, yields sieve {0 1 3 5 6 9}, shown in Figure 3.7. We see that both sieves, as well as the one they originate from (Figure 3.3) share a

[27] This is true for both sieves with an even and an odd number of intervals (when these are a simple union of modules). For example, if $m_1 = 3$ and $m_2 = 5$, the amount of intervals is 7.

similar representation in the matrix: their symmetric nature is evident as a simple intersection of a row and a column that correspond to the residues of each residual class. As I have shown, the rows and columns correspond to the two modules whose union yields the sieve. In our examples, these two unions are (3, 0) + (4, 3) for the former transposition and (3, 0) + (4, 1) for the latter. The two complete modules (i.e. the complete column and row) have an element that is common to both. These are elements 3 and 9, and they correspond to the degrees of the effectuated transpositions [with (3, 0) + (4, 0) as the starting point]. If these two sieves were arranged to start on 3 or 9 respectively, the resulting intervallic structure would be the original (3 1 2 2 1 3). The common element of the two modules corresponds to the initial point of the original's period. In this way symmetry is established by the sieve's appearance in the matrix without having to resort to multiple cyclic transpositions.

Symmetric intervallic structure is a general property of sieves formed by a union of different modules. However, this does not imply that the reverse would also be true: sieves that have symmetric intervallic structure might not always be expressed as unions of modules. Let us consider sieve {0 1 3 4 6 8 9 11} with period 12 semitones (Figure 3.8). Its structure is 1 2 1 2 2 1 2 1 semitones, which is palindromic and contains an even number of intervals (8). The notation of this sieve, although symmetric, is not possible without involving intersection. Its appearance in the matrix is not as simple as the case of the previous examples. That is, we do not observe only a straightforward combination of modules (complete columns or rows) but also two additional elements: C# and B as represented by numbers 1 and 11. These additional elements are not part of a complete column or row and therefore need to be notated as simple, unique points. The only way to notate unique points is by involving intersection:

$$(3, 0) + (4, 0) + (3, 1)\cdot(4, 1) + (3, 2)\cdot(4, 2).$$

Nonetheless, the symmetric structure is still apparent in the matrix. Apart from the combination of the two modules (3, 0) and (4, 0) the two additional points 1 and 11 lie at a symmetric position in relation to each other. As a general rule, symmetric are also the sieves that additionally to the union of modules there are points in pairs that their sum equals the period. Thus, starting from the symmetric intervallic structure of (3, 0) + (4, 0), if any of the pairs of elements {1, 11}, {2, 10}, or {5, 7} is added then it will remain symmetric (see matrix). This observation is also true for sieves that appear in the matrix as a combination incomplete columns and rows, such as sieve {0, 4, 6, 8} with period 12 and intervallic structure 4 2 2 4 (Figure 3.9). In this case the two missing elements are 3 and 9, whose sum also equals the period. Therefore the structure remains symmetric.[28]

3.4.1.1.2 Symmetric Sieves with Odd Number of Intervals

In the case that an intervallic structure has an odd number of intervals there is no axis of symmetry proper. Such a sieve might as well symmetric: there can still be a central element that replaces the axis of symmetry. Odd-symmetric sieves do not present as a straightforward structure as even-symmetric sieves.

[28] In the case of cyclic transpositions the same relations hold when the transposition index is added to the sum of the elements; this should be (modulo the period) equal to the period or one of its multiples.

From all the possible cyclic transpositions of an odd-symmetric sieve only one is palindromic (instead of two, in the case of even-symmetric sieves). All the intervallic structures that are palindromic under cyclic transposition are considered to be equivalent. As Xenakis states, ‘the white keys on a piano constitute a unique sieve’ (FM 268). In the light of Sieve Theory then, all the modes (such as D, E, or the natural minor) can be derived from the cyclic transpositions of the intervallic structure of the major diatonic scale. These modes and scales are therefore versions of one and the same sieve. In particular, this sieve is the mode of D, with intervallic structure 2 1 2 2 2 1 2. If we transpose this intervallic structure two semitones we get the major diatonic. Thus, the major diatonic is palindromic under cyclic transposition and therefore considered symmetric. The general appearance of such a sieve in the matrix does not change with cyclic transposition. The synoptic view they offer is evident when one compares the matrices of two transpositions. Figure 3.10 compares the matrices for the mode of D and the major diatonic. We observe that the two matrices show that two modules are incomplete by one and the same element. For the mode on D element 6 is missing from modules (3, 0) and (4, 2). For the major scale, element 8 is missing from modules (3, 2) and (4, 0). The elements’ difference shows the transposition index (2 semitones). The two matrices are similar to the one of Figure 3.8: if element 8 was included in the matrix for the mode of D then these two sieves would be the same under cyclic transposition. We see, that although matrices might not always verify positively a sieve’s internal structure, they can often provide a synoptic view that might compare either two different sieves or two versions of the same sieve.

3.4.1.2 Asymmetric Sieves

Sieves with a non-palindromic intervallic structure are called asymmetric. The theoretical representation of an asymmetric sieve is possible either according to its period or, if decomposed, as a series of (unions of) intersections.[29] Using only the period in the notation of a sieve is not considered a decomposition: it is rather equivalent to merely notating all the 'pitch-classes' involved. In the case of the decomposition of the harmonic minor scale we need to involve the logical operation of intersection (or intersection and negation). This is due to its non-palindromic intervallic structure: 2 1 2 2 1 3 1. Again, we use for this moduli 3 and 4. Figure 3.11 shows the matrix for the minor scale. We observe that the scale can be conceived as an alteration of the simple union of (3, 2) and (4, 3). There is an additional element, 0, that destroys its symmetry and produces a new structure. The formula for this sieve is (3, 2) + (4, 3) + (3, 0)·(4, 0).

In order to decompose a modulus into simpler ones, one has only to replace this modulus with two factors of it. Given the possibility of multiple notations of the same sieve, we can decide upon which factors to choose in order to get a desirable result. However, not all intersections are possible. Xenakis clearly points out that in order for an intersection to exist, the difference between the residues must be divisible by the Greatest Common Divisor (GCD) of the moduli (FM 272). The safest way to avoid an empty intersection is therefore to always work with moduli that are coprime (i.e. their GCD is equal to 1). This explains, at least partly, why Xenakis would decompose a modulus into

[29] Although we can decompose a modulus into more than two different ones, I will here refer to decomposition into two moduli, since the same rules hold for decomposition into more than two.

coprime factors. Coprimality of the factors is also a necessary condition for the construction and function of the matrices.

3.4.2 Periodicity

The criterion of Periodicity relates to the decomposability of the sieve's period. Making an obvious historical reference to the Sieve of Eratosthenes, I will use the terms *prime sieve* for sieves with a prime period and *composite sieve* for sieves with a composite period. Furthermore, and as will become obvious further on, this decision is also closely related to Xenakis's application of the theory.

3.4.2.1 Prime Sieves

Although the notation of symmetric sieves allow intersection and the notation of asymmetric ones necessitate it, in terms of factorial decomposability all types of sieves I have demonstrated so far belong to the same category. However, decomposing the period is not possible for all sieves. Factorial decomposability refers to intersections of modules and depends on whether the period of the sieve is a prime or a composite number. It follows that sieves whose period is prime cannot be decomposed into factors and therefore cannot be notated except according to the period only. Of course, a prime period can be found in any sieve (either symmetric or asymmetric).[30] A prime asymmetric sieve is the one used in *Jonchaies* (1977, for orchestra); it is shown in Figure 3.12. Its intervallic structure is asymmetric (it is non-palindromic in any cyclic

[30] Prime symmetric sieves can only have an odd number of elements, since all prime numbers greater than 2 are odd.

transposition) and its period is one octave and a perfect 4th, i.e. 17 semitones, which is a prime. This type of sieves can be notated according to their period only, which is equivalent to writing down all its 'pitch-classes': {0 1 4 5 7 11 12 16}. In other words, the sieve of *Jonchaies* can be written only as a union of eight modules (one for each element of one period of the scale) that share modulus 17 (where 0 = A2):

(17, 0) + (17, 1) + (17, 4) + (17, 5) + (17, 7) + (17, 11) + (17, 12) + (17, 16).

In terms of notation, decomposability of sieves is related to the two options of notation according to their internal or their external aspect; the above formula is obviously of the second type.[31] This is the only way of notating a prime (non-decomposable) sieve, while decomposable sieves can be also notated using two or more modules. In other words, the possibility of the two alternative types of notation, stems from the possibility of a period to be expressed as a combination of simpler moduli.

3.4.2.2 Composite Asymmetric Sieves

Xenakis's intention to arrive at a 'more hidden' symmetry refers therefore to composite asymmetric sieves (see FM 269-70).[32] An example of such a sieve is the harmonic minor. But Xenakis's general aesthetic led him to much more complex asymmetric sieves with significantly larger periods, such as 60 or 90 semitones.[33] Having defined internal

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[31] These types of notation refer to the decomposed and simplified formulae that I discuss in Section 4.3.

[32] This idea of 'hidden symmetry' is not necessarily found only through the decomposition of the period. I will discuss this in Part III.

[33] Xenakis has defined the audible range as extending to 11 octaves or 132 semitones (see MA 67).

symmetry as a palindromic intervallic structure and non-symmetry as non-palindromic, this level of 'hidden symmetries', curiously perhaps, seems to refer to intermediate stages between the two extreme poles. These two poles are occupied by symmetric sieves on the one side (either prime or composite) and prime asymmetric sieves on the other; the former are too regular to offer any interesting properties and the latter seem to escape the potentiality of the theory. In a note of Xenakis's article we read that 'it is sometimes necessary and possible to decompose' a modulus (FM 381). This is an obvious reference to composite asymmetric sieves: 'possibility' here refers to the period's factorial decomposition (if it is a composite number) and 'necessity' to the unveiling of a hidden symmetry whenever it exists. I will clarify this point further in the discussion of the Unique Factorisation Theorem and its importance to the theoretical representation and analysis of sieves (Sections 4.1 & 4.2). This discussion, like Xenakis's practice, will refer to composite asymmetric sieves.

See Figure 3.13 for a synoptic view of the four types of sieves. Rows refer to the criterion of (internal) symmetry and columns to that of periodicity. The table also provides a description of each type's internal and external aspect and examples for each type. All examples have already been mentioned, apart from that of the prime symmetric type: sieve {0 2 5 6 9} with period 11, has a palindromic intervallic structure (2 3 1 3 2) and does not allow factorial representation.

3.5 Metabolae of Sieves (Transformations)

The possibility of theoretical representation of sieves is inherently connected with the decomposition of a sieve's modulus. One of the aims of decomposition is to enable transformations that might not be as obvious in the actual scale. This can be done by altering some aspect of the sieve. Generally, as demonstrated by Xenakis, there are several possible ways of modifying a sieve. Mainly, transformations might be applied either to the modulus *m* itself or to the residue *r*, or to both. These ways have been adequately demonstrated by Squibbs (1996: 57-67) and Gibson (2001; 2003: 58-72). Additionally to Xenakis's demonstration, both scholars have noted the potentialities and limits of the transposition of sieves (modifications on *r*) with the aid of matrices that provide an accurate alternative to Xenakis's conception of sieves as points on a straight line. After having decomposed the sieve, it is possible to modify only one (or some) of its elementary modules in a desired way. In the diatonic scale, we can choose, for example, to alter only the modules that share modulo 4 – and even alter modules that belong to a different group, in a different way.

3.5.1 Residues

Affecting the residues of each module does not change the period of the sieve and concerns primarily the transposition of the sieve.

3.5.1.1 Inversion

Unlike a series, a sieve is an ordered structure outside of time. This means that the notion of order is not related to time, but is inherent to the structure. (Recall Xenakis's comment that the outside-time domain does not include the notions of ‘before’ and ‘after’.) In other words, a sieve is arranged from the lower to the higher pitch. Due to this outside-time order (and like all scales in general) a sieve exhibits certain differences in the forms it can take. While in twelve-tone technique the retrograde of the inversion is equivalent to the inversion of the retrograde (under transposition), in the outside-time domain the inversion and the retrograde are equivalent.

The *inversion* of a sieve is effected by using the original intervallic succession in a downward sieve, thus changing the pitches. But if we arrange the new pitches in ascending order the resulting sieve has the retrograde of the intervallic structure of the original. Since in the outside-time domain the retrograde is annulled by the inversion, we have only two forms: the original sieve and the inversion. Note that these two forms apply only to the intervallic succession. The inversion of a sieve can be achieved by replacing all the residues in a formula by their negative value and then reduced according to the modulus. Module (17, 5) would then become (17, -5) and finally (17, 12). When this is applied to all modules, the resulting formula produces the retrograde. These steps are followed for all modules which are at the end placed in ascending order (note that the same procedure would be applied in a formula that involves intersection):

(a) (17, 0) + (17, 1) + (17, 4) + (17, 5) + (17, 7) + (17, 11) + (17, 12) + (17, 16)

(b) (17, 0) + (17,-1) + (17,-4) + (17,-5) + (17,-7) + (17,-11) + (17,-12) + (17,-16)

(c) (17, 0) + (17, 16) + (17, 13) + (17, 12) + (17, 10) + (17, 6) + (17, 5) + (17, 1)

(d) (17, 0) + (17, 1) + (17, 5) + (17, 6) + (17, 10) + (17, 12) + (17, 13) + (17, 16).

But, as I will show in Chapter 5, this is not possible for all types of formulae. Therefore, I will for now limit the discussion of the inversion to the actual sieve (scale). To produce the inversion then, we simply have to reverse to order of the intervals and construct a new sieve on the pitch level. Let us take for example the sieve of *Jonchaies*:

{0 1 4 5 7 11 12 16 17}

Its intervallic succession is

1 3 1 2 4 1 4 1

and if we reverse the order of the intervals it is

1 4 1 4 2 1 3 1.

The sieve produced by the new, reversed intervallic succession is

{0 1 5 6 10 12 13 16 17}.

The elements of the original sieve, reading from left to right, and the elements of the inversion, reading from right to left, sum to the period of the sieve: 0 + 17 = 1 + 16 = 4 + 13 = … = 17.

3.5.1.2 Cyclic Transposition

If we add a value to the residues of a sieve with coprime moduli we obtain a cyclic transposition of the sieve equal to the added value.[34] Taking into account the period, this transposition might not always have a traditional meaning. If after transposition all

[34] If we add a value larger than the period, we have to take this value modulo the period.

elements are reduced modulo *m*, then this is rather a 'cyclic' transposition. If we add 10 semitones to all residues of the major diatonic scale the resulting sieve is:

(4, 10)·(3, 10) + (4, 12)·(3, 12) + (4, 10)·(3, 11) + (4, 11)·(3, 12) + (4, 13)·(3, 11) + (4, 11)·(3, 10) + (4, 13)·(3, 12)

We now perform modular reduction: 10(mod4) ≡ 2 and 10(mod3) ≡ 1 for the first intersection of modules, 12(mod4) ≡ 0 and 12(mod3) ≡ 0 for the second intersection, and so on. The resulting scale is (as I have shown in Section 3.4.1.1.2) the mode of D:

(4, 0)·(3, 0) + (4, 2)·(3, 2) + (4, 3)·(3, 0) + (4, 1)·(3, 2) + (4, 3)·(3, 1) + (4, 1)·(3, 0) + (4, 2)·(3, 1).

We notate this cyclic transposition as T_{10}(mod12).

In this way cyclic transposition is achieved by altering the residues. In the previous section, cyclic transposition was applied to the intervallic structure of the sieve (wrapping around the intervals), whereas now it is applied to the formula of the sieve. This touches upon the issue of the types of formulae I will discuss later: cyclic transposition may be achieved only through the formula when the desired modulo operator is equal to the LCM of the moduli. For example, if one wished to apply a cyclic transposition of 8 semitones, to the 9-semitone segment of the major diatonic from C to A, i.e. a T_8(mod9) transposition, then this may not be achieved through the formula that involves moduli 3 and 4. One would simply have to take all elements of the C-A segment of the major diatonic and replace them by their T_8(mod9) values. Thus, like the inversion, cyclic transposition is included in Sieve Theory as one particular case of transformation, but only when a certain type of formula is used. For a correspondence of all intersections of modules in the formula with the pitches of the mode of D see Figure 3.14. Cyclic transposition is different from the actual transposition that can be applied to the scale itself. In the former case, when we take each element modulo *m*, the scale appears in the same register (i.e. within the scope of the period) whereas in the latter the scale is actually being transposed to a higher or lower register. Squibbs has demonstrated that we can foresee the common points of two transposed versions of a sieve with the aid of transposition matrices. These matrices can be of two kinds: one showing the actual, conventional transposition and the other the cyclic, modular one (which renders more common points between the two transpositions) (see Squibbs 1996: 54-5).

If we add a different value to residues r_i, r_j, we still have a cyclic transposition. For example, starting with the mode of D we add value 2 to residues that share modulo 3 and value 1 to residues that share modulo 4. We obtain the following sieve:

$$(4, 0)\cdot(3, 0) + (4, 2)\cdot(3, 2) + (4, 3)\cdot(3, 0) + (4, 1)\cdot(3, 2) + (4, 3)\cdot(3, 1) + (4, 0)\cdot(3, 2) + (4, 2)\cdot(3, 1)$$

It yields points {0 2 3 5 7 8 10} with intervallic structure 2 1 2 2 1 2 2. As shown by the shift from the original intervallic structure, it is a 5-semitone transposition of the mode of D. In general, the transposition index is found in the matrix as the intersection of the column and row that correspond to the added values: when *k* is added to r_i and *l* to r_j, the sieve has been transposed to the number found in the cell corresponding to column k and row l. In our example $k = 1$ and $l = 2$. Element 5 is found at the intersection of modules (4, 1) and (3, 2) (see the matrix in Figure 3.2).[35]

Cyclic transposition does not produce different intervallic structures. The number of different structures that can be produced is equal to the GCD of the modules. In the case of moduli 3 and 4 the GCD is equal to 1 and therefore no different intervallic structures can be produced by adding a single or more values to the residues.

Although the matrix is very useful in finding the overall transposition index, it does not consist the only way of doing so. Squibbs has shown that this possibility is offered by transposition matrices. Transposition matrices do not involve any decomposition of moduli; in such a matrix it is sufficient to represent the sieve as a succession of its elements (see Squibbs 1996: 55); this facilitates their use with prime sieves. Cyclic transposition therefore, does not necessitate the use of Sieve Theory: it merely refers to the transposition of a modular set. From this we can conclude that the use of Sieve Theory – although applicable – might not always be necessary if cyclic transposition is the desired transformation. On the other hand, it might be needed in order to combine cyclic transposition with other kinds of transformations.

3.5.1.3 Variables

In order to produce a different intervallic structure we have to add a value that changes according to the context. Of this transformational type is a process that includes

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[35] For a critique on Xenakis's assertion that different values added to different residues would produce new intervallic structures see Gibson 2003: 62.

variables. For example, in an intersection $(m_1, r_1)\cdot(m_2, r_2)$ we can apply the process $r_2 = r_1 + r_2 + 1$. The values of r_2 will also depend on the values of r_1 in each intersection. If we apply this to the major diatonic scale, the result is

(4, 1)·(3, 1) + (4, 2)·(3 ,2) + (4, 3)·(3, 0) + (4, 0)·(3, 1) + (4, 1)·(3, 2) + (4, 0)·(3, 2) + (4, 3)·(3, 2) + (4, 1)·(3, 1)

and the resulting scale is shown in Figure 3.15. The period is still 12, but its intervallic structure is now asymmetric: 1 1 1 1 3 4 1.

3.5.2 Moduli

In order to transform a sieve's structure without resorting to variables we have to affect the moduli. Whereas changing the residues does not affect the period of the sieve, changing the moduli produces a new period. In the formula of the major diatonic, if we add 2 semitones to modulus 3, the period changes to 20 semitones and the intervallic structure now contains intervals up to a perfect 4th. The new formula is

(4, 0)·(5, 0) + (4, 2)·(5, 2) + (4, 1)·(5, 0) + (4, 3)·(5, 2) + (4, 3)·(5, 1) + (4, 0)·(5, 1) + (4, 1)·(5, 2) + (4, 0)·(5, 0)

and the result is shown in Figure 3.16.

3.5.3 Unit

This transformation is effectuated by changing the unit distance. This would increase or decrease the values. For example, from semitone to quarter-tone, from semiquaver to quaver, etc. The technique of altering the unit distance, although might seem too obvious, is very important in Sieve Theory. Especially, the idea of different unit distances can be traced in Xenakis's attitude towards the tempered chromatic. Further to the discussion of the tempered chromatic and its equivalence to natural numbers (presented in Chapters 1 & 2), the unit distance is for Xenakis a basic concept that can be traced even further. He stresses an observation Bertrand Russell made in relation to the axiomatics of numbers and considers the tempered chromatic as having 'no unitary displacement that is either predetermined or related to an absolute size' (FM 195). We can have 'chromatic' scales of quarter-tones or semitones, but also of tones, prefect 4ths, and so on. The 'chromatic' scale of the perfect 4th is no other than an elementary module with modulus 5, such as (5, 0). By extension, Xenakis represents the total chromatic as module (1, 0). More than a simple tool for metabolae, this idea of the extension of the tempered chromatic scale became very important in the mature phase of sieve-construction; as I will discuss later on, it is closely related to the existence of symmetries and periodicities in a sieve.

4 Sieve Theory and Primes

The discussion made so far might raise some issues as to the very character of the theory – or more accurately, to the purpose of Sieve Theory as a *method*. I will provide at this point a summary of what has been covered so far and which can be considered as central to this method. Firstly, it is concerned with sieves that employ composite moduli (from those Xenakis favoured asymmetric ones). We can decompose such sieves and notate them as a series of unions of intersections. The resulting formula, which shows the internal symmetry of the sieve, depends on our decision on which factors to use in the decomposition of the modulus (period). This decision is crucial as regards to the transformation of the sieve; a combination of different moduli engenders a different outcome of the transformation procedure. There are two levels of a redundancy here that need to be overcome: (a) a formula for a given sieve must be chosen among its alternative ones according to certain criteria, and (b) after having applied the same metabola to formulae that incorporate an alternative combination of moduli (e.g. 4 and 6 instead of 3 and 4)[36] different sieves are produced, which cease to be equivalent.

4.1 Canonical Form

Xenakis mentions that decomposition of a period offers the possibility of 'comparison among different sieves'. And this in turn will enable one to (a) 'study their degree of difference' and (b) 'define a notion of distance' (FM 270). The treatment of composite

[36] Recall that the period of the sieves is equal to the LCM of the moduli. This is found by dividing the product of the moduli with the GCD. The GCD of 4 and 6 is 2 and their LCM is $4 \cdot 6/2 = 12$.

sieves presupposes the decision on which factors are going to be employed in the decomposition of the period. We can choose among several alternative decompositions into two or more factors. At first it seems that the only restriction is a matter of convenience: any combination of moduli whose LCM equals the period is sufficient, as long as the difference between the residues in an intersection is divisible by the GCD of the moduli. This is helpful in order to secure intersections without having to be careful in our selection of the residues. When the moduli are coprime then the difference between the residues can be of any value – and this is necessary in order to be able to apply transformational processes without any restrictions.

In fact, decomposing a modulus is for Xenakis not unlike decomposing any integer.[37] In that sense we can choose to decompose 12 as either using 4 and 6 or 3 and 4. From the two options the second is obviously preferable (the GCD of 4 and 6 is 2 and therefore, in order to represent intersections, the difference between the residues of each module is restricted to even numbers). However, the reason to prefer factors 3 and 4 is not only their co-primality but something more essential. As I have mentioned, for Xenakis the decomposition of a modulus responds not only to a possibility but also to a necessity. A necessity to decompose a modulus into constituent elements (elementary moduli). This is not unlike the elementary role primes have in Number Theory.

The rationale behind the decomposition of a composite modulus is related to Prime Factorisation, as this is aimed at rendering a decomposed form of a composite number. In Sieve Theory the same principle is applied in order to render the building

[37] See FM 194-5 for the relationships that Xenakis establishes between the tempered chromatic scale and Peano's axiomatics of numbers.

blocks of a modulus. In order to unveil this rationale we need to refer to Number Theory's Unique Factorisation Theorem; according to this theorem, every natural number either is a prime number itself or can be written as a unique product of primes. Moreover, when these primes are written in a specific order, all composite numbers can be expressed in a unique form, which is called *canonical form*. Therefore, it is valid to say that any integer a larger than 1, can be uniquely written in the following form:

$$a = p_1^{k} \cdot p_2^{l} \cdot \ldots \cdot p_k^{m}$$

where $p_1, p_2, \ldots, p_k$ are prime numbers and $p_1 < p_2 < \ldots < p_k$

and this is called the canonical form of a.[38] Appendix 1 shows the canonical form of the first 200 integers.

Unique Factorisation, or Prime Factorisation, is the fundamental theorem of arithmetics and its aim is to represent a number in its unique form, decomposed to its constituent elements. This is evident in the Sieve of Eratosthenes in its sole preoccupation with primes – the building blocks of all numbers. The same is also reflected in Xenakis's Sieve Theory: any scale can be decomposed to its canonical form. The 'hidden' element he referred to is located there.

It is easily implied from the theorem that since two numbers a, b are coprime then a^m, b^n are coprime as well (with m, n being any positive integer other than 0). In the canonical form, all numbers ($p_1, p_2, \ldots, p_k$) are primes themselves and therefore any

[38] This theorem is valid when all numbers involved are positive integers.

combination of any two or more of them forms a set of coprime numbers. Consequently, any combination of any power of these numbers is still a set of coprime numbers. Thus we see that through unique factorisation co-primality is secured. Note that coprime numbers might not only be the ones derived from such a procedure. There might be several combinations of two or more coprime numbers. The introduction of Unique Factorisation (or Prime Factorisation) in the process of breaking down a modulus has a multiple significance; one aspect of this significance is that it secures co-primality.

The canonical form of 12 is $2^2 \cdot 3$. When these two factors correspond to moduli, the literal intersection would have to be $(2, r_1) \cdot (2, r_2) \cdot (3, r_3)$. It is obvious that $(2, r_1) \cdot (2, r_2)$ is not a valid option. In the case that $r_1 = r_2$ (either as such or after modular reduction) the intersection of this module with itself can represent only the original single module: $(2, r_1) = (2, r_2)$. In the case they are different the intersection is empty.[39] This would of course mean that the intersection is simply reduced at $(2, r_1) \cdot (3, r_2)$ which in turn derives from $2 \cdot 3$, the canonical form of number 6. We should therefore resolve any exponentials before treating prime factors as the elementary moduli of a period: $12 = 2^2 \cdot 3 = 4 \cdot 3$. Therefore, $(12, r) = (4, r_1) \cdot (3, r_2)$.[40]

[39] After modular reduction of the residues, there can be no intersection of modules that share the same modulus.

[40] Here I demonstrate only the decomposition of a modulus in order to give an *intersection* of elementary moduli. Of course, the same holds for the *union* of elementary moduli that derive from the canonical form of the period.

4.2 Limitations

There is one practical limitation regarding the application of the canonical form to the construction of a module. The resolution of all powers prior to the decomposition of the moduli is not possible for all composite moduli. It is exactly this possibility that Xenakis talked about. A modulus that cannot be decomposed is either a prime number, or a *prime power* (the power of a single prime). Such a number, i.e. a prime power, is 16: its canonical form is 2^4. Therefore, although modulus 16 is a composite number, it cannot be decomposed. (This is also apparent in Xenakis's program, discussed in Section 4.4.) Although they are not prime numbers, moduli such as 4 (2^2), 8 (2^3), 9 (3^2), 16 (2^4), 25 (5^2), 27 (3^3), 32 (2^5), or 81 (3^4), cannot be decomposed; we therefore need to use the original moduli: 4, 8, 9, 16, 27, 32, 81. These numbers are all prime powers. A sieve whose period is equal to a prime power is non-decomposable and therefore belongs to the same category as prime sieves. Thus, both primes and prime powers represent periods (or moduli) that are non-decomposable. Primes and prime powers are shown in Appendix 1 in bold typeface.

4.3 Types of Formulae

4.3.1 Decomposed Formula

The *decomposed formula* is the one that employs only moduli that are primes or prime powers. These are the elementary moduli that derive from the canonical form of the sieve's period. As I have shown, of this type is the formula that uses moduli 4 and 3 to express a sieve whose period is 12. A more complex decomposed formula is that of the sieve of *Nekuïa*. The sieve is shown in Figure 4.1 and its formula is[41]

$$8_0\cdot(11_0 + 11_2 + 11_4 + 11_5 + 11_6) + 8_1\cdot(11_2 + 11_3 + 11_6 + 11_7 + 11_9) + 8_2\cdot(11_0 + 11_1 + 11_2 + 11_3 + 11_5 + 11_{10}) + 8_3\cdot(11_1 + 11_2 + 11_3 + 11_4 + 11_{10}) + 8_4\cdot(11_0 + 11_4 + 11_8) + 8_5\cdot(11_0 + 11_2 + 11_3 + 11_7 + 11_9 + 11_{10}) + 8_6\cdot(11_1 + 11_3 + 11_5 + 11_7 + 1_8 + 11_9) + 8_7\cdot(11_3 + 11_6 + 11_7 + 11_8 + 11_{10}).$$

The two elementary moduli here are prime power 8 and prime 11 and the period of the sieve is 8·11 = 88 semitones; it produces 41 points which are shown in the matrix of Figure 4.2.

The combination of any two or more moduli does not necessarily suggest a decomposed formula, in the way I define it here. The elementary moduli must derive from the canonical form of the period (which itself is the unique decomposition of a number).[42] The above formula is the decomposed one because modulus 8 is a prime power and 11 a prime, and more specifically they are derived from the canonical form of

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[41] In the interest of clarity, the residues in this formula appear in subscript; thus, instead of (m, r), a module is notated m_r. I will notate all long formulae similarly.

[42] Recall that it is not necessary to use prime factorisation in order for the elementary moduli to be coprime. One could always construct a formula with elementary moduli that are not prime or prime powers; e.g. period 60 can be broken down to 5·12 instead of 4·3·5 ($60 = 2^2\cdot3\cdot5$). This decision depends on whether one wishes to express the period as the product (or the LCM) of two factors instead of three. This particular decomposition (i.e. 60 = 5·12) was used by Xenakis for the rhythmic sieves of *Persephassa* (1969, for six percussionists) (see Gibson 2003: 58ff).

$88 = 2^3 \cdot 11$. ‘Decomposed’ means that the combination of these elementary moduli reflects the Unique Factorisation Theorem; therefore, an intersection that involves moduli 4 and 6 is not a part of a decomposed formula (because 6 is neither a prime nor a prime power), whereas one that involves 4 and 3 is.

4.3.2 Simplified Formula

A *simplified formula* consists only of unions of single modules.[43] In the example of the diatonic scale it is the formula that is based on the periodicity (the octave). But a simplified formula does not necessarily represent a sieve according to a single modulus that corresponds to the period. This is the case with the Sieve of Eratosthenes. The simplified formula for this sieve employs single modules that correspond to each of the stages in the sieving procedure for integers 1-50: (2, 2) + (3, 3) + (5, 5) + (7, 7).[44] Here there is no obvious period; the theoretical period is the LCM of all moduli involved: 2, 3, 5, 7, which is equal to 210. It might be the case that a simplified formula is derived from a decomposed one, where there are several intersections with different moduli; each one of these intersections represents a single point and can be replaced by a single module with its own modulus (the product of the elementary moduli in each intersection). The

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[43] Ariza refers to ‘sieve models’ instead. What I refer to as ‘decomposed formula’ and ‘simplified formula’, he refers to as ‘complex sieve’ and ‘simple sieve’. He indicates that the latter model ‘fails to incorporate aspects of the original [the former]’ (2005: 44). The logical expression is the primary object in his discussion; he offers a complete software implementation and does not discuss Xenakis’s aesthetic of sieve-construction.

[44] This formula is a hypothetical one: it does not yield the points of the Sieve of Eratosthenes (i.e. the set of primes). Here it is used merely as an example.

result would be a series of unions of single modules that came up by the combination of all the moduli in each one of the original intersections. Starting with a formula that possibly contains all three logical operations, we can transcribe it into a series of unions of intersections. Each intersection might involve two or more moduli. We can arrive at the simplified formula by reducing each intersection to a single module. The modulus of each single module is equal to the LCM of the moduli in the intersection where it derived from. This simplified formula for the sieve of *Nekuïa* is produced by reducing all intersections into 41 single modules; and the result is of the same type with the one of the sieve of *Jonchaies* as (shown in Section 3.4.2.1): a series of unions of 41 modules with modulus 88:

$$88_0 + 88_2 + 88_3 + 88_4 + 88_7 + 88_9 + 88_{10} + 88_{13} + 88_{14} + 88_{16} + 88_{17} + 88_{21} + 88_{24} + 88_{25} + 88_{29} + 88_{30} + 88_{34} + 88_{35} + 88_{38} + 88_{39} + 88_{43} + 88_{44} + 88_{47} + 88_{48} + 88_{52} + 88_{53} + 88_{57} + 88_{58} + 88_{59} + 88_{62} + 88_{63} + 88_{66} + 88_{67} + 88_{69} + 88_{72} + 88_{73} + 88_{77} + 88_{78} + 88_{82} + 88_{86} + 88_{87}.$$

This type of simplified formula is based on the overall periodicity of the sieve. This is because all its moduli are equal to 88. If the decomposed matrix of Figure 4.2 appeared with some of its rows or columns complete, then the formula would also include moduli 8 or 11, which are congruent with the period. But nothing prevents one from constructing an alternative formula for the same sieve that includes alternative moduli that are not congruent modulo 8 or 11. The following simplified formula is equivalent to the preceding one, as it produces the same sieve, but with different moduli:

(24, 0) + (14, 2) + (22, 3) + (31, 4) + (28, 7) + (29, 9) + (19, 10) + (25, 13) + (24, 14) + (26, 17) + (23, 21) + (24, 10) + (30, 9) + (35, 17) + (29, 24) + (32, 25) + (30, 29) + (26, 21) + (30, 17) + (31, 16).

We see that although equivalent, this formula does not depend on the period of the sieve. Moduli 14 (2·7), 22 (2·11), 24 (2^3·3), 26 (2·13), 28 (2^2·7), 30 (2·3·5), and 32 (2^5) are congruent with the period (modulo a different value each), but moduli 19, 23, 25 (5^2), 29, 31, and 35 (5·7) are not. The theoretical period of this formula is the LCM of all its moduli: 943,814,071,200 semitones. This formula is constructed by applying the smallest possible modulus for each point of the sieve. Xenakis constructed an algorithm (and a computer program based on it) that produces a formula by assigning each point of the sieve the smallest possible modulus. I will analyse more options for constructing such a simplified formula, as well as explore the problem of the redundancy of simplified formulae, in Chapter 5. What is important here is that a simplified formula can have different forms, depending on what one wishes to indicate.

The two simplified formulae of the sieve of *Nekuïa* differ fundamentally in their moduli, by including moduli that are congruent with the period or not. Therefore there is another distinction between two types of formulae: the one that is based on the external period, and the one that ignores it. This distinction is parallel to the decomposed/simplified one: we can start with a simplified formula that ignores the external period and decompose each modulus, if it is a composite number:

$(8, 0)\cdot(3, 0) + (2, 0)\cdot(7, 2) + (2, 1)\cdot(11, 3) + (31, 4) + (4, 3)\cdot(7, 0) + (29, 9) + (19, 10) +$
$(25, 13) + (8, 6)\cdot(3, 2) + (2, 1)\cdot(13, 4) + (23, 21) + (8, 2)\cdot(3, 1) + (2, 1)\cdot(3, 0)\cdot(5, 4) +$
$(5, 2)\cdot(7, 3) + (29, 24) + (32, 25) + (2, 1)\cdot(3, 2)\cdot(5, 4) + (2, 1)\cdot(13, 8) +$
$(2, 1)\cdot(3, 2)\cdot(5, 2) + (31, 16)$.

The result is a formula that involves the operation of intersection and that includes only moduli that are primes or prime powers. It is a decomposed formula that is not based on the external period of the sieve. The information that a formula reveals when it is not based on the external period of the sieve is a very important issue which is pertinent both to the aesthetics of sieve-construction and to sieve-analysis.

4.4 Program: Generation of Points

Xenakis provided two computer programs for the treatment of sieves. They are found both in his article of 1990 and as Chapter XII of the 1992 edition of *Formalized Music*. Their titles are indicative of their function: ‘A. Generation of points on a straight line from the logical formula of the sieve’, and ‘B. Generation of the logical formula of the sieve from a series of points on a straight line’. I will refer to these programs using the labels ‘A’ and ‘B’.[45] Xenakis's demonstration of Program A reveals certain crucial aspects of Sieve Theory, that might not be very obvious in his writings. For this we need to look at the behaviour of the program. I will be discussing Program B in Chapter 5, as part of my methodology of sieve-analysis.

[45] Unfortunately, the code of both programs contains several typographical errors. A corrected version appears appended in Squibbs (1996: 291-303).

The intention to decompose a modulus into its prime factors is evident in Program A, used to generate a sieve from the logical formula. It goes through the following stages:

(a) The user is asked to enter the formula in the form of unions and intersections (of two or more modules).

(b) It reduces any groups of intersections to a single module, providing in this way the simplified notation for each intersection.

(c) It presents the simplified formula of the sieve as a series of unions of single modules.

(d) The program asks the user whether it should decompose the modulus into prime moduli.

(e) It displays the simplified formula again.

(f) Finally it generates the points.

Decomposition into prime moduli (stage [d]) might seem to be superfluous in the course of the program. Decomposition is part of a process that simplifies and then decomposes the moduli, only in order to display again the simplified formula of the sieve as unions of single modules. This does not affect the sieve in any manner and seems to be there just to provide the user with two alternative formulae. But greater emphasis seems to be given to the decomposed form of the modulus in each single intersection. The behaviour of the program is interesting when it is given a formula that includes moduli that do not derive from the canonical form: the program reduces the intersections of two or more modules into one and then provides a decomposition into prime modules. For example, if the input for an intersection involves moduli 4, 5 and 6, then the program suggests a reduction of these moduli to a single module with modulus 60 (the LCM). Afterwards, it suggests a decomposition into *prime* moduli 4, 3 and 5, and finally displays the simplified notation again, reduced to a single module with modulus 60 (the product of the coprime 4, 3 and 5). This is because 4, 5, and 6 cannot be derived from the canonical form of any number. Specifically, 6 is neither a prime nor a prime power.

At a first glance, it might seem strange that decomposition into *prime* moduli includes modulus 4, which is not a prime. Xenakis provides a demonstration of the program (as well as of the inverse one, discussed below). Recall that it was first published in the 1990 edition of his article and then in 1992 when the article was included in the revised edition of *Formalized Music*. In the former edition the program's prompts appear in French and in the latter in English. However, this is not the only difference; in the French edition we read: 'decomposition into *coprime* modules?' (Xenakis 1990: 69; italics added),[46] whereas in the English: 'decompression into *prime* modules?' (FM 278; italics added). Although the two expressions appear to be inconsistent, they are both correct. If we prompt the program to decompose modulus 12 the result involves moduli 4 and 3. At first, the French expression about co-primality seems to be true: 4 is not a prime, but the two moduli are co-prime. In this sense the English expression is not valid. But the two versions seem to refer to different sub-stages of the process. The French expression is true only after having resolved the powers of the primes as found in the canonical form. In other words, the French expression is true because for Sieve Theory primes and prime powers are equivalent.

[46] '[décomposition] en modules premiers entre eux ?'. According the terminology I have chosen for this dissertation, the word 'modules' (both in the French and the English versions) should read as 'moduli'.

As I have demonstrated, co-primality is not achieved through a free selection and combination of numbers, but is arrived at through the canonical form of the modulus. The canonical form of a number, refers to the *order* prime factors appear. It is not merely an unordered collection of its prime factors, but these are put in the order demonstrated in Section 4.1. This is the reason why the program's output at this stage is 4·3 and not 3·4 ($12 = 2^2 \cdot 3 = 4 \cdot 3$). Therefore, the English expression refers to the stage before the actual output, which is no other than the canonical form after the resolution of the exponentials. So, in the latter expression we should read $12 = 2 \cdot 2 \cdot 3$. This stage (the stage of the canonical form) is neither explicit in the program, nor in the theory as demonstrated in the article. It remains a hidden element, but implied both in the former and the latter. Consequently, the (more recent) English expression is also true.

4.5 Sieve Theory and Sieves

Although Xenakis's article dates from 1990, the first extended reference to Sieve Theory is found as the final section of 'Towards a Metamusic'; an article of 1967, whose unpublished manuscript dates from 1965 and is titled 'Harmoniques (Structures hors-temps)' (see Solomos 2001: 236 & Turner 2005). The more recent publication reflects a re-orientation of Xenakis's attitude towards sieves; or at least an increased interest that led him to publish an article solely devoted on sieves. There are certain differences between the two demonstrations of the theory. The section of 'Metamusic' is titled 'Sieve Theory' whereas the 1990 article simply 'Sieves'. The latter article includes a thorough explanation of the theory, but is also aimed at a practical demonstration of creating, analysing and treating sieves; this was realised by the inclusion of the two computer programs. One might argue that computers were not as widely accessible in the 1960s as in the 1980s and this an obvious reason for the inclusion of the programs in the more recent publication. However, the analytical algorithm of ‘Sieves’ (FM 274-5) does not necessitate the use of a computer.

These two writings reflect two periods of sieve-theoretical and compositional attitude. During the first, the type of formula he used was based on the external period. Xenakis would rely much more on the decomposition of the sieve’s period, in order to study its structure and generate transformations. The alternative of a simplified formula appears only in the 1990 article (both in the algorithm and the two programs). More than just a progression from the decomposed to the simplified formula, it is a matter of progressing from the a certain kind of sieve-aesthetics to another.

4.6 Symmetries/Periodicities

Throughout most of Xenakis’s implementation of Sieve Theory, the internal structure of a sieve was more important than its external aspect. The intervallic structure is highly irregular and asymmetric, while the period of the sieve is rarely intended to be audible. Sieve Theory was developed in order to study internal symmetries, and reveal more hidden ones. It is important to underline that these symmetries are not found in the intervallic succession as such. As I have mentioned, Xenakis demonstrated that an elementary modulus is a kind of tempered chromatic scale, with unit an interval other than the semitone (FM 195). Furthermore, the elementary moduli are thought both as symmetries and as periodicities (FM 270). In a decomposed formula these periodicities are shown to coincide (intersection) and join (union) in order to produce the points of the sieve. In a simplified formula the elementary moduli have the form of periodicities that are joined by union; in other words, they would all be complete columns and rows in a hypothetical, multi-dimensional decomposed matrix, of dimensions equal to the total number of moduli. In musical terms, a combination of elementary modules in a simplified formula would be a combination of several chromatic scales with different units (other than the semitone) and different starting points. This will be much clearer after a more elaborate exploration of the transition to a 'simplified' conception of sieves.

The turning point in Xenakis's evolution in sieve-based composition is marked by his orchestral work *Jonchaies* (1977). In the preface to the score he clearly states that the work 'deals with pitch "sieves" (scales) in a new way'. As I have already shown the sieve of this work is prime asymmetric, which means that a decomposed formula is excluded (its simplified formula is shown in Section 3.4.2.1). It is the first time that such a type is used and this clearly verifies that Xenakis did not rely on the decomposition of the period anymore. The sieve in question is a rare case where the period is so small that is easily audible. However, the transition to new aesthetics is seen in the way the sieve is used inside time. In *Jonchaies* it is treated with a particular technique which Makis Solomos has termed 'halo sonority' (see Solomos 1996: 84). Xenakis himself had not commented extensively on this technique but described only in the preface to a work from the same period, *Nekuia* (1981), as 'multiplicities of shifted melodic patterns, like in a kind of artificial reverberation'. The result is a kind of heterophony, where the outcome is not of any traditional treatment of pitch such as melody, polyphony, etc; not even of the type of set-theoretical treatment that Xenakis had used in *Herma* and which he termed 'symbolic music'. Indeed, for Xenakis sieves became timbres rather than pitch sets or scales.

The idea of 'multiplicities of shifted patterns' can be seen in the structure of the sieves following *Jonchaies*. These sieves share a certain aesthetic: they are characterised by an irregular distribution of a set of intervals which are dispersed over the whole range of pitches. The size of these intervals is contained between a semitone and a major 3rd; furthermore, in most sieves there are no more than three consecutive chromatic elements. The selection of these intervals is related to an aesthetic criterion that seems to have influenced most of Xenakis's recent output: the construction of sieves is inspired by the Javanese *pelog* with its interlocking fourths; hence the characteristic interval succession 1 4 1 semitones that is abundant in his sieves of the 1980s. This type of intervallic structure was used for the first time in the sieve of *Jonchaies*. In an interview of 1989 Xenakis said that, further to his inspiration by the pelog, he was interested in choosing intervals that produce some tension. Tension for him is conceived as a kind of objective category and it can be produced through

> the opposition of large and small intervals – that is, the contrast between something very narrow and something much larger. To maintain this tension along the sieve – in other words in the scale you have chosen – is a tall order. It is also an intriguing problem: none of the parts is to be symmetric – that is, periodic; nor are the ranges to be periodic as compared to the higher or lower ranges, maintaining tension all the while in a different way (Varga 1996: 146).

The opposition of large and small intervals is exemplified in the intervallic structure of the two interlocking fourths that Xenakis mentions, and it is precisely this idea of the interlocking fourths that suggests a simplified formula. The intervallic succession of 1 4 1 would be produced by points {0 1 5 6}, which in turn are produced by modules (5, 0) and (5, 1); i.e. two shifted perfect 4hs. Nothing prevents one to add more 'interlocking' modules that extend throughout the whole range of the sieve. Each of these modules would thus be equivalent to a different chromatic scale. The result would be a multiplicity of shifted chromatic scales (each having a different unit distance). A simplified formula would be more indicative of such a multiplicity of elementary modules (periodicities). It is exactly the same idea that Xenakis used for the basic principle of *Jonchaies*. In the following quotation he talks about the rhythmic structures, but the same was applied to the pitch domain as well:

> We can illustrate regular events by points an equal distance apart. On a second, lower parallel line, more points represent other regular patterns with a different time unit, so they are shifted with respect to the first line's points even if they start together. This procedure can be repeated with regular points on other lines. When we hear all these lines together, we obtain a flow of events which consists of a regular intervallic series, but which as a whole is impossible to grasp. Our brain is totally unable to follow such a complicated flow (Xenakis 1996: 148).

Unlike the ones that follow, the sieve of *Jonchaies* is not itself constructed according to this principle. However, it marks Xenakis's general approach to sieves as timbres. The inspiration for this orchestral work, the composer comments, comes from the results of his research in sound synthesis for *La légende d'Eer*. Especially towards the ending of *Jonchaies* there is a striking aesthetic resemblance with this electroacoustic composition, composed in the same year (1977). In the preface to the score of *Jonchaies*, apart from the comment on the novelty in the treatment of sieves, Xenakis briefly describes his inspiration by his view on sound synthesis: 'one starts from noise and […] periodicities are injected to it'. Admittedly, this is a possibility offered by stochastics and is in particular related to his application of random walks and Brownian movements, which exemplified the reversal of traditional sound synthesis. However, the inspiration from electroacoustic to instrumental composition (and vice versa) can be seen, metaphorically, in relation to Sieve Theory as well. The idea of individual periodicities is not extremely different from the original idea of stochastics: individual elements are distributed in such a way that are not intended to be perceived as such, but to create a 'multitude of sounds, seen as a totality' (FM 9).

At the end of his article, Xenakis argues that the inverse, that is the application of Sieve Theory to sound synthesis, is 'quite conceivable' (FM 276). Using the metaphor of the injected periodicities we could say that the simple modules in a simplified formula represent the individual, internally hidden periodicities (or symmetries, or regularities) of a sieve. In the same sense, it is more than conceivable, instead of starting from noise, to start from the total chromatic throughout the audible range and 'inject periodicities' to it in order to construct a sieve that produces a certain timbre. A formula that accounts for these periodicities is the one whose starting point is not the overall period of the sieve. It might be the case that the inner periodicities also account for the external periodicity: this is true for symmetric sieves that can be formed by the union of simple modules. But this is an extremely simple case. A formula can be given a form such that it represents the multilayered structure of a sieve. When a formula ignores the overall period, the elementary moduli are applied straight to the points of the sieve. This type is the one that reflects Xenakis's more recent approach to sieve-construction, where elementary modules represent 'chromatic' scales, periodicities, or symmetries. Symmetry here has a general, abstract meaning: it stands for regularity in general. If each module in a formula represents a regularity, then every irregular scale (in its abstract form, an irregular arrangement of points on a straight line) can be broken down into a multiplicity of regularities. This is also the fundamental idea behind the Sieve of Eratosthenes. The sequence of prime numbers has no known pattern; it appears as a purely random, irregular sequence, and the Sieve of Eratosthenes provides a simple method of achieving this irregularity by an algorithmic process that deduces all regular patterns.

This part presented Xenakis's method initially from a purely theoretical perspective and later from a more practical one that also involved matters of aesthetics. The theoretical approach of Chapter 3 concerned mainly the structural characteristics of sieves in general and the possible methods of their transformation (metabolae). Chapter 4 presented the relationship of Sieve Theory and Number Theory. As I have shown, this relationship affects the theoretical representation of sieves (formula), which in turn is related to the structural characteristics of sieves as well as the aesthetic approach to sieve construction. This approach reflects Xenakis's use of the simplified formula in his later period. We saw, in particular, that Xenakis progressed towards a more practical way of achieving irregularity: the more the number of regular simultaneous events, the more irregular the overall effect. The following part of this thesis is preoccupied exactly with this idea of multiple regularities, employed firstly in the development of an analytical methodology (Chapter 5) and secondly in its application to the analysis of the sieves of the later period (Chapter 6).

PART III

5 Methodology

5.1 Inner Periodicities and Formulae Redundancy

One of the basic problems in Sieve Theory is the redundancy of formulae for a single sieve. As I showed in Chapter 4, there might be different simplified formulae for a given sieve. That is, whereas the problem of decomposed formulae redundancy is overcome by prime factorisation, the redundancy of simplified formulae is not as straightforward to resolve. This depends on the type of information one wishes the formula to provide and consequently the properties of a sieve one wishes to consider. The simplified formula can either be based on the overall period of the sieve, or not. In the latter case, the formula reveals other periodicities, not congruent with the overall one. But first we need to probe the information that a simplified formula offers in relation to the decomposed one. In fact, the simplified formula derived from the decomposed one provides the same, if not less, information than the decomposed one. When the simplified formula is derived from the decomposed, we replace all intersections of elementary moduli with the period (the LCM of the elementary moduli); the unions of complete modules, if they exist, keep their elementary modulus. The result is a series of unions of modules. But if there are no complete modules (complete rows or columns in the decomposed matrix), the moduli of all (incomplete) modules are equal to the period; thus they produce only a single point (in the range of the period). Therefore, such a simplified formula fails to reveal any aspects of the sieve's internal structure, as it represents each single point with a single module (see the simplified formula of the sieve of *Nekuïa* in Section 4.3.2, p. 89). The simplified formula then, provides different information than the decomposed only when it is not based on the overall period of the sieve. We therefore need to determine another level of periodicity, different from the overall period of the sieve.

The period of a sieve is external to it and symmetry is an internal property; but when a sieve is asymmetric, 'a more hidden symmetry' might exist. This hidden symmetry has the form of 'moduli (symmetries, periodicities)' (FM 270). The *external periodicity* is none other than the sieve's overall period. A simplified formula based on it presupposes the determination of the sieve's period in advance; the result is a notation that simply represents the sieve according to its period. In general, a simplified formula based on external periodicity is one that indicates the period by including only moduli that are congruent (or equal) with the period. In this sense, when a simplified formula is based on external periodicity it belongs to the same category as the decomposed one – at least as far as the information it provides is concerned.

The *inner periodicities* of the sieve are shown by a simplified formula in the form of elementary modules. Each one of these elementary modules has a single modulus whose multiples produce some of the points of the sieve. These points are conveniently shown by a matrix, which I will refer to as *simplified matrix*. To construct a simplified matrix we write all the sieve's elements in the top row and all the modules of the simplified formula in the leftmost column (such that this column represents the simplified formula).[47] The intervallic structure of the sieve (in semitones) is shown under the actual

[47] The simplified matrix, although not termed as such, has also been used by Gibson in his demonstration of Xenakis's computer program B (2003: 55).

points of the sieve. We then mark all the cells where the elements of each module (rows) meet a point (columns) of the given sieve.

The simplified matrix of the sieve of *Nekuïa* is shown in Figure 5.1. The label (*M*, *I*, *R*) stands for: *Modulus*, *Initial point*, *Reprises of the Modulus*. The matrix shows that each of the twenty modules cover several points of the sieve. When a simplified formula is based on the inner periodicities there is no information indicating the overall period, apart from the possibility of assuming a period as the LCM of all moduli included in the formula. This means that the focus is now on the sieve's internal structure. It is an *inner-periodic* simplified formula. Therefore, the simplified formula is valuable only when it represents a sieve whose external periodicity is not taken into account. When the period is not known (or not taken into account), a simplified formula is naturally inner-periodic. Note that this simplified inner-periodic formula can be used to deduce an inner-periodic decomposed formula (cf. the simplified and decomposed formulae of the sieve of *Nekuïa* in Section 4.3.2, pp. 90, 91). But we could not arrive at an inner-periodic decomposed formula without first representing the inner periodicities as simple modules.

The process of constructing an inner-periodic formula ignores the period and takes into account only the intervallic structure of the sieve. The redundancy of inner-periodic formulae can be overcome by checking every single point of the sieve and assigning to it the smallest possible modulus. More specifically, we can find for each point, the module with the smallest modulus, that either starts on this point or produces it later. These are in fact two different approaches. In the former, every point is considered a point of departure; we then find the smallest modulus that starts on this point. In the latter, every point is considered as part of a module; we then find the module with the smallest modulus that includes this point (this point might not necessarily be the starting point of the module). These two methods, although different, share the same principle that is intended to produce a unique formula: they apply the smallest possible modulus. In fact the former method was for Xenakis an earlier stage before arriving at the latter, implemented in his final analytical algorithm.

5.2 Construction of the Inner-Periodic Simplified Formula

By progressing to a simplified notation, Xenakis showed a new way of revealing the hidden symmetry of sieves, which are now viewed as multiplicities of periodicities. In order to demonstrate the progression to this inner-periodic conception of sieves, let us take for example the sieve of *Akea* (1986, for piano and string quartet) as found in Xenakis's pre-compositional sketches.[48] As I have mentioned, this sieve derives from that of *Nekuïa*, but for the moment I will analyse it as such, using it as an example that shows Xenakis's own treatment of it. The sieve is shown in Figure 5.2. It consists of 37 points and its range is 80 semitones (6 octaves and a minor 6th). The intervallic structure of the sieve is asymmetric and consists of intervals between a semitone and a major 3rd (with one exception, the perfect 4th); as with most sieves of this period, there are no strings longer than three semitones in its intervallic succession. Therefore, the greatest interval in the complement of the sieve would also be a major 3rd. In *Akea* the sieve is not used in any way that might reveal the existence of a periodicity; this is confirmed by the fact that,

[48] This sieve was also used in *Ata* (1987, for orchestra). For an analysis of the inside-time treatment of the sieve of *Akea* see Section 7.2.

although for the original sieve of *Nekuïa* Xenakis used a decomposed formula (as I will show later), for *Akea* he used only an inner-periodic simplified formula.

The most common use of the external periodicity in the sieves of the later music is related to cyclic transpositions. In order to perform a cyclic transposition, the range of the sieve can be considered as a period; thus, when the sieve is transposed the elements that exceed its range would be re-located inside its original range. In the case of the sieve of *Akea* we would have to consider 80 as the period and decompose it according to its canonical form: $80 = 2^4 \cdot 5 = 16 \cdot 5$. Thus a 16 by 5 matrix will indicate the sieve's structure. The decomposed matrix for the sieve of *Akea* is shown in Figure 5.3. Number 0 is here set to correspond the lowest element of the sieve, C1. But in *Akea* there are no cyclic transpositions of the sieve, and therefore there is no evidence of such a period. Furthermore, the decomposed matrix shows the hidden symmetry only by two moduli; and the fact that there is no complete column or row in the matrix accounts for the incompleteness of the two inner periodicities of 5 and 16 semitones (perfect 4th, and 8ve+M3rd). The most populated row (5, 0) shows that the periodicity of a perfect 4th exists between points 0 and 10, and then between points 40 to 80. The most populated column (16, 7) shows the periodicity of 8ve+M3rd missing element 39. The decomposed matrix shows these two periodicities starting at all possible points, but never manifesting themselves completely. This accounts for the asymmetric structure of the sieve (in terms of moduli 5 and 16). Had some columns or rows in the matrix been complete, this would suggest a certain higher degree of hidden symmetry. However, all analytical conclusions on its symmetry would still depend on the two periodicities (moduli) that the external periodicity indicates. But the external periodicity of a sieve does not prevent noncongruent periodicities of taking place internally. For example the periodicity of 15 semitones starting at point 10. That is, the internal structure is not seen as a multiplicity of elementary, individual periodicities. This is shown by the inner-periodic simplified formula.

The sketches of *Akea* provide the simplified formula of the sieve, which is shown in Figure 5.4. There are 17 modules in the sieve and they share 10 moduli. The modules are classified according to the size of their modulus and are shown under the label (*M*, *I*, *R*). We see that this formula includes modulus 5, which was shown to be a constituent part of the hidden symmetry in the decomposed matrix too. But now, only its continuous segment is shown – the one between points 40 and 80. The initial segment of this periodicity, between points 0 and 10 is not shown, as it is interrupted at point 15. One significant aspect of this specific formula is that the residues are not reduced according to the modulus. They are now simply considered as starting points (*I*). As the starting point of a module (i.e. of an inner periodicity) can be located anywhere in the sieve, the starting point is kept as such even if it is greater than the modulus. Thus, instead of substituting (5, 40) with (5, 0) [since 40(mod5) ≡ 0], Xenakis indicates that a periodicity of 5 semitones, or of a perfect 4th, is initiated at point 40, which is E4, and extends to the upper edge of the sieve. This is one of the aspects of Xenakis's more practical approach to sieve-construction, that characterises his more recent output. The third entry in the brackets (*R*) shows the number of repetitions of each module.[49] The leftmost column

[49] The number of points covered by a module is then $R + 1$. This is slightly different from Program B, which uses R to denote the *number of points* each module covers (in our case $R + 1$). Following Xenakis's own practice in his pre-compositional sketches, I will use R to denote the occurrences of a modulus, instead

shows the canonical form of each modulus, and the rightmost column the interval each modulus represents in musical terms. The numbers in grey denote the number of modules. The table offers synoptic information on the periodic intervals that make up the sieve.

5.3 Analytical Algorithm: Early Stage

The way Xenakis arrived at the simplified formula of the sieve of *Akea* is not identical to the algorithm he presented in Xenakis (1990). It is, in a way, a precursor of this algorithm, and shows a slightly different aspect of the sieve. This earlier algorithm goes through the following steps:

(a) Each point is considered as a point of departure (I) of a modulus (M). We start testing the first point (I_n) with $M = 2$ and check if:

  (i) its multiples produce only points (greater than the starting point) that belong to the given sieve, and

  (ii) it produces at least one of the not-yet-covered points of the given sieve.

(b) If (i) is not satisfied we pass on to $M + 1$. If it is satisfied we keep the module and check if (ii) is satisfied: if yes, we keep the module and pass onto the next point (I_{n+1}); if not, we ignore the module and pass onto I_{n+1}.

(c) We stop when each point of the sieve has been covered by a module.

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of the number of points covered. This indicates more effectively the contribution of each modulus to the inner-periodic structure of a sieve.

This algorithmic process, as it was applied in the calculation of the formula of the sieve of *Akea*, is provided in a separate page in the sketches and is here reproduced in Figure 5.5. The first stage consists of considering each point of the sieve as a point of departure of a modulus. For this, we write down all the points of the sieve and, one by one, we find the smallest modulus that departs from it. For point 0, the smallest modulus that meets points of the sieve is 25. The points that it covers are shown in the table at the right of the module: 0, 25, 50, 75. The not-previously-covered points that a module covers are shown in bold typeface. For the first three modules, all the points covered are in bold, because none of it has been covered in a preceding step. The fourth module, (14, 9) covers six points of which only four have not already been covered: 9, 37, 51, 65. On the contrary, points 23 and 79 have already been covered by modules (18, 5) and (24, 7) respectively. Modules that are subsets of already-found modules are shown in grey typeface, with the indication of the module of which they are a subset. For example, the smallest modulus that starts at point 23 is 14; but (14, 23) is a subset of (14, 9), shown at its left; i.e. all the points that (14, 23) covers have already been covered by its superset, (14, 9). Therefore, (14, 23) is rejected and we pass on to test the following point of the sieve, 25. Similarly, the smallest modulus that starts at 25 is 15, but (15, 25) is a subset of (15, 10).

Although the process can stop when all the points of the sieve have been covered by a modulus, Xenakis did in fact test every single point of the sieve of *Akea*. This is an indication that he calculated the formula entirely manually, finding the smallest modulus for every single point of the sieve and afterwards checking how many not-yet-covered points each module covers. The numbers in bold typeface in the column under the label 'Final Choice' denote the choice of modules Xenakis made for this specific sieve. Note that the last module he chose (14, 52) is the one that covers the only not-yet-covered point remaining, 52.

The modules that did not make it to the final choice are of two kinds: the ones that are included in already-found modules (grey typeface) and modules that simply do not cover any of the not-yet-covered points (normal typeface). A subset of an already-found module has the same modulus, which means that its starting point is a multiple of the modulus of the already-found module – such as (14, 23) and (14, 9). A module whose every point has already been covered by previous ones can have a modulus of any size. As with subsets of already-found modules, these modules can be located anywhere in the process. That is, they can be found either before all points have been covered or after. In Figure 5.5 there is one such module before the last remaining point of the sieve is covered by (14, 52); this is module (15, 41). As concerns the subsets of already found modules, we see that, peculiarly, Xenakis kept (19, 46) in the final version, although this module is included in (19, 27). There is no apparent reason for such a decision, apart from a mistake while transferring the modules to the final formula. Of course, the inclusion of (19, 46) in the formula does not alter the resulting sieve: the periodicity of 19 semitones starting at point 46 is present in the sieve; it covers pitches A#4 and F6. We should therefore exclude module (19, 46) from the final version of the formula and allow for a minimal representation of the sieve, by sixteen modules.

The simplified matrix based on the above formula is shown in Figure 5.6. The modules are shown in the order they were found during the calculation of the formula. In fact this is the order based on the size of *I*, since the formula was constructed by assigning a modulus to the lowest point of the sieve and proceeding to the next one. So each row of the table shows the assigning of a modulus to a starting point. To find the next point assigned a modulus move one row down and look for the next column that is marked. For example, after having assigned modulus 19 to point 22, the next point assigned a modulus was point 27 (again modulus 19). The intervening points, 23 and 25, have not been assigned a module because the smallest modulus that starts at each one of these points is equal to the modulus assigned to each one of them in a preceding step (moduli 14 and 15 respectively). These points would have been only assigned a new modulus if this was smaller than the already-assigned ones. In the formula for the sieve of *Akea*, the greatest number of points covered by a single module belong to module (5, 40), which covers 9 points; this means that a periodicity of a perfect 4th (5 semitones) is repeated 8 times; therefore we note (5, 40, 8).

5.4 The Condition of Inner Periodicity

The modules in a simplified formula of a sieve can be only considered as inner periodicities when they repeat at least twice. In other words, a module must cover at least three points in order for the modulus to repeat twice. That is, three equally distant elements produce two equal intervals (modulus) that make it possible to compare them and identify them as (two occurrences of) a periodicity. This is precisely Xenakis's view of sieves as outside-time structures. Recall the stages of temporal perception he referred to in 'Symbolic Music': three successive events *a*, *b*, *c*, 'divide time into two sections [that] may be compared and then expressed in multiples of a unit' (FM 160). Although he talks about time here, the temporal algebra time-intervals require is identical to that of the outside-time algebra (Section 1.3). Consequently, in order for $R \geq 2$ the modulus must be of a size less than half the distance between the module's starting point and the highest point of the sieve. If n is the highest point of the sieve, this *condition of inner periodicity* is formulated as follows: for each module in a simplified formula it must be true that

$$M \leq \frac{n - I}{2}$$

in order for $R \geq 2$. If this is true for all modules in a sieve, then this sieve is *inner-periodic*. In the sieve of *Akea* the highest point is $n = 80$. If the starting point of a module is 0, then according to the condition of inner periodicity, it must be true that

$$M \leq \frac{80 - 0}{2} \Rightarrow M \leq 40 .$$

Similarly, a module starting at point 52 can have a modulus up to (80–52)/2 = 14 semitones. This is found in the last entry in the matrix as module (14, 52). In these two cases, a modulus greater than 40 or 14 semitones respectively, cannot be considered as an inner periodicity of the sieve. The sieve of *Akea* is shown to be carefully constructed to include only moduli that repeat for at least twice. This corresponds to a an 'inner-periodic' conception of sieves that Xenakis maintained throughout his application of Sieve Theory in his later music.

5.5 Inner-Periodic Analysis

Xenakis's calculation of the formula enables a certain type of analysis, facilitated by the existence of smaller sizes of moduli. As mentioned above, the greatest number of repetitions of a single module in the formula of the sieve of *Akea* belong to the interval of the perfect 4th. The sketches suggest that Xenakis was pleased by the perfect 4th having a decisive contribution to the sieve's inner-periodic structure. Another characteristic of the sieve he noted was that there are no moduli of octaves, except perhaps of the three double octaves shown in Figure 5.7.[50] If we add all the repetitions of a modulus found in more than one module, we see that the interval of 8ve+5th is found 10 times in the sieve [if we exclude module (19, 46) in the formula of Figure 5.4]. However, this does not suggest a more decisive contribution of the interval of 8ve+5th to the sieve's structure other than the perfect 4th, since it is not a case of successive repetitions of an interval. Such information is valuable only in order to give a more general character of the sieve's periodic intervals. Moreover, the repetition of a perfect 4th eight times is a much more perceptible characteristic because of its relatively small size. The largest interval in the (successive) intervallic structure of the sieve is the major 3rd (with one exception) and therefore the perfect 4th is the smallest interval one can expect to find as a modulus (in the sense that a module of a major 3rd would confine the sieve's intervallic structure to intervals of minor 3rd or smaller).

[50] 'Peu d'octaves (3 doubles) etc. Mais (8) pas mal de 4tes et des 5tes'. In my translation: 'Few octaves (3 doubles) etc. But (8) not bad in 4ths and 5ths'. Xenakis, Iannis, Pre-compositional sketches of *Akea* (Bibliothèque Nationale de France).

Xenakis put forward the idea of the decomposition of a modulus as a method to compare different sieves: in particular, we can study their degree of difference and define a notion of distance between them. Distance can be expressed as multiples of a unit, which is not pre-defined. A modulus – a ‘chromatic’ scale with unit distance other than the semitone – might be a composite number, which can be decomposed in order to allow comparison between all inner periodicities in the intervallic structure of a sieve. If there are moduli with small sizes, we can express greater moduli as multiples of the former.

Distance between two moduli can be defined through their decomposition. By this means we can study the structure of a single sieve before moving on to the comparison of different ones. In the sieve of *Akea* the interval of the perfect 4th has been shown to be the most characteristic. The main reason for this is that it is the smallest modulus in the simplified formula. In general, the unit distance between congruent moduli can be defined by their GCD. An easy way to find the GCD is through the canonical form: the GCD of numbers written in their canonical form is equal to the product of their common factors and with each factor taken in the lowest power that appears. For example, the canonical form of 12 is $2^2 \cdot 3$ and of 18, $2 \cdot 3^2$. Their common factors are 2 and 3. The lowest power these appear in either forms is 2^1 for factor 2, and 3^1 for factor 3. Consequently the GCD of 12 and 18 is equal to $2^1 \cdot 3^1 = 6$. For numbers 12 and 18, 6 can be defined as the unit distance: the two numbers are 1 unit apart. This can be also useful in grouping more than two moduli in a formula.

The interval of a perfect 4th is a prime number (5) and can be found as a constituent element of other moduli in the sieve of *Akea*. These moduli are the ones whose canonical form includes factor 5 (i.e. they are multiples of 5). These are moduli

15, 20, and 25. Therefore, four moduli out of ten (there are ten moduli and sixteen modules) are congruent modulo 5. These are the intervals of 4th, 8ve+m3rd, 8ve+m6th, and 2·8ve+semitone. If the unit distance of these moduli is the perfect 4th, then their size can be thought of in terms of how many 4ths they contain. Therefore, the above intervals can be written as follows: 4th, 3·4th, 4·4th, and 5·4th; these would be the multiples of a unit Xenakis referred to. Figures 5.8 and 5.9 indicate all the four moduli that are congruent modulo 5 – the perfect 4th as found in module (5, 40). In Figure 5.9 the perfect 4th is shown on a first level above the pitches; below the pitches the 8ve+m3rd; on a second level above the perfect 4th, the 8ve+m6th; and on the top level the 2·8ve+semitone. The four moduli are found in 6 modules in total: (5, 40), (15, 10), (15, 36), (20, 31), (25, 0), and (25, 16). These 6 modules produce 18 points of the sieve. In particular, the modules congruent modulo 5 are more than the one third of the modules in total (16 modules), and produce almost half of the sieve's points (18 out of 37). Another interesting observation is that from these eighteen points, eight are produced by the perfect 4th itself. The importance of the this interval is found elsewhere in Xenakis's music and comments. As the analysis shows, this interval is also found in the inside-time treatment of sieves (see especially Section 7.4).

The grouping of moduli could potentially be carried out for other intervals taken as the unit distance. For example, there is another combination of four moduli that are congruent with the interval of a tone. These are moduli 14, 18, 20, and 24, which correspond to the following intervals: 8ve+tone, 8ve+tritone, 8ve+m6th, and 2·8ve. In terms of the tone as a unit distance, these intervals can be written as 7·tone, 9·tone, 10·tone, and 12·tone. However, the tone does not appear in the sieve as such. If it appeared as part of a module it would have reduced the intervals of the sieve's intervallic structure (or of a part of it) to a succession of semitones and tones. Unlike the tone, the perfect 4th is present as an inner periodicity that extends for half of the sieve's range, and secondly, in the sieves of Xenakis's more recent output, the intervallic structure contains intervals up to a major 3rd.

Apart from modulus 5, prime numbers in the sieve of *Akea* are moduli 17 (8ve+4th), 19 (8ve+5th), and 23 (8ve+M7th). As a general characteristic, when several elementary moduli produce a sieve these are expected to be coprime; the unit distance between coprime moduli is the semitone (their GCD is 1). The attempt to define a unit distance is aimed at grouping moduli according to a unit greater than 1. But such a procedure is not irrelevant from decomposing a modulus into its constituent elements. Therefore, a unit must not be decomposable itself. As in the Sieve of Eratosthenes, the possible intervals for the unit distance must be sought among the primes. If for example, there was a module covering a great range of the sieve with modulus 6 semitones (a tritone), then this would have to be decomposed to $2 \cdot 3$, and then decide which of the two intervals (the tone or the minor 3rd) would be the unit distance. This of course, would raise questions of the validity of a unit distance that is not present on the sieve's structure. Therefore, a unit distance that is not present could be thought of as a deeper level of symmetry/periodicity. On the other hand, Xenakis analysed his sieves according to the appearance of the modules as they were given by his algorithmic analysis. This description of a sieve depends upon general characteristics such as the number of modules, the number of points in the sieve, the number of repetitions of a modulus, or the average size of the moduli.

Inner-periodic analysis offers one way of grouping moduli as multiples of a unit. However, this unit might not always be present. And if it is present, it must be sufficiently small (as modulus 5 in the sieve of *Akea*), so that it can be thought as the unit of greater moduli. If such a unit is not present, inner-periodic analysis cannot offer more information than the formula itself or a matrix-representation of the sieve. This is presented here as one way of employing the early version of Xenakis's analytical algorithm, as it favours smaller sizes of moduli when examining higher points of the sieve (I will come back to this property later). Xenakis's comment on the 8 repetitions of modulus 5 is related to the structure of the sieve as an intervallic succession whose greatest interval is a major 3rd; but it is important to note that since analysis is based on the formula it depends on the algorithm used. Xenakis's algorithm does favour smaller sizes of moduli and might facilitate comparison of other moduli as multiples of the smallest. But his final version does not allow for such small moduli; therefore, what I have termed here as inner-periodic analysis is dependant on an analytical process that Xenakis himself (gradually) abandoned up to 1990.

5.6 Interlocking Periodicities

The method Xenakis used to produce the formula of the sieve of *Akea* differs from the final algorithm of 1990 in one significant respect, already mentioned: the starting point of a module is not necessarily smaller than the size of the modulus. This contradicts the basic property of modular arithmetics, but reflects a more practical way of analysing sieves. As I will demonstrate, the final algorithm detects only modules with $M > I$. In any case, we see that Xenakis's suggestion to study the hidden symmetry of a scale is fundamental both when the external period is taken into account and when the analysis is based on the inner periodicities (inner-periodic analysis). In the former case the period is decomposed into two or three factors, thus limiting the study of symmetries to two or three elementary moduli. For example, the decomposition of the octave in the major diatonic scale (Section 3.2.2) reveals that there is an elementary periodicity of 3 semitones between pitch-classes B, D and F; E and G; and A and C; and an elementary periodicity of 4 semitones between C and E; F and A; G and B. These two elementary moduli (periodicities) intersect and can be heard in any direction, either upwards or downwards.[51] This is because the decomposition of a period into elementary ones does not depend on any notion of direction. With inner-periodic analysis this is not the case. We start at the lowest point of the sieve and look for the smallest modulus that departs from this point. If a modulus produces a point (beyond the departure point) that is not included in the sieve, it is discarded. In order for a modulus to be valid, all of its multiples need to meet a point of the sieve: in other words, it needs to extend to the upper edge of the sieve.

Let us take the example of the cyclic transpositions of the major diatonic (i.e. the different modes on the white keys of the piano); although it is a symmetric scale (its intervallic structure is palindromic under cyclic transposition) we can still use it as an example and look at the interval of the minor 3rd. The modulus of 3 semitones, between pitch-classes B, D, and F, is not in all cases validated as an inner periodicity. It is

[51] Recall that a sieve-theoretical examination of the major diatonic ignores the cadential, inside-time relations between the degrees.

validated only when its last point, F, is placed at a distance from the upper edge of the sieve smaller than 3 semitones:

A, **B**, C, **D**, E, **F**, G.

On the contrary, when F lies at a greater distance from the highest point of the sieve, it is not validated as an inner periodicity:

B, C, **D**, E, **F**, G, A.

This is because the modulus of 3 semitones produces G#, which is missing from the latter sequence (while the former stops before G#). The reason for this is related to Xenakis's inspiration of the interlocking 4hs with an leading-note sense. In his later sieves this idea is essentially extended to use all intervals and not only the perfect 4th. It is a case of *interlocking periodicities*. Sieves therefore are conceived as a multiplicity of interlocking periodicities; and a periodicity must be continuous in order to be considered as such.

By validating only moduli that extend to the upper edge of the sieve, we account for the moduli that end at a distance from the upper limit of the sieve smaller than their own size. That is, we account only for modules whose final point is greater than $n - M$. For example, if the highest point of the sieve is $n = 80$ semitones, modulus 5, irrelevant of its starting point, will be validated only if it reaches a point after 75 (= 80 – 5). One consequence of this limitation is that if 75 (or a smaller multiple of 5) does not belong to the given sieve, modulus 5 could be detected only by inner-periodic analysis in a cyclic transposition. Let 70 be a point in the sieve. A cyclic transposition of 6 semitones would place 70 at a distance of 4 semitones from the highest point of the sieve (80 – 76 = 4); thus modulus 5 would be validated as an inner-periodicity only for this specific cyclic transposition.

5.7 Analytical Algorithm: Final Stage

The above limitation of inner-periodic analysis is probably an expected one, in the sense that a periodicity must be as continuously present as possible; the greater the number of occurrences of a periodicity the more decisive its contribution to the inner-periodic structure of the sieve. If this is our starting point, we can examine another limitation of the process Xenakis applied to calculate the formula of the sieve of *Akea*. If it is enough for a modulus to end at a distance from n smaller than its own size, it is not necessary to depart as close to the lowest point. Such is the case of module (5, 40) in the simplified formula of the sieve of *Akea*. Modulus 5 is present only for the second half of the sieve. This is a consequence of not limiting the size of I to be smaller than M (as the residue naturally is smaller than the modulus in Modular Arithmetic). As the algorithmic process goes through all the points of the sieve and assigns the smallest periodicity that departs from each point, the possibility of finding smaller periodicities increases. This is obvious in Figure 5.5: beyond point 56 all moduli are smaller than 10. But even before the end of the process at point 52 the size of the moduli range from 25 at the first point to 5 at point 40. The fact that $I > M$ for half of the moduli in the modules of Figure 5.5, means that, although all moduli are present until the top of the sieve, only half of the moduli are present close to the bottom.

This one-sidedness of the process raises a fundamental issue in inner-periodic analysis: if an inner periodicity is validated as such according to a minimum number of occurrences, then it also has to extend to a minimum range in the sieve. For example, although modulus 5 repeats eight times in the sieve of *Akea*, it is present only beyond the middle of the sieve's range. This means simply that this particular modulus is characteristic for only a part of the sieve – unlike module (25, 0) whose range is 75 semitones, from the bottom almost to the top. The difference between (5, 40) and (25, 0) is not only their difference in size, but the fact that $M < I$ in the former and $M > I$ in the latter. In fact, the term 'module' is used only by extension for (5, 40). More specifically, both the initial and the final points of (25, 0) lie at a distance from the edges of the sieve smaller than the size of the modulus. This is another criterion of validating moduli, that is, relevant to the presence of a modulus in the sieve and to the idea of sieves as being outside time. Bearing in mind that the notions of symmetry and periodicity have been used by Xenakis as equivalent when talking about moduli, a sieve is conceived as a multiplicity of periodicities and their analysis should not depend on an unbalanced favouring of smaller moduli when progressing towards the higher points. This is the reason why Xenakis added one more step in the final version of his analytical algorithm. The additional condition in the final algorithm re-introduces the idea of the residue: in all modules it must be true that $M > I$. This is shown in the final step of the algorithm as published in the 1990 article:

> we ignore all the [modules] (Q, I) which, while producing some of the not-yet-encountered points of the given series, also produce, *upstream* of the index I, some parasitical points other than those of the given series (FM 275; italics added).

Therefore, the algorithm does not merely look for the smallest modulus that departs from the point under consideration, but for the smallest modulus that starts at a point smaller than its own size and that *produces* the point under consideration (unless this point is located early enough in the sieve that is itself the starting point).

This is the algorithm that computer program B is based on. The sieve is entered as a sequence of numbers; the program checks each point and computes the smallest periodicity that either starts at or covers this point. In other words, the program checks for the residue class (which means that $M > I$) with the smallest modulus, whose members belong to the sieve and include the point under consideration. For example, if the point under consideration is 22, the program starts checking, from the bottom, the residue classes whose members include 22: (1, 0), (2, 0), (3, 1), (4, 2), (5, 2) and so on, passing on to $M + 1$, until it finds a residue class whose members: a) all belong to the given sieve and b) produce point 22. If no such module is found with $M \leq 22$ (while $M > I$), then it looks for $M > 22$ with $I = 22$. Afterwards, it checks the redundancy of the module; when all points have been covered, it computes the period of the sieve (as the LCM of all moduli) and finally displays the formula. The formula that program B suggests for the sieve of *Akea* is shown in Figure 5.10. We see that the moduli are now in average greater than the ones in the formula Xenakis used. Before exploring the properties of this formula, I will focus on the central difference between the two alternative simplified formulae: the former indicates the smallest possible periodicities *beyond* the point under consideration and the latter the smallest possible periodicities *both after and before* the point under consideration.

The simplified matrix for the formula that program B suggests is shown in Figure 5.11. In the matrix, we see a less 'diagonal' arrangement than in the one of Figure 5.6. This is because each modulus departs from and arrives at a distance from the edges of the sieve smaller than its own size. Due to the increased value of M, this formula shows one module to occur only once: (28, 27). Therefore, (28, 27) cannot be considered as an inner periodicity. The reason for this is that $I = 27$ pushes the smallest possible modulus beyond the limit of satisfying the condition of inner periodicity for $n = 80$. I will explore this limit later on. Figure 5.12 shows the two alternative ways of producing a formula for the sieve of *Akea*: the one that Xenakis actually applied in the pre-compositional sketches (1986) and the one that the final algorithm suggests (1990). The modules in bold are the ones that where actually introduced to cover the point they are next to. Thus, point 40 was assigned module (5, 40) by the 1986 algorithm; module (15, 10) does cover point 40, but was assigned to cover point 10 (where it appears in bold). In the table, each module appears in bold typeface only once, which means that for each point under consideration only one module is assigned, although several modules might intersect at a point. The two versions of the algorithm suggest identical formulae up to point 18. After this point, the condition of $M > I$ in the 1990 version produces greater moduli than the earlier one, which produces gradually smaller moduli. There is therefore an opposing tendency in the two methods. The earlier version finds smaller moduli as I increases, since the smaller the range the more likely for a small modulus to cover it (given that the density of the sieve remains roughly the same in all its range); and the final version finds gradually greater moduli simply because the value of the starting points increases. This is shown in the two graphs of Figure 5.13. These graphs are intended to show a synoptic and approximate picture of the size of the modulus as I increases in the two methods. The earlier method, considers a point as a point of departure and finds the smallest modulus that departs from it. In the final method, the points are considered either as starting or as subsequent points of a module; this explains why in the graph of the 1990 version some starting points appear more than once (belonging to different modules).

5.8 Inversion

Given that in the more recent formula the modulus of each module is the smallest one that covers the *whole range* of the sieve (including each point that is considered), all modules extend as far as possible to both directions. This can be additionally confirmed with the aid of the inversion of the sieve. Recall that, in general, the inversion of a sieve can be achieved by replacing the starting points in a formula with their negative value and consequently reducing them according to the modulus (see Section 3.5.1.1). However, this can be done only in the extent of the whole period of the sieve (the LCM of all M). Practically speaking, this method can rarely be applied to the inner-periodic formula (as the LCM is frequently extremely high).

From a musical point of view, the low-to-high arrangement is a conventional one. We could equally decide that the standard arrangement of the elements is from high-to-low (this would still be an ordering independent of time). Xenakis's decision of low-to-high arrangement is not crucial on the theoretical level and is an obvious choice in scale-construction in general. This upward conception of sieves also reveals an aesthetic criterion. When Xenakis described his influence from the *pelog* he stressed the 'leading-note' sense of two interlocking 4hs a semitone apart. He gave the example of 'G and C going up, and F# and B going up. The B is a leading note to C and F# is a kind of leading note to G' (Varga 1996: 145). Inversion is not included in the transformations Xenakis applied to his sieves of the later music. But it is important to study it in order to reveal certain characteristics of his sieves.

We can construct the inversion and, reading from right to left, correspond each one of its elements to an element of the original sieve, reading from left to right. Subsequently, we replace all the final points of a module in the simplified formula of the original sieve with their corresponding points in the inversion. The result is a formula that produces the inversion.[52] Note that such a formula is identical to the one of the original sieve, but with different starting points. The final point of a module can be easily calculated when the R-value is known: it is equal to $R \cdot M + I$. Therefore, to construct the inversion of an inner-periodic simplified formula, we replace each I by the difference between the highest point of the sieve and the final point of the module: $n - (R \cdot M + I)$. For example, module (24, 22) in the simplified formula of the sieve of *Akea*, would be replaced in the inversion by (24, 10). The I-value is calculated as such: $I = 80 - (2 \cdot 24 + 22) = 80 - 70 = 10$. Module (24, 10) covers all the points in the inversion that correspond to the points of (24, 22) in the original sieve: 10, 34 and 58. On the contrary, a module where $M \leq I$ would not produce the corresponding points in the inversion; it would

[52] By this I do not mean that such a formula would be identical to the one Xenakis's algorithm would suggest for the inversion of a sieve. In fact, the formula that the algorithm suggests is not always identical to the one derived straight from the original. However, in most cases it includes the same moduli (but not necessarily the same modules) with the original. This depends on the structure of the sieve in question. But as I will show later, the formula derived from the inversion of the sieve would still satisfy the condition of inner symmetry.

produce at least one point that does not belong to the sieve. In other words, when the simplified formula of a sieve contains a module where $M \leq I$, it means that in the inversion its modulus would not extend to the upper limit of the sieve, and thus could not be validated as an inner periodicity. If it is true that $M > I$ for all modules in the simplified formula of the original sieve, we can apply the same procedure and get the simplified formula of the inversion, as shown in Figure 5.14. The simplified matrix for this formula is shown in Figure 5.15. The arrangement of the modules and the points that they cover are symmetrically related to the matrix of the original (Figure 5.11).

5.9 The Condition of Inner Symmetry

When all the inner periodicities extend as far as possible to both directions in a sieve, and therefore could also produce its inversion, then the sieve bears a certain kind of symmetry, on a higher level than that of the inner periodicities. But this symmetry is still a hidden one: the intervallic structure of such a sieve might very well be asymmetric, i.e. non-palindromic in any of its cyclic transpositions (unless of course we consider the theoretical period as the range of the sieve, which can be extremely large when several moduli are involved). This observation reveals another level of symmetry, not obvious on the surface: it is a kind of symmetry revealed through the distribution and setting off of the inner periodicities. In analogy with the inner-periodic nature of Xenakis's later sieves, I will refer to this kind of symmetry as *inner symmetry*[53] and to sieves that exhibit inner symmetry as *inner-symmetric*.

[53] In (1996: 149) Xenakis refers to 'inner symmetries'; this is what I here refer to as 'inner periodicities'.

The fact that the algorithm suggests a formula where in all modules $M > I$ is a necessary condition to indicate inner symmetry whenever it exists, but not an adequate condition for its existence. This is because the algorithm always finds a modulus that meets at least two points when the starting point (I) is sufficiently small and the highest point (n) of the sieve is sufficiently large (in absolute values). But with the condition of inner periodicity a modulus must repeat at least twice, i.e. $R \geq 2$, which affects the maximum size of M. I will refer to modules with $R \geq 2$ as *periodic modules*. According to the basic property of the formula suggested by both the earlier and the final version of Xenakis's analytical algorithm, M should be thought of as the *smallest possible modulus* that covers each point under consideration. This smallest possible modulus is in turn limited by the condition of inner periodicity to a maximum value, depending on the size of n and I: $M \leq (n - I)/2$. With the additional limiting of the size of the modulus to values greater than the starting point, M depends on the varying values of I for both its minimum and maximum values. This minimum is the size of the starting point: $I < M$; and since we operated in discreet space, this is equivalently expressed as $I + 1 \leq M$. Therefore, the *condition of inner symmetry*, which incorporates that of inner periodicity, is formulated as follows:

$$I + 1 \leq M \leq \frac{n - I}{2}. \tag{1}$$

This implies that both the minimum and the maximum values of M depend on the value of I.[54] For example, if $n = 80$ and $I = 22$ the minimum permissible value of M is 23 and its maximum permissible value is 29. That is,

$$22 + 1 \leq M \leq \frac{80 - 22}{2} \Rightarrow$$

$$\Rightarrow 23 \leq M \leq 29$$

Therefore, modules (23, 22), (24, 22), (25, 22), (26, 22), (27, 22), (28, 22), or (29, 22), if valid, would be part of the inner-symmetric structure of the sieve. But a module like (19, 22), although it is periodic (since $R = 3 \geq 2$) it does not satisfy the condition of inner symmetry (since $M = 19 < I = 22$). Therefore, an inner-periodic sieve is not necessarily inner-symmetric. Inner periodicity was defined only as a condition for inner symmetry. It is important to stress here that both of them are integrated into one. I used the former to demonstrate the earlier stage of Xenakis's method, which was extended by his final algorithm. Had the final algorithm been the starting point of this analysis, the conditions of inner periodicity and symmetry would still hold. Xenakis's prompt to study the hidden symmetry of a sieve referred to both symmetries and periodicities, in the form of moduli. These two notions are then themselves integrated; inner symmetry is achieved by the analysis/synthesis of inner periodicities. Thus, an outside-time characteristic (symmetry) is achieved through the treatment of an inside-time one (periodicity). In this sense, it is presupposed that a sieve must be inner-periodic in order to be inner-symmetric.

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[54] The maximum value of M also depends on the size of n.

5.10 Inner-Symmetric Analysis

In fact, most of Xenakis's sieves are inner-symmetric in almost all of their moduli; in most cases they include one or two moduli that are not periodic. In the sieve of *Akea* this is the case with module (28, 27). The maximum permissible value for M, when $n = 80$ and $I = 27$, is $(80–27)/2 = 53/2 = 26.5$; i.e. $M \leq 26$. The points that module (28, 27) covers are 27 and 55 (D#3 and G5); from these points, 27 is covered only by this module, but 55 is also covered by two additional moduli, which are periodic (see Figure 5.12 – 1990 version). Therefore, D# is the only element of the sieve of *Akea* that does not belong to an inner periodicity. I will refer to elements that are produced only by non-periodic modules as *non-periodic elements* (or *points*); and to elements that belong to an inner-periodicity, as *inner-symmetric elements*. 36 out of 37 elements though, do account for the high degree of inner symmetry in the sieve. This slight deviation from complete inner symmetry is perhaps one of Xenakis's typical aesthetic criteria; I will examine this further after I formulate in more detail the consequences of the condition of inner symmetry.

5.10.1 Extreme Modular and Residual Values

The condition of inner symmetry also implies that the minimal and maximal permissible values of M for a given I, increase and decrease respectively as I increases. This allows for the determination of the absolute maximum permissible value for I, given a constant n. The smallest permissible I is naturally 0. The maximum permissible value of I in inner-symmetric sieves is found at the point of convergence of the two tendencies of the value of M as I increases. At this point of convergence the maximum and minimum of M for a given n are equal. Then from (1) follows that

$$I + 1 = M = \frac{n - I}{2}.$$

Consequently,

$$I + 1 = \frac{n - I}{2} \Rightarrow 2I + 2 = n - I \Rightarrow 3I = n - 2 \Rightarrow I = \frac{n - 2}{3}.$$

Therefore, the maximum value of I for any inner-symmetric sieve, where n is its highest point, is

$$I = \frac{n - 2}{3}.$$

For the sieve of *Akea*, or for any sieve where $n = 80$, the greatest point of departure is $(80–2)/3 = 26$; therefore, it is true for sieve with $n = 80$ that $I \leq 26$.

The maximum value of I is found at the point of convergence of the maximum and minimum M for a given I. Let us indicate the value of M at this point as M_c. Since the value of I at this point is the maximum and since it depends only on n, we can locate the value of M at this point (M_c), according to (1), as follows:

$$I + 1 = M_c = \frac{n - I}{2}.$$

Consequently,

$$I+1=M_c \Rightarrow I=M_c-1$$

and

$$M_c=\frac{n-I}{2} \Rightarrow M_c=\frac{n-(M_c-1)}{2} \Rightarrow 2M_c=n-M_c+1 \Rightarrow 3M_c=n+1 \Rightarrow M_c=\frac{n+1}{3}.$$

That is, in inner-symmetric sieves, the *M* of the greatest *I* is equal to the one third of (*n* + 1).[55] For any sieve whose highest point is 80, the inner-periodicity starting at the maximum *I* is (80+1)/3 = 27. In the sieve of *Akea*, module (28, 27) has the smallest *M* that could be assigned to *I* = 27 but the value of *I* exceeds by one semitone the maximum limit for *n* = 80. This maximum value of *I*, as well as the maximum and minimum permissible values for *M* at every possible value of *I* in the sieve of *Akea*, are shown in the graph of Figure 5.16. The *x*-axis is equipped with the points of the sieve of *Akea* that can function as starting points (i.e. the points smaller than or equal to 26); the *y*-axis shows the values of *M* for every *I*. The two lines indicate the limitation posed by the condition of inner symmetry: the lower line shows the minimum permissible *M* for every *I* [left part of (1)] and the upper line shows the maximum permissible *M* for every *I* [right part of (1)]. The basic consequence of the condition of inner symmetry is easily seen in the graph: the fact that *I* is present in both the leftmost and rightmost members of the inequality in (1) means that, as *I* increases, the minimal and maximal permissible values of *M* converge to

[55] Since *M* here represents the smallest possible modulus that can be assigned to a point, when the result is not a whole number, M_c is equal to the lower whole number; i.e. it is equal to the whole number without the decimals. When *n* = 88, (88+1)/3 = 29.66, therefore M_c = 29.

$$M_c = \frac{n+1}{3} = 27\,.$$

The point of this convergence, M_c, is the one that defines the maximal permissible value of I, which is

$$I = \frac{n-2}{3} = 26\,.$$

The maximal value of $I = 26$ is shown on the x-axis in the graph of Figure 5.16, but 26 does not belong to the sieve of *Akea*; it is shown there as the maximum limit I can have in an inner-symmetric sieve with $n = 80$; beyond this point, the maximum (upper line) and the minimum (lower line) permissible values of M would swap, having the former below the latter.

The values M can take range between 1 and $n/2$.[56] The critical point M_c is the value that determines, not merely the highest permissible I, but the behaviour of the values of I as M increases. In fact, M_c is the value of M that determines which part of the inequality in the condition of inner symmetry defines the maximum values for I. When

$$M \le M_c$$

then

$$I + 1 \le M \Rightarrow I \le M - 1\,.$$

[56] Although Xenakis defines the first step of his algorithm to start at $M = 2$, the program starts testing $M = 1$. The elementary module (1,0) is then by extension the total chromatic.

And when

$$M_c \leq M$$

then

$$M \leq \frac{n-I}{2} \Rightarrow 2M \leq n-I \Rightarrow I \leq n-2M \ .$$

As the value of M increases towards M_c, the maximum value of I also increases; as the value of M increases beyond M_c, the maximum value of I decreases. The graph of Figure 5.17 shows the maximal values of I for every possible M in any sieve of $n = 80$. This graph is an extension of the graph in Figure 5.16, but the two axes are now inverted. The x-axis represents all the consecutive values of M between 1 and 40 – the highest permissible value of M in inner-symmetric sieves of $n = 80$; the y-axis represents the values of I. The red curve shows the maximal values of I for every M. We see that the values of I increase linearly up $M_c = 26$, where $I = M - 1$. As the value of M increases beyond $M_c = 26$, the values of I decrease linearly, but faster than they increased: $I = n - 2M$. I will explore this in more detail in relation to the values of R. The graph demonstrates a general property of inner-symmetric sieves; for a given n, the extreme values of M and I are related as follows:

(a) When M is minimal ($M = 1$), I is minimal ($I = 0$).

(b) When M is maximal ($M = \frac{n}{2}$), I is minimal ($I = 0$).

(c) When I is minimal ($I = 0$), M can take any permissible value ($1 \leq M \leq \frac{n}{2}$).

(d) When I is maximal ($I = \frac{n-2}{3}$), M is equal to M_c ($M = \frac{n+1}{3}$).

The graph of Figure 5.17 is a chart for the values of M and I, that accounts for the inner symmetry of a sieve. Since these values depend only on the size of n, the above relations of the values and their limits, enable one to construct sieves that satisfy the condition of inner symmetry. If a module appears below the red line of this chart it repeats at least twice. Therefore, when all modules appear under this limit, the sieve is inner-symmetric. The formula of the sieve of *Akea* is shown in this chart in Figure 5.18. The chart offers a synoptic view of the inner symmetry of the sieve: as I have shown, module (28, 27) repeats only once and therefore lies outside the range of the values of M and I for any inner-symmetric sieve of $n = 80$.

5.10.2 Reprises of the Modulus

In inner-symmetric sieves each inner periodicity can repeat for the maximum of possible times in the scope of the sieve's range (or period). The absolute maximum number of reprises of a modulus is equal to the quotient of n divided by M; e.g. if $n = 80$, $M = 25$ can repeat at most 3 times (i.e. the amount of times 25 'fits' into 80). This absolute maximum of R is achieved when I is sufficiently small. In particular, it is achieved when $I \leq n(\text{mod}M)$; i.e. if the starting point (I) of a modulus (M) is equal to or smaller than the residue of n divided by M. If the starting point is greater than this residue (and smaller than the size of the modulus), the modulus repeats for one time less than the maximum of the possible reprises. For example, $M = 25$ repeats for the maximum possible times (R = 3), if its starting point is $I \leq 80(\text{mod}25) \Rightarrow I \leq 5$. In the case I is greater than 5 and smaller than 25 it repeats only twice. Note that this upper limit of 25 is already imposed by the condition of inner symmetry.[57] Therefore, while the condition of inner symmetry determines the minimal R, the size of I determines the maximal. R is consequently limited between two consecutive values, its maximal and its sub-maximal one:

if

$$I \leq n(\mathrm{mod}\, M)$$

then

$$R = \frac{n - n(\mathrm{mod}\, M)}{M};$$

if

$$n(\mathrm{mod}\, M) < I < M$$

then

$$R = \frac{n - n(\mathrm{mod}\, M)}{M} - 1.$$

The value of R for a given module can be seen easily with the help of the graph that I used to demonstrate the relationship of the values of I and M. To the two variables of the graph of Figure 5.17 we can add the values of $n(\mathrm{mod}M)$ for each M. This is shown in Figure 5.19. The values of $n(\mathrm{mod}M)$ for every M are shown by the dotted zigzag curve. In order for R to be maximal, the point that corresponds to the values of M and I of a module, must lie on or below the $n(\mathrm{mod}M)$ curve [which implies that $I \leq n(\mathrm{mod}M)$]. For

[57] Unconditionally, this upper limit is equal to $M + n(\mathrm{mod}M)$.

example, for module (14, 9) R is maximal since the point that corresponds to $M = 14$ in the x-axis and $I = 9$ in the y-axis, is located below the dotted curve. The value of R is

$$R = \frac{n - n(\mathrm{mod}\, M)}{M} = \frac{80 - 80(\mathrm{mod}\, 14)}{14} = \frac{80 - 10}{14} = 5.$$

On the contrary, for module (15, 10) R is sub-maximal, because (15, 10) lies above the dotted curve:

$$R = \frac{n - n(\mathrm{mod}\, M)}{M} - 1 = \frac{80 - 80(\mathrm{mod}\, 15)}{15} - 1 = \frac{80 - 5}{15} - 1 = 4.$$

When a module is located towards the right part of the chart, its M is higher and it naturally appears less times than the ones located towards the left. The additional $n(\mathrm{mod}M)$ curve helps one to locate in a glance the most periodic modules in the sieve: these would be located towards the left and under the dotted curve. When the modules of a simplified formula tend to concentrate to this area they are more periodic (their R-value is higher). In fact, each peak of the dotted curve stands for the transition to a group of consecutive values of M that repeat for one more time moving to the left and one less time moving to the right. We see in the graph, that beyond M_c (= 27) the $n(\mathrm{mod}M)$ curve coincides with the curve of the maximum values of I (i.e. the dotted curve coincides with the red). The rightmost area of the chart is for modules with the minimum R (= 2). Starting on the right, the highest M is 40 and appears naturally twice. The same holds for 39: it repeats twice with a residue of 2. Note that the sequence of the $n(\mathrm{mod}M)$-values, while M-values decrease, increases by 2. Moving to the left towards the previous peak, the $n(\mathrm{mod}M)$-values increase by 3 (that is between M-values 26 and 21). This area of the chart between the red and the dotted curves would accommodate modules with sub-maximal $R = 2$. The area bellow the dotted curve for the same M-values (26 to 21) is for the maximal $R = 3$. In the chart we see that R-values increase as we move to the previous peaks of the $n(\mathrm{mod}M)$ curve. Only values between 2 and 5 are shown in the chart of Figure 5.19, but the reader can easily see that R would reach up to $R = 40$ at $M = 2$ and $R = 80$ at $M = 1$ (the total chromatic). I will refer to this type of chart as *inner symmetry chart*. Figure 5.20 shows the inner symmetry chart for the sieve of *Akea* in relation to this additional aspect. Seven modules lie beneath (or on) the $n(\mathrm{mod}M)$ curve and therefore their R is maximal. Module (28, 27) is the only non-periodic one; since Xenakis's algorithm always finds a module that covers two points (when I is sufficiently small), its R-value is equal to 1.[58]

5.10.3 Density and Modules

In inner-symmetric sieves, the greater the size of the modulus, the smaller the number of its repetitions. The value of R depends firstly on the value of M and only secondly on the

[58] This is true when I is smaller than half the size of n. The reason for this is that modules are in fact residue classes and therefore the absolute value of I affects the size of M. For example, if we analyse a sieve with a range of 40 semitones and start the sieve at point 40 (so that the sieve's points range between 40 and 80), the modules would appear to cover only the starting point [the module at the lowest starting point would be (41, 40), which covers only point 40 and therefore $R = 0$]. In order to avoid this we should transpose the sieve 40 semitones downwards. In general, we can avoid this by calculating the formula with the sieve's lowest point set to 0.

value of I. Specifically, the value of I determines whether R will have either its maximal or its sub-maximal value for a given M. In general then, a smaller modulus naturally appears more times than a larger.[59] If sieves are viewed as a multiplicity of periodicities, then all periodicities in a sieve are equally characteristic, even if the R-value of a module is much greater than the average value of R's in the formula. But the value of R also depends on the total number of modules in a formula. In the sketches of *Akea*, although he used an early version of the algorithm, Xenakis noted that there are many modules in the formula, and this means that only few of them repeat continuously.[60] If there was a large number of inner periodicities that would repeat for a large number of times each, the sieve would tend to chromatic saturation. For example, we can construct a sieve with modules (3, 1), (4, 2), (7, 2) and (10, 0), where $n = 80$. With the help of the inner symmetry chart of Figure 5.19, the values of n, M and I are sufficient to calculate the R-values of these modules: 26, 19, 11 and 8, respectively. These four modules, because of their high values of R, are sufficient to produce a sieve of 51 elements up to $n = 80$. Therefore, should we need to construct a less dense sieve, we would have to either use less modules or increase the size of the moduli. This is because the R-value depends

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[59] This observation is loosely related to a property of the Sieve of Eratosthenes: each prime number eliminates a proportion of the remaining integers equal to its reciprocal. That is, 2 eliminates half of the remaining numbers, 3 one third, 5 one fifth, and so on (see Hawkins 1958: 108).

[60] 'Beaucoup de (M, I, R) car #S = 17 vue[?] 37 points du crible. Donc peu de périodes d'une seule traite'. In my translation: 'Many (M, I, R), since #S = 17, given the 37 points of the sieve. Therefore, few periods without stopping once' (Xenakis, Iannis, Pre-compositional sketches of *Akea*, Bibliothèque Nationale de France). Here Xenakis uses 'S' to denote the number of modules. The question mark denotes illegible handwriting.

(mainly) upon the *M*-value: for a given *n*, when *M* increases, *R* decreases (to either its maximal or sub-maximal value).

The largest interval of most of the sieves analysed here is a major 3rd. Additionally, and again with certain exceptions, there is no succession longer than three elements a semitone apart. These two observations practically mean that neither more than three chromatically consecutive elements appear, nor more than three chromatically consecutive elements are missing from the sieve. These two characteristics are general and affect the density of the sieves of Xenakis's later music. The density can be given by the ratio of the number of elements of the sieve, to its range. The sieve of *Akea*, with its 37 elements and $n = 80$,[61] has density D = 37/80 = 0.46.[62] Note that the number of modules in the simplified formula do not reflect this density, since a point might be covered by more than one module But when the density among different sieves is constant, the average value of *M* is directly proportional to the number of the modules. According to the aforementioned observations on the size of *R*, the larger the number of the modules in a formula, the lower the average value of *R* (with constant density); and the lower the average value of *R*, the larger the average size of *M*. Consequently, the size of the moduli depends on the number of modules: for a given density, the large number of modules is compensated by large moduli. Sieves with similar density to the one of *Akea* are expected to appear either similarly in the chart of Figure 5.20, or with less modules but concentrated to the left.

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[61] The range of the sieve is the difference between its highest and its lowest point. Note that when the lowest point of the sieve is set to zero, the highest point, *n*, is also equal to the range.

[62] By approximation to the second decimal.

The large number of modules in a formula accounts for the 'hidden' symmetry (whenever it exists). If we start with an approximate desired density, e.g. D ≈ 0.5, we can construct a sieve with either a small number of (inner) periodicities or with a larger one. We can start with the most symmetric (regular) sieve. The most elementary sieve with D = 0.5 is the whole-tone scale: module (2, 0), which in the inner symmetry chart is located to the left extreme (see Figure 5.21). If we wish to construct a more complex sieve, we can use two periodicities; but to achieve the desired density these two modules need to have greater *M*-values. We can use modules (3, 1) and (4, 0). For $n = 80$ they produce 41 points. The regularity of this sieve is less obvious, but still perceptible (12 semitones). The two modules would be still located at the very left of the inner symmetry chart, but to the right of (2, 0) (see Figure 5.22). An even more complex sieve would have to use more modules with greater moduli: e.g. (5, 0) + (6, 1) + (7, 2). They produce a sieve with 37 points, but its regularity is now much less obvious: the period of the sieve is 320 semitones. In the inner symmetry chart they would be located further to the right than the previous sieves (see Figure 5.23). We see that this process would gradually lead to a sieve with several modules of greater size, located towards the right of the chart. When we are still in the limits imposed by the condition of inner symmetry (the red curve in the chart), symmetry still exists, but it is not as obvious. The leftmost part of the chart indicates a more superficial symmetry and the rightmost part a deeper one, so to speak (when comparing sieve with the same density).[63] Note that the limits of inner symmetry

[63] Note that a different density would not necessarily have implications relating to the inner-symmetric structure of the sieve. That is, a more symmetric sieve might have either higher or lower density than a less symmetric one.

are of two kinds. The ascending side of the red curve represents a condition of analytical method: we allow only for moduli greater than the size of the starting point ($M > I$). The descending side of the red curve represents the condition that validates a module as periodic or not. Therefore, if there is a module that lies outside the limits of the red curve, this can be only beyond the right side of the curve. Although validating a module as periodic is a clear decision, a sieve (as a multiplicity of inner periodicities) might not be inner-symmetric in its entirety. The sieve of *Akea* is one among several sieves that include one or two non-periodic modules. However, Sieve Theory offers the way of indicating both the periodic and the non-periodic elements in a sieve.

When the inner-periodic simplified formula of two sieves with the same density include only periodic modules ($R \geq 2$), then the notion of distance between the two can be defined by the number of modules in each formula. A formula with a large number of modules shows a sieve which is less symmetric than one with fewer modules. For example, let two sieves with the same density and range (n), with 18 and 20 periodic modules each. The first sieve is then more symmetric than the second, by 2 modules (in the sense the absolutely symmetric sieve is the one with 1 module). This notion of distance can be applied only to sieves whose all modules are periodic. But the sieves of Xenakis's later music are shown to include one or two non-periodic modules; and in general, whenever there is inner symmetry, this is marginal: the modules appear mostly on the rightmost side of the inner symmetry chart. Analysing the inner symmetry of a sieve is not aimed merely at classifying sieves in one of the two categories: inner-symmetric / inner-asymmetric. Both notions of symmetry and asymmetry are crucial, in the aesthetics of Xenakis's music. As he himself put it, they are 'the two poles between which music goes back and forth, and the first suggestion of a solution comes from distributing points on a line' (Xenakis 1996: 147). In fact, the principal aesthetics of Xenakis's sieves are very clear and underlie (with very few exceptions) all the sieves in his oeuvre: they are absolutely non-repetitive. But they are not constructed by hand; rather, their non-repetitiveness, their irregularity is secured by sieve-theoretical means. In this respect, a general characteristic of an irregular sieve is the relatively large number of modules. The number of modules is relative both to the number of points in the sieve and to its range (i.e. to the sieve's density). But since there might be several intersections of modules (i.e. points that are produced by more than one module) the relationship between this density and the number of modules cannot be rigorously formulated. However, if the sieve is our starting point, i.e. when the sieve's density is given, we can express a high or low degree of symmetry as a small or large number of modules (regularities). Therefore, when there exist non-periodic modules in the formula, the irregularity of the intervallic structure has outreached the limit of inner-symmetry. With the help of the inner symmetry chart, we can see not only whether a module is periodic or not (for this the R-value would suffice), but also how distant a module is from the limits of inner symmetry. The notion of distance from inner symmetry is then related to the individual modules. Finally, with the inclusion of $n(\mathrm{mod}M)$ curve, the inner symmetry chart retains the discreteness required for defining a notion of distance.[64] In this way the inner symmetry chart shows both a general and a more detailed picture of the play between symmetry and asymmetry.

[64] For the higher R-values this discreteness between regions of the map is not as distinct. However, in irregular sieves R is not expected to reach extremely high values.

6 Sieve Analysis

6.1 Sieves and Versions

Many works of the period following *Jonchaies* make use of scales that are common to several of them. The sieve of *Jonchaies* was used in *Pléïades* (1978), *Palimpsest* (1979), *Anemoessa* (1979) and other works, in combination with other sieves. The most prominent throughout the greatest part of the period is the sieve of *Nekuïa* (1981), which is used in more than fifteen subsequent works: up to1987, in *XAS*, and transformed to a greater degree up to 1991, in *Krinoïdi*. Other frequent sieves are the ones that derive from *Keqrops* (1986) and *Epicycle* (1989), while less frequent are the ones issuing from *Komboï* (1981), *Tetras* (1983), *Thalleïn* (1984), *Kyania* (1990), *Dox-Orkh* (1991) and *Paille in the Wind* (1992). The frequency of the use of a sieve is an obvious indication of its importance for the composer. Therefore, a detailed analysis of more frequently employed sieves is necessary.

Sieves that appear in different compositions are in most occasions versions of the original. An interesting aspect of sieve analysis is encountered when analysing different versions of the same sieve, as this might reveal compositional decisions relating to certain aspects of the sieves. The transformations that Xenakis applied to his sieves range from cyclic transposition, which maintains the intervallic structure, to manual alterations such as omitting, adding, or changing one or more elements or segments (thus changing the intervallic structure). In the latter case, analysis can reveal properties common to different versions and thus enable results concerning the construction of sieves. For example in 1981, in the case of the sieve of *Nekuïa*, Xenakis had not yet started using his algorithm with its simplified formula; he provided the sieve with a theoretical period that was useful in relation to the several cyclic transpositions he used in this work. The decomposition of its period appears on the score as a label that denotes the elementary moduli the period is decomposed into (detailed analysis will follow). When a version of the same sieve was later taken in *Akea* of 1986, he was already working with the simplified formula of an early version of his algorithm (as the pre-compositional sketches suggest). Therefore, the progression towards a simplified conception of sieves must have influenced the creation and selection of the new versions. In this chapter I analyse 15 sieves that are more prominent in the period starting with *Jonchaies*. These sieves are either used in more than one work, or their treatment in the pre-compositional sketches provides insight to our understanding of Xenakis's approach to sieve-construction.

6.2 *Jonchaies* (1977, for orchestra)

Two years after *Jonchaies* Xenakis used its sieve in *Palimpsest* (1979, for ensemble). This work it does not properly belong to the later period of systematic sieve construction. However, it is considered by Solomos as a 'precursor of the writing that would dominate the ulterior period' (1996: 85).[65] Indeed, it is a composition that makes relatively little use of glissandi and with no microtones – both elements that the composer would gradually abandon.

In the sketches to *Palimpsest* the sieve appears in three versions: the first one is a transposition T_{-1} that extends for seven octaves, shown in Figure 6.1. The second is a T_{-7}

[65] Cette pièce de 1979 [*Palimpsest*] est très annonciatrice de l'écriture qui dominera dans la période ultérieure.

transposition along with a variation of the intervallic structure – the minor 3rd is replaced by the 4th, which results in the following succession: 1 5 1 2 4 1 4 1 (Figure 6.2). Finally, the third version, also a transposition (T_{+1}) and a variation of the intervallic structure, omits a semitone: the last element of the original sieve is omitted and there are now eight pitches instead of nine; and the intervallic structure is 1 3 1 2 4 1 4, which turn results into a period of 8ve+tritone, instead of the original 8ve+4th – see Figure 6.3.

From these three versions Xenakis used only the first one, i.e. the sieve of *Jonchaies* transposed downwards a semitone. Interestingly, Xenakis used the same transposition of the sieve eleven years later, in *Kyania* (1990, for orchestra) and in *Roaï* (1991, for orchestra).[66] In a sense, although the sieve is not conceived as a non-repetitive structure, it marks the whole late period of sieve-based composition, from the very beginning and almost until the last works that are based on sieves.

6.3 *Mists* (1980, for piano)

Squibbs (1996: 64) has demonstrated that Xenakis developed the sieve for *Mists* and then produced its formula. Still working by hand and with decomposed formulae, Xenakis considered three different decompositions before arriving at the final decision: 3, 4, and 7; 3, 5, and 7; 3, 5, and 8.[67] The periods that these decompositions presuppose are 84, 105, and 120 semitones respectively. He finally settled on another decomposition, 2, 5, and 9, which gives a period of 90 semitones (7·8ve + tritone). Since the work is scored

[66] The sketches of *Idmen A*, show that he also considered the same transposition for this work, but he did not finally use it.

[67] Dr Ronald Squibbs kindly provided me a copy of the page of the sketches with the sieve of *Mists*.

for piano, the period of 90 semitones, like the provisional ones of 105 and 120 semitones, exceeds the range of the instrument. It is clear that for Xenakis the period of this sieve was primarily a matter of theoretical nature; different periods would result in different decompositions and these in turn would produce different (decomposed) formulae. Therefore, although the sieve itself would not change depending on his decision on the decomposition, the resulting formula would, and his choice was made according to certain criteria concerning the theoretical representation of the sieve and therefore its subsequent transformations.

The final version is slightly different from the original sieve (Figures 6.5 & 6.4 respectively). There are two alterations in the final sieve: G1 is replaced by F#1 and C8 is replaced by C#8 (which exceeds the range of the piano by one semitone). The reason for the first alteration might be that Xenakis wanted to exclude the perfect 5th from the intervallic structure of the sieve; in the original this appears as a unique entry of 7 at the second position in the intervallic succession, replaced by 6 in the final version.

In the final version, the sieve contains six intervals, from the semitone to the tritone, and its top pitch is one semitone over the range of the piano. The size of the intervals determines, to an extent, the density of the sieve. Therefore, the sieve of *Mists* is somewhat less dense than most of the sieves of subsequent compositions, whose largest interval is the M3rd (which also appears less frequently in the sieve than the smaller intervals). The sieve has 30 elements; over the range of 90 semitones its density is one third of the total chromatic (D = 0.33). Although the sketches of *Mists* do provide the formula of the sieve, the alterations made on the original require a new formula, constructed with the aid of a decomposed matrix (see Figure 6.6).[68] The resulting formula is

$$9_1 \cdot 10_1 + 9_3 \cdot 10_3 + 9_0 \cdot 10_9 + 9_2 \cdot 10_1 + 9_5 \cdot 10_4 + 9_8 \cdot 10_7 + 9_4 \cdot 10_2 + 9_6 \cdot 10_4 + 9_1 \cdot 10_8 + 9_4 \cdot 10_1 + 9_1 \cdot 10_7 + 9_3 \cdot 10_9 + 9_6 \cdot 10_2 + 9_8 \cdot 10_4 + 9_0 \cdot 10_5 + 9_5 \cdot 10_0 + 9_7 \cdot 10_2 + 9_8 \cdot 10_3 + 9_4 \cdot 10_8 + 9_5 \cdot 10_9 + 9_0 \cdot 10_3 + 9_4 \cdot 10_7 + 9_6 \cdot 10_9 + 9_0 \cdot 10_2 + 9_1 \cdot 10_3 + 9_5 \cdot 10_7 + 9_8 \cdot 10_0 + 9_1 \cdot 10_2 + 9_5 \cdot 10_6 + 9_7 \cdot 10_8.$$

With the help of sub-matrix (Figure 6.7) we can construct the formula that includes all three factors:

$$2_1 \cdot 5_1 \cdot 9_1 + 2_1 \cdot 5_3 \cdot 9_3 + 2_1 \cdot 5_4 \cdot 9_0 + 2_1 \cdot 5_1 \cdot 9_2 + 2_0 \cdot 5_4 \cdot 9_5 + 2_1 \cdot 5_2 \cdot 9_8 + 2_0 \cdot 5_2 \cdot 9_4 + 2_0 \cdot 5_4 \cdot 9_6 + 2_0 \cdot 5_3 \cdot 9_1 + 2_1 \cdot 5_1 \cdot 9_4 + 2_1 \cdot 5_2 \cdot 9_1 + 2_1 \cdot 5_4 \cdot 9_3 + 2_0 \cdot 5_2 \cdot 9_6 + 2_0 \cdot 5_4 \cdot 9_8 + 2_1 \cdot 5_0 \cdot 9_0 + 2_0 \cdot 5_0 \cdot 9_5 + 2_0 \cdot 5_2 \cdot 9_7 + 2_1 \cdot 5_3 \cdot 9_8 + 2_0 \cdot 5_3 \cdot 9_4 + 2_1 \cdot 5_4 \cdot 9_5 + 2_1 \cdot 5_3 \cdot 9_0 + 2_1 \cdot 5_2 \cdot 9_4 + 2_1 \cdot 5_4 \cdot 9_6 + 2_0 \cdot 5_2 \cdot 9_0 + 2_1 \cdot 5_3 \cdot 9_1 + 2_1 \cdot 5_2 \cdot 9_5 + 2_0 \cdot 5_0 \cdot 9_8 + 2_0 \cdot 5_2 \cdot 9_1 + 2_0 \cdot 5_1 \cdot 9_5 + 2_0 \cdot 5_3 \cdot 9_7.$$

[68] In order to do this we first construct a matrix with dimensions 9 and 10 ($2 \cdot 5 = 10$). A sub-matrix is constructed for the two smaller factors, 2 and 5. Note that it is not necessary that the larger matrix corresponds to the largest factor. We can choose among any of the three original factors for the one dimension and the product of the remaining two for the other dimension. The choice depends on the periodicity the analyst desires to show. In the matrix of Figure 6.6 moduli 9 and 10 have been chosen, but one could have chosen to construct a matrix with dimensions e.g. 5 and 18 ($2 \cdot 9 = 18$). The final formula is not affected by such decisions. I have chosen the dimensions of 10 and 9 following Xenakis's own treatment in the sketches.

The alternative decompositions that Xenakis rejected could similarly produce a formula for the sieve by following the same process; that is, by constructing decomposed matrices and sub-matrices for each decomposition. However, the sketches do not provide any points past the period of 90 semitones. Therefore the formulae for the rejected decompositions that have larger periods would be incomplete. Were these formulae to be constructed, even though they would be incomplete, they would result to different cyclic transpositions.

Figure 6.8 shows the simplified formula of the final sieve. It consists of sixteen modules, with relatively low *R*-values. This is also shown in the inner symmetry chart of Figure 6.9: the modules are located towards the right of the chart, with four of them outside the red curve. 16 modules produce 30 points of an asymmetric sieve. The number of modules in the sieve of *Mists* is then relatively large for the number of its points. The work that followed *Mists* was *Aïs*, and from this work onwards the sieves are more dense and with smaller intervals.

Figures 6.10 to 6.19 show the inner symmetry charts of all the transpositions used in *Mists*.[69] We see that all of the transpositions have less non-periodic modules than the original sieve; also, they have either the same or smaller number of modules than the original. The fewer modules belong to T_7(mod90) and T_8(mod90), both with 12 modules in their simplified formula. Although all cyclic transpositions have less non-periodic modules than the original, none of them reveal an inner-symmetric sieve. There are two transpositions with only one non-periodic module: T_7(mod90) and T_{36}(mod90). The

[69] All cyclic transposition indices are provided in the score by the composer. For a detailed listing see Squibbs (1996: Vol. 2, 113-14 & 2002).

former has two modules located far from the majority: (14, 7) is the most periodic (R = 5) and (41, 35) is the only non-periodic (R = 1). T_{36}(mod90) is different. It has a larger number of modules, 15 instead of 12, and therefore the points of the sieve are produced by modules with no higher-than-average R-values (all R-values are smaller than 3). These two transpositions are the less asymmetric ones, but their inner symmetry chart reveals that the latter is more heterogeneous than the former: in T_{36}(mod90) there are more modules and with (sub-)minimal contribution each, whereas in T_7(mod90) there is one module with R = 5 and the rest contribute with the minimal or sub-minimal R-value (2 or 3). In this sense, and excluding the non-periodic modules of both transpositions, T_7(mod90) is more inner-symmetric than T_{36}(mod90) (or than the any of the rest transpositions).

6.4 *Aïs* (1980, for baritone, percussion and orchestra)

The sketches of this work provide three sieves, one of which was selected for the final composition. The first one, shown in Figure 6.20, does not exhibit the characteristic intervallic structure of the inner-periodic sieves one would expect to find. It is not related to the other two. Xenakis considered it but did not use it in the composition. The second sieve is shown in Figure 6.21. It is a typical sieve that extends throughout the whole range, with no more than three chromatically consecutive pitches present or absent. The sieve that actually made it into the composition is a T_{+7} (perfect 5th) transposition of this one (see Figure 6.22). However, there are certain differences between the two sieves that would prevent one from speaking about a strict transposition. Three pitches in the high range of the transposed sieve are missing, D6, F#6 and F#7, while there is an additional F7.

Xenakis mentioned that this sieve is a slightly modified version of the one of *Jonchaies*, ‘in order to make it less recognizable, to be different and yet retain a kind of specific tension’ (Varga 1996: 164-5). Indeed, there is a segment in the middle range of the sieve that exhibits the intervallic structure of the 1977 sieve.[70] This is the eight-note segment starting on F#4 and ending on G5. The intervallic succession of the sieve of *Jonchaies* is 1 3 1 2 4 1 4 1. This segment of the sieve of *Aïs* is a cyclic transposition that omits the last element: 1 1 3 1 2 4 1.

The three additional intervals of 3 semitones at the lowest range of the transposed sieve suggest a cyclic transposition. These are precisely the three final intervals in the intervallic succession of the original sieve (compare Figures 6.21 and 6.22). Therefore, there is an implicit period of 86 semitones.[71] It is obvious that this period, although not audible, serves as a tool for cyclic transpositions. The existence of a period suggests that a decomposed formula might have been used by the composer. The two prime factors of 86 are 2 and 43; thus a 2 by 43 decomposed matrix, inconveniently perhaps, would be required for this sieve.[72] The decomposed formula of the sieve of *Aïs* is:

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[70] In the sketches of *Epicycle* Xenakis designates the ranges of a sieve as low, middle, and high, where the middle register extends for two octaves starting on E4.

[71] The sketches provide only an 83-semitone segment the original sieve (Figure 6.21). The period of 86 semitones is implied only by the cyclic transposition.

[72] The only divisors of 86 are 1, 2, 43, and 86. Therefore, there is no option of using alternative, non-prime factors that would possibly be more convenient for this period.

$2_0 \cdot (43_0 + 43_3 + 43_6 + 43_7 + 43_{12} + 43_{15} + 43_{16} + 43_{19} + 43_{20} + 43_{21} + 43_{29} + 43_{30} + 43_{34} + 43_{35} + 43_{36} + 43_{37} + 43_{39} + 43_{40} + 43_{42}) +$

$2_1 \cdot (43_2 + 43_3 + 43_4 + 43_8 + 43_9 + 43_{10} + 43_{14} + 43_{18} + 43_{19} + 43_{23} + 43_{24} + 43_{25} + 43_{28} + 43_{29} + 43_{32} + 43_{37} + 43_{39}).$

The decomposed matrix and its corresponding formula offer a synoptic view of the sieve's internal structure and serve as a means to applying transformations (by changing the moduli and/or the residues). But since the transformation in question is cyclic transposition, a decomposed formula is useful but not necessary for this particular treatment. Cyclic transposition refers to properties of modular sets. From the two basic components of Sieve Theory, Modular Arithmetic and Set Theory, only the former is required to performed cyclic transpositions. It suffices to replace any number outside the range of the modulus with its equivalent modulo the period. In the present case, the highest pitch of the original, F#6, is point 85. When 7 is added it becomes 92 in the transposed sieve, exceeding the range of the period. Therefore it is replaced by $92 (\mathrm{mod} 86) \equiv 6$, which D#1 in the transposed sieve.

Due to the slight differences between the original and the final, transposed sieve we should examine both. In the final sieve, the 37 elements that occupy a range of 86 semitones produce a density D = 0.43. The simplified formula of the final sieve is shown in Figure 6.23. It consists of 19 modules with moduli that range between 21 and 37 semitones. This is itself a first indication of its non-symmetric structure and it is confirmed by the inner symmetry chart of Figure 6.24. There are two non-periodic elements (pitches): F#5 (point 54) and E6 (point 67) that belong to modules (32, 25) and (37, 30) respectively.

Figure 6.25 shows the simplified formula of the original sieve of *Aïs*.[73] Although the original sieve has one more point (D = 0.45), its formula contains fewer modules and the average size of moduli is smaller: as in the final sieve, the smallest modulus is 21 semitones, but the greatest is 33 semitones (instead of 37). However, the inner symmetry chart of the original sieve (Figure 6.26) suggests that this difference in not enough to characterise the sieve as symmetric. Although the two non-periodic modules are closer to the red curve, their *R*-value is still 1. These are modules (31, 25) and (30, 28). The former module covers G5 which in the transposition would be D6; but D6 is missing from the final sieve. The latter covers A5, which in the final transposition is E6, the one of the two non-periodic elements. In fact, the two non-periodic elements in the final sieve are not affected if we construct a faithful transposition from the original. Figure 6.27 shows this transposition: we see that the non-periodic modules are the same as in the final sieve. Otherwise, the faithful transposition has one less module (18) than the final sieve; there are 6 modules with $R = 3$, whereas in the final sieve only four modules have $R = 3$.

6.5 *Nekuïa* (1981, for choir and orchestra): Original Sieve and Versions

The sieve of *Nekuïa* (shown in Figure 4.1) is the one that Xenakis explored most in his later music. It was used in numerous subsequent works and new sieves were constructed

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[73] Recall that in order to secure the smallest possible size of moduli that covers each point of the sieve, we always have to set the lowest point of the sieve to zero. Therefore, in the final sieve of *Aïs* 0 = A0 and in the original 0 = B0.

with this one as their starting point. I will analyse individually its versions as found in other works, after I analyse the original and its transformations used in *Nekuïa*. Itself bears a straightforward relationship with the sieve of *Jonchaies* which was naturally passed on to the derived sieves as well.

The sieve was used partially in *Serment* (for mixed choir), completed shortly afterwards in the same year,[74] in 1986 in *Keqrops* (for piano and orchestra) and in *Tracées* (1987, for orchestra). Harley has indicated the symmetric (palindromic) intervallic structure for the part of the sieve in *Serment*, the fragment from F#2 to G#4 (2004: 135). With the C4-E4 major 3rd as the axis of symmetry, the intervallic structure is a palindrome extending from the A2-A#2 semitone to the F#4-G4 one. Also, this part of the sieve is restricted to three of the four expected intervals, omitting the tone. This is the reason why Gibson considers this part of the sieve as a perfect example of interlocking 4ths. He also indicates the fragment between A4 and C5, which has eight out of the nine pitches of the sieve of *Jonchaies* (2003: 73).

Xenakis notes the label of this sieve on the score of *Serment*. This note, more than a simple label, reveals the function that he had used in order to produce the sieve. We read an intersection of two moduli: $8_i \cap 11_j$.[75] In this transitive period, Xenakis was still working with decomposed formulae. The sieve of *Nekuïa*, in aesthetic terms belongs to the mature phase of sieve-construction; but on the sieve-theoretical level, Xenakis treated it according to its period of 88 semitones. The sieves of this phase therefore, require an analysis that illustrates both their external periodic nature and their inner-periodic one.

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[74] Xenakis frequently provided the completion date at the end of the score.

[75] The symbol ∩ stands here for intersection, for which the symbol · is used in this dissertation.

In the decomposed matrix of the sieve of *Nekuïa* (shown in Figure 4.2) we can observe an irregular arrangement of the elements. Of course, this is not to suggest that other types of sieves (e.g. the early ones) would follow a regular pattern; but it might be the case that a decomposed matrix contains complete columns or rows, which would indicate continuous inner-periodicities. The decomposed matrix of the sieve of *Nekuïa* reveals two aspects relating to the distribution of the sieve's elements. Firstly, the limiting of the size of intervals to the major 3rd and, secondly, the exclusion of strings of chromatic pitches longer than three, affect the way elements are distributed in the decomposed matrix. This means that there is neither a diagonal succession of elements nor a diagonal gap longer than three. Secondly (and as the simplified matrix will show), the periodicities of 8 and 11 semitones (minor 6th and major 7th respectively) are not part of the sieve's inner-periodic structure: no row or column of the table is complete. The ones that might seem more prominent are scattered in a way that does not suggest a continuous periodicity.[76] We see that in the decomposed matrix of the sieve of *Nekuïa* the most populated column is (11, 3). But the periodicity of 11 semitones here appears scattered: it is interrupted at point 36 and stops before reaching its final point, 80.

On the contrary, the simplified formula of the sieve of *Nekuïa*, shown in Figure 6.28, does not include moduli 8 or 11 (this is also the formula shown in the matrix of Figure 5.1). There are 20 modules that produce its 42 points, which gives a density D = 0.48. Figure 6.28 shows that the smallest and most frequent modulus is 14 (8ve+tone),

[76] Recall the decomposed matrix of the sieve of *Akea* with its prominent periodicity of a perfect 4th [module (5, 0) in the decomposed matrix of Figure 5.3].

repeating 6 times.[77] The inner symmetry chart for the sieve of *Nekuïa* is shown in Figure 6.29. The chart accounts for the large number of moduli and the small value of *R* in each of the modules: since the modules are located towards the upper right part of the chart, the number of reprises of each modulus is small. If it was more (inner-)symmetric it would have less modules with smaller *M*-values and greater *R*-values. The two non-periodic modules are (32, 25) and (30, 29): they are located outside the limit of inner symmetry and occur only once. They cover pitches A#2, D2, F#4 and G#4. These pitches belong to the two sides of the palindromic structure at the middle of the sieve. The two latter ones are covered only by the two non-periodic modules (points 57 and 59).

The indices of the cyclic transpositions used in the work are shown in Figure 6.30. As I have mentioned in the case of the sieve of *Aïs*, when analysing a sieve, we always have to set its lowest point to be equal to zero. This is because the size of *M* (i.e. the smallest possible modulus) depends on the absolute value of *I*. Therefore, when we analyse a cyclic transposition whose lowest point is greater than zero we have to actually transpose the sieve down so that its lowest point is equal to zero. This means that, in terms of inner-symmetric analysis, any sieve has as many distinct cyclic transpositions as the number of its elements. For example, in the sieve of *Nekuïa* the lowest pitch of the T_{56}(mod88) transposition corresponds to number 2 [34 + 56 = 90 and 90(mod88) ≡ 2]; therefore, in its numeric version, we have to transpose it two semitones down, in order for its lowest point to be equal to zero, and the resulting cyclic transposition is equivalent to T_{54}(mod88). Consequently, the cyclic transpositions of T_{54}(mod88), T_{55}(mod88), and

[77] This is the formula shown in 4.3.2 and in the simplified matrix of Figure 5.1.

T_{56}(mod88), are equivalent in terms of inner-symmetric analysis.[78] The transposition that appears most frequently in the work is $8_i + 11_{j+8}$, with effective index 8(mod88) semitones. In this transposition the intervallic succession is displaced three places to the right. While a decomposed matrix reveals only the two elementary moduli that the period is decomposed into, a simplified matrix renders the different moduli in each cyclic transposition. Figure 6.31 shows the simplified formulae of the cyclic transpositions of the sieve of *Nekuïa*.[79] We see that the ones with the smallest number of modules are T_{33}(mod88) and T_{40}(mod88). These two transpositions are more distant than the rest. Note that this notion of distance refers to how far the intervallic succession is displaced in relation to the original. In this sense, T_{40}(mod88), where the intervallic structure is displaced eighteen places to the right, is more distant from the original than T_{77}(mod88), where it is displaced seven places to the left. We see that a cyclic transposition might render different inner periodicities. For example, modulus 10 (minor 7th) is shown to participate only in one of the seven versions of the sieve, T_{40}(mod88). This means that the interval of the m7th might be present in other transpositions, but not continuously. For example, in the original sieve it exists fragmentarily, between points 4, 14, 24, 34 and 44 (C#1, B1, A2, G3 and F4) and between 52, 62, 72 and 82 (C#5, B5, A6 and G7). In T_{40}(mod88) point 52 (C#5) becomes point 4 (C#1), which is the starting point of the continuous periodicity of the minor 7th: this is module (10, 4) which covers C#1, B1, A2, G3, F4, D#5, C#6, B6 and A7.

[78] An equivalent cyclic transposition of T_{56}(mod88), T_{55}(mod88) was used in *Tracées*. T_6(mod88) [equivalent of T_8(mod88)] was used in *Keqrops*.

[79] The fact that the cyclic transpositions of a sieve have different formulae with different amount of modules, confirms that the relationship between density and the number of modules is not constant.

Figures 6.32 to 6.37 show the inner symmetry charts for all the cyclic transpositions used in *Nekuïa*. Of the six transpositions, four exhibit an inner-symmetric structure (i.e. $R \geq 2$): T_{33}(mod88), T_{56}(mod88), T_{77}(mod88), T_{80}(mod88). From the four inner-symmetric cyclic transpositions, T_{33}(mod88) has the smallest number of modules (seventeen). Consequently, this particular cyclic transposition has the highest *R*-values in average and therefore it is the most inner-symmetric of the transpositions used in the work. This also is evident in the chart of Figure 6.33: its modules are located towards the left, in relation to the rest of the inner-symmetric transpositions. T_{40}(mod88) is shown in its inner symmetry chart (Figure 6.34) to also have its modules towards the left but, additionally, there is one non-periodic module: (32, 25). Whereas in the inner symmetry chart of T_{33}(mod88) the modules tend to concentrate (except perhaps one or two modules that lie towards the right), in the chart of T_{40}(mod88) the modules are shown to spread both to the left and the to the right. The degree of symmetry (or asymmetry) of the sieve, is maintained in the latter transposition by the inclusion of both highly periodic (left) and less, or non-periodic modules (right). It is interesting here that these two transpositions are only a perfect 5th apart. The less inner-symmetric transpositions are T_8(mod88) and the original (T_0) with two non-periodic modules.

6.5.1 Sieve of *Nekuïa*: First Version

The first version of the sieve of *Nekuïa* is the one used in *Naama* (1984, for harpsichord), *Alax* (1985, for 3 ensembles), and *à r.* (1987, for piano). It differs from the original only in two pitches: A2 is replaced by G#2 and there is an additional F3 – see Figure 6.38. In *Naama* the sieve is used between G1 and E6; in *Alax*, a shorter segment, between F#2 and D5, and in *à r.* it is used in all its range. We see that segments of different length were occasionally used. According to Xenakis's suggestion, the application of his algorithm should take into account as many points as possible (in order to secure a precise logical formula). The simplified formula of Figure 6.39 is the logical expression of the complete version of the sieve of *Nekuïa* as used in the aforementioned works. It has 19 modules, instead of 20 in the original. The inclusion of F3 (point 32) brings about modules (15, 2) with $R = 5$ and (25, 7) with $R = 3$. The latter also produces F#4 and therefore the non-periodic module (32, 25) of the original sieve is redundant. But at the same time, the absence of A2 (point 24) brings about a non-periodic module in the new version: point 53 is now covered by the non-periodic (36, 17) instead of (29, 24) in the original. This results in a formula with two non-periodic modules, as shown in the inner symmetry chart of Figure 6.40, that is not much different from that of the original. The only difference is that in the new version there are two modules that stand out as more periodic than the average (instead of one in the original), whereas the total number of modules is not significantly smaller.

We can compare the above analysis with the analysis of one segment of the sieve, as it is used in a work. If we examine only the segment of the sieve that is used in *Naama* the simplified formula suggested by the algorithm is the one shown in the inner symmetry chart of Figure 6.41. We see that this segment has only periodic modules. In the complete version of the sieve pitches D5 and G#5 where covered only by non-periodic modules (36, 17) and (30, 29) respectively; in the segment they are covered by periodic modules (15, 13) and (15, 4). This is an expected characteristic of sieve analysis: when only a segment of a sieve is examined, it is natural that different elements are periodically associated with each other differently than when analysing the complete sieve. What is important here is that the segment used in *Naama* retains the same general characteristics of the complete sieve. Firstly, their density is the same: D = 43/88 = 0.49 for the complete sieve and D = 28/57 = 0.49 for the segment. Secondly, they share a similar intervallic structure. However, the two inner symmetry charts of Figures 6.40 and 6.41 have a different arrangement of modules: the latter has only periodic modules with $R = 2$ and $R = 3$, whereas the former has two non-periodic modules and two modules located further to the left (which means that they are more periodic than the average).

6.5.2 Sieve of *Nekuïa*: Second Version

In *Alax* Xenakis also used another, slightly different version of the sieve of *Nekuïa*. G#2 and A#2 of the previous version are now replaced by G2 and A2, whereas there is an additional C#3. This version and its inner symmetry chart are shown in Figure 6.42. There are here 18 modules (while the previous version had 19) and there is only one non-periodic module. We also see that the addition of a single element (along with the alteration of two other elements) in the sieve caused the modules to concentrate more to the left in relation to the original sieve and the first version. Although there is still one module outside the red curve, this version has two modules with $R = 4$ and two with $R = 5$. Since this inclusion of more periodic modules is not combined with a corresponding inclusion of less periodic ones, we can see a progression to a sieve with more inner-symmetric elements than its previous versions.

6.5.3 Sieve of *Nekuïa*: Third Version

In the version used in *A l'île de Gorée* (1986, for harpsichord and ensemble) A2 is replaced by G#2, like the first version, but the additional pitch is now F#3 (instead of F3); also, there is an additional G6. It is shown, along with its inner symmetry chart, in Figure 6.43. We see that only two different pitches change the way the formula is shown in the chart. There is now one more module: 20 modules instead of 19 cover the 44 points of the sieve. Having in mind that both in the first and the third version there are two non-periodic modules, the fact that more modules are required for the latter one, means that its inner symmetry is now even less obvious. The segment actually used in the work extends from E1 to G6 (see Figure 6.44). The density of this segment is the same as the one of the complete sieve, $D = 32/63 \approx 0.5$, but the chart of the segment shows an inner-symmetric structure with no modules outside the red curve. As with the segment used in *Naama*, this one too, has the same general character of the complete sieve (in terms of density and intervallic structure); but here the segment is shown to be inner-symmetric whereas the complete sieve is not. In general, the inner-symmetric character of a sieve does not necessarily imply that any of its segment will be similar in terms of inner symmetry. Therefore, analysing the original, complete sieve (when it is known), is preferable to analysing only a segment of it. This is the reason why Xenakis suggested that we should take into account as many points as possible. Once the complete sieve has been analysed, segment analysis might also be desirable in order to examine the properties of the sieve.

6.5.4 Sieve of *Nekuïa*: Fourth Version

6.5.4.1 Sieve

The fourth version of the sieve of *Nekuïa* is a T_{84}(mod88) cyclic transposition of the first (or equivalently, a T_{-4}(mod88) cyclic transposition). It is used in *Horos* (1986, for orchestra) and *Jalons* (1986, for ensemble). The sieve of *Akea* (analysed in Chapter 5) is also based on this version, but is identical to it only between pitches G1 and D#6. This version of the sieve is shown in Figure 6.45. There are 18 modules in the simplified formula – two modules less than that of the original sieve. Since the number of modules is smaller, their average *R*-value is greater and consequently they appear in the chart further to the left than the modules in the formula of the original. More importantly, this version exhibits an inner-symmetric structure: all modules satisfy the condition of inner symmetry and are located inside the limits of the red curve. 9 modules are in the 'R = 2' region of the chart, 6 in the region where R = 3, for (19, 6) R = 4 and two modules, (15, 13) and (14, 12), repeat for 5 times. Recall that the arrangement of the modules in the inner symmetry chart for the original sieve (Figure 6.29) has most of them concentrated to the region of the chart where R = 2.

The actual segment of this version that was used in *Horos* is shown in the sketches. It is the segment between F#1 and D#7 (69 semitones). The inner symmetry chart of the segment used in *Horos* is shown in Figure 6.46. It has one non-periodic module, with most of its periodic modules having the minimal *R*-values. As in the complete version of the sieve, there are 18 modules but they produce 33 points. In this sense, the number of modules is relatively larger than the number of modules in the complete sieve (18 modules that produce 42 points). Therefore, the periodic modules in the inner symmetry chart of Figure 6.46 appear to the extreme right, which denotes a smaller degree of symmetry. Again here we see that although a sieve might be a segment of an inner-symmetric one, it does not necessarily retain the same degree of inner symmetry.

6.5.4.2 Complement

In *Akea* the complement of the sieve appears as frequently as the original, and is also used in *Horos*, and partially in *Jalons*. It is shown in Figure 6.47, along with its inner symmetry chart. The formula contains 19 modules, and produces 45 points in a range of 80 semitones (D = 0.56). In comparison to the 17 modules of the original sieve (Figures 5.2 & 5.20) that has 37 points (D = 0.46), the complement has only periodic modules. Furthermore, the modules in the complement appear more concentrated; this means that they all contribute almost equally to the structure of the sieve (their R-value is 2 or 3 with one $R = 4$). The fact that the number of modules in the complement is larger does not contradict the fact that it is inner-symmetric; since the number of points is larger for the complement, more modules are required (given that the intervallic structure is as irregular as that of the original).

6.5.5 Sieve of *Nekuïa*: Fifth Version

The same cyclic transposition is where another version of the sieve is based; it is used in *XAS* (1987, for saxophone quartet) , and in relation to the previous version C#3 is omitted and there is an additional B3 (see Figure 6.48). There are 20 modules, just like the original sieve, but all modules are periodic. Unlike the fourth version, the modules are mainly concentrated in the region of the chart where $R = 2$, which is a consequence of the larger number of modules in relation to the fourth version. In fact, the literal cyclic transposition T_{84}(mod88) of the original is itself inner-symmetric, as its inner symmetry chart of Figure 6.49 suggests. There are 21 modules and they are all periodic. But since their number is larger than the fifth version, they are concentrated more to the right. Therefore, the literal T_{84}(mod88) (not actually used in any composition) is less inner-symmetric than the fifth version (20 modules), which itself is less inner-symmetric than the fourth (18 modules).

6.6 *Komboï* (1981, for harpsichord and percussion)

The opening sieve of *Komboï* was also used partly in another work of the same year, *Pour la Paix*. Is deviates from Xenakis's usual practice of constructing sieves with the greatest interval the major 3rd. As shown in Figure 6.50, there is only one major 3rd, the lowest interval of the sieve. It is therefore a sieve with greater density than one would expect: D = 0.52 (44 elements occupy a range of 84 semitones). The simplified formula includes 19 modules. Recall that the same number of modules produced the sieve of *Nekuïa* which is slightly less dense, with D = 0.48. In this respect then, the opening sieve of *Komboï* is more inner-symmetric than that of *Nekuïa*. Indeed, whereas in the latter there are two non-periodic modules (Figure 6.29) in the chart of Figure 6.50 only one module is outside the inner-symmetry curve, and more periodic modules are concentrated in the region of the chart where $R = 3$ than that where $R = 2$. But if we compare it with the T_{80}(mod88) transposition of the sieve of *Nekuïa*, which has 20 modules all of which are periodic, we see that in the sieve of *Komboï* most of the modules are more periodic, with a non-periodic breaking the inner-symmetry.

There are several sieves in the duo, but most of them appear only partly, and this prevents a complete analysis. However, later in the work one sieve appears in a great range; it is shown in Figure 6.51. This sieve is less dense (it includes intervals of major 3rd); there are 37 points in a 82-semitone (D = 0.47) range with 14 modules, one of which is non-periodic. The small number of modules does not imply that these are, in average, more periodic than those of the opening sieve, due to the fact that the density and range of the sieve (as it appears in the work) are smaller than that of the opening one.

6.7 *Shaar* (1982, for string orchestra)

In the sketches of *Shaar* Xenakis experimented with four different sieves, labelled ***α***, ***β***, ***γ***, and ***δ*** (shown in Figure 6.52), from which he used sieves ***γ*** and ***δ***. Sieve ***α*** bears an irregular, non-repetitive structure, but the three that follow are periodic with periods 5 for ***β*** and 7 for ***γ*** and ***δ***. The inner symmetry chart of ***α*** is shown in Figure 6.53. It has relatively low density D = 29/73 = 0.40, and the 12 modules that produce the sieve are enough to reach and exceed the limit of inner symmetry.

Xenakis commented on the page of the sketches: ‘Sieves based on a period, difficult for 12 (5, or 7)’. Sieve ***β***, is a perfect example of interlocking 4ths. When two 4ths interlock the periodicity is the 4th itself (with intervallic structure: 1 4 and so on). But after 12 reprises of modulus 5 the sieve starts repeating in terms of octave-equivalent pitches. This would give a greater period of 5·12 = 60 semitones. The case of sieves ***γ*** and ***δ*** is not as straightforward. Sieve ***δ’*** is a reconstruction of ***δ***, according to the intervallic pattern evident up to D#5 (the identical part of the two sieves). Both ***γ*** and ***δ'*** are based on the period of the perfect 5th. In both these two sieves, the octave-equivalent pitches would appear at point 7·12 = 84 semitones. But the period of the 5th is not apparent, because the intervals it is broken down to do not always appear in the same order. In sieve ***γ*** each cycle of 5th consists of a different permutation of three intervals that the 5th is broken down to: 1, 2, and 4; and in sieve ***δ'*** of permutations of the four intervals 1, 1, 2, and 3. The sketches provide the simple way of determining the order the permutations appear in sieve ***δ*** up to D#5 (see Figure 6.54); sieve ***δ'*** is precisely the reconstruction of ***δ*** according to this system of intervallic permutations. The number of all the possible permutations of 1, 1, 2, and 3 is 12. Xenakis wrote down in a column the six permutations that correspond to all the possible positions of the interval of the semitone, and with 3 always preceding 2; in a second column to the right, he wrote the remaining six permutations where 2 precedes 3. There is therefore a symmetric relation between the two columns. He started on the top left entry: 3 2 1 1. These are the first four intervals in sieve ***δ'*** (see Figure 6.52). He then used the permutation on the bottom right: 2 1 1 3. The numbers next to each permutation in the table of Figure 6.54 show the order of appearance of these permutations. The process continues similarly: the second permutation of the left column is followed by the second to the last permutation in the right column, and so on. The last permutation marks the end of the period of the sieve; the first permutation would appear again at point 84, and the sieve would be reproduced at the octave equivalent after 7 octaves. We see that the intervallic structure of sieve ***δ*** is identical to that of ***δ'*** but the former has an interval of 3 instead of 2 semitones (D#5-F#5 in ***δ*** instead of D#5-F5 in ***δ'***) and an additional minor 3rd (F#6-A6). Xenakis used, peculiarly, a sieve that is based on the system of intervallic permutations noted in the sketches, but with these two deviations.

The sketches do not provide the system for the permutations of sieve ***γ***, but we can reconstruct it. The possible permutations of three distinct elements are 6. In Figure 6.55 the left column shows five of these permutations, whereas at its bottom the initial permutation (4 1 2) re-appears. In the second column each permutation is the 'retrograde' of the corresponding one in the left column. Again, in the right column there are only five distinct permutations, with the top and bottom entries being the same. The order of appearance of each permutation in the sieve is the same as that of sieve ***δ'***. The reason for having only five distinct permutation in each column is related to this order. If the bottom left entry was the 6^{th} remaining permutation (that would be 2 1 4), its retrograde at the bottom of the right column would be the same at the initial one (4 1 2); thus the two first permutations in the intervallic structure of the sieve would be identical. In order to avoid this, we enter the retrograde of the remaining permutation instead (which is equivalent to swapping the two permutations at the bottom of the two columns). The consequence of this is that the initial permutation (4 1 2) appears again as the 11^{th} permutation, before the sieve reaches its period (i.e. it appears at point 70 instead of 84). Thus, the three final intervals in sieve ***γ*** (4 1 2) do not denote the recurrence of the period; this would actually happen at point 84, after all the 12 permutations of the table in Figure 6.55 have appeared.

6.8 *Tetras* (1983, for string quartet)

Similarly to the sieve of *Komboï*, the sieve of *Tetras* is mainly based on the three intervals equal to or smaller than the minor 3rd. As shown in Figure 6.56, there is only one major 3rd; its density is D = 0.57 (38 points with n = 67).[80] However, this sieve is even more dense than that of *Komboï* (D = 0.52). The fact that the density (as well as the range) of the two sieves is different does not facilitate comparison. Had the sieve of *Tetras* been checked for the length of the opening sieve of *Komboï*, it would appear with all its modules inside the limits of inner symmetry [module (25, 20) would have R = 2]. But, although we could allow its 16 modules to cover a range up to the range of the sieve of *Komboï*, the resulting sieve would not be identical (this would have shown in its formula). However, the inner symmetry chart shows the position of each module in relation to the extreme values in any inner-symmetric sieve of the same length. If we compare the charts of the two sieves (Figures 6.50 and 6.56) we see that they have a similar degree of symmetry. Although the two sieves have different length and density, their inner symmetry charts show, in approximation, the relationship between the two: most of the modules are in the R = 2 and R = 3 region, one stands out as more periodic, and one is non-periodic.

A slightly different version of the sieve of *Tetras* is used in the opening of *Khal Perr* (1983, for brass quintet and two percussionists); it is shown in Figure 6.57. Its range and density are even smaller: 31 points over 60 semitones, give D = 0.52. The number of modules is 13, and the arrangement in the chart is not very different. The one non-

[80] This is based on the sieve as it appears on the score. Naturally its range is limited to the string quartet's range; however, it is still a relatively extensive scale, that allows analysis and comparison with other ones.

periodic module, (22, 17) is by one semitone so: in inner-symmetric sieves the maximum M-value for $I = 17$ is $M = 21$; and the maximum I-value for $M = 22$ is $I = 16$. In general, this sieve shows slightly more leftward arrangement, but not enough to suggest a different, more symmetric structure.

6.9 *Lichens* (1983, for orchestra)

The sieve of *Lichens* presents the least inner-symmetric structure from the sieves examined so far (apart from the sieve of *Mists*). Compared with the sieve of *Akea*, which has similar density and range, with 16 modules, the sieve of *Lichens* would not, at a first glance, be expected to be more asymmetric; it has 35 points in a range of 78, with 14 modules (for the sieve of *Akea* these values are 37, 80, and 16 respectively). But the appearance of the modules in the chart of Figure 6.58 shows that even at the lower part of the sieve there is a non-periodic element. There are four non-periodic points (while in the sieve of *Akea* only one). Module (39, 2) covers two points that are not covered by any other module: F#1 and A4 (points 2 and 41). Modules (28, 27) and (31, 27) produce pitches B5 and D6 respectively (points 55 and 58). This observation verifies that the number of modules is a secure index of the degree of inner-symmetry of a sieve, only for periodic modules. This is because the algorithm always finds two points for a given module, when the value of I is less than half of the value of n (whereas the validating of periodic modules depends on the intervallic structure itself). For $I = 2$, in the sieve of *Lichens*, any M-value between 3 and 38 would validate a module as periodic. Therefore, although the sieves of *Akea* and *Lichens* have similar size, density, and number of modules, they are not similar in terms of inner symmetry; the intervallic structure of the latter is what differentiates it from the former.

6.10 *Thalleïn* (1984, for ensemble)

The sieve used in *Thalleïn* is among the very few sieves of the later period that includes five intervals – from the semitone to the perfect 4th (see Figure 6.59). Along with the sieve of *Nekuïa* it was also used later in *Keqrops*. Its inner symmetry chart shows that its modules are arranged to the right region, with the smallest *R*-value (2). From the 18 modules that produce its 32 points (when n = 78 and D = 0.41) three are produced only by the three non-periodic modules that lie outside the red curve; these are pitches F3, D#5, and E5 (points 30, 52, and 53). If compared with the sieve of *Lichens* (which has the same *n*-value and the same number of non-periodic modules), although it is less dense, the sieve of *Thalleïn* has more modules. This is reflected in the inner-symmetry chart, as the periodic modules are located more to the right; this means that the periodic modules of *Thalleïn* produce a less inner-symmetric structure than the periodic modules of *Lichens*.

A four-octave segment of the sieve of *Thalleïn*, was used, with slight differences, in *Alax*; the segment from C#2 to C6 appears with an additional D2; G2 instead of A2; and D 5 instead of D#5. Figure 6.60 shows the complete sieve of the former work with the alterations that appear in the latter. The inner symmetry chart shows that this version of the sieve still has three non-periodic modules. Especially, the substitution of the D#5 with D5 has replaced non-periodic module (27, 25) by (28, 23). The total number of modules is now 17, with D = 0.42. These two slight differences could suggest a more symmetric version of the original; however the general character of the sieve remains predominantly inner-asymmetric.

6.11 *Keqrops* (1986, for piano and orchestra)

Keqrops is perhaps the work with the most extensive employment of sieves up to the time. It is based on three sieves: the sieve of *Nekuïa*, the sieve of *Thalleïn*, and a sieve that is used for the first time in *Keqrops*. From the two former sieves, that of *Nekuïa* is used either as such or in two of its cyclic transpositions: T_6(mod88) and T_{64}(mod88). T_6(mod88) is, in terms of inner-symmetric analysis, equivalent to T_8(mod88), whose inner symmetry chart is shown in Figure 6.32. T_{64}(mod88) is one of the inner-symmetric cyclic transpositions of the sieve of *Nekuïa*, as shown in the chart of Figure 6.61.

The main (more frequent) sieve of *Keqrops* is shown in Figure 6.62. The sieve has the same range as that of *Aïs*, but is slightly more dense, with D = 39/86 = 0.45 (whereas for *Aïs*, D = 37/86 = 0.43). If we compare the charts of the two sieves we see that whereas in both of them most of the modules are in the R = 2 region, the sieve of *Keqrops* has less modules by 1, and some of its modules have R = 4, 5, and 6. The one non-periodic module is (31, 25) and the point that causes this asymmetry is A#2 (point 25). The sieve of *Keqrops* was used in three subsequent works, *A l'île de Gorée*, *Tuorakemsu* (1990, for orchestra), and *Kyania*. The sieves and their charts are shown in Figures 6.63 to 6.65. They all appear in a relatively long range, so they are analysed as such (instead of analysing the reconstruction of the original sieve). All three versions cover slightly different range and have almost the same density (D = 32/65 = 0.49, D = 34/70 = 0.49, and D = 38/79 = 0.48 respectively), but the version in *A l'île de Gorée* is the most inner-symmetric one (even more symmetric than the original sieve); it is the only version with all its modules being periodic and it has a smaller number of modules than the rest (12).

6.12 SWF

In the sketches of *XAS* Xenakis noted a sieve, labeled SWF, which he did not finally used in the quartet, but in a work of the same year, *Ata* (1987, for orchestra).[81] SWF, loosely related to the T_{84}(mod88) transposition of the first version of the sieve of *Nekuïa*, is shown in Figure 6.66. Its range is 72 semitones and the number of points is 35 (which gives a density D = 0.49). In the chart we see an arrangement with 3 of the 15 modules in the more symmetric region (where R = 4), 4 in the region where R = 3 and the rest with R-value 2. One element with R = 1 does not satisfy the condition of inner symmetry. Although the SWF derives from the T_{84}(mod88) transposition of the sieve of *Nekuïa*, there are several differences among the two. The segment of the T_{84}(mod88) transposition that corresponds to the range of SWF has 32 points, and 7 points of SWF do not belong to the transposition. Therefore, the distance between the original and the SWF is much greater that of earlier versions and an analysis of the T_{84}(mod88) transposition would not be appropriate for SWF.

The complement of this sieve, labelled CSWF, was used in *Ata* and with slight alterations in *Kyania*. It is shown in Figure 6.67. It is more dense (D = 39/73 = 0.53) and all of its modules are periodic. The number of its modules is 14 and since they are all

[81] *Ata* was a commission by Südwestfunk (‘Southwest Broadcasting’, abbreviated SWF), in Baden-Baden, Germany.

periodic, this number accounts for the degree of its inner symmetry. Especially, module (10, 0) repeats for 7 times and there are only 5 modules with $R = 2$. In fact, this is not the literal complement of SWF; the expected C#7 is missing and F7 should be F#7. The formula of the complete CSWF would have module (22, 16), with $R = 2$, in the place of (19, 16), and two extra modules: (24, 2), with $R = 3$, and (24, 21) with $R = 2$. We see that this slight alteration does not change the inner-symmetric nature of the sieve. On the contrary, since the less the number of modules the more symmetric the sieve, this alteration increases the degree of symmetry.

6.13 SWF'

In the sketches of *Waarg* (1988, for ensemble) Xenakis notes another version of the previous sieve, labeled SWF'. For this sieve, as well as for its complement, Xenakis used an intermediate stage of his algorithm. Whereas in the algorithm of 1986 he validated any size of M, and in the final one he validated only $M > I$, for the SWF' the sketches provide a formula where $M \geq I$:

(19, 0) + (18, 2) + (24, 5) + (29, 6) + (27, 10) + (16, 11) + (22, 13) + (21, 14) + (26, 16) + (21, 21) + (24, 24) + (33, 30) + (34, 34) + (35, 35) + (45, 45) + (49, 49) + (51, 51) + (66, 66).

Like with the sieve of *Mists* and Squibbs's observation, SWF' is likely to have been constructed before calculating its formula. In the sketches Xenakis wrote down the sieve up to $n = 72$, calculated the formula, and then applied the formula for 150 points. In other words, he extended the initial 37-element to a 150 one, through the formula of the former. Afterwards, he checked the sieve past point 72, to see whether it retains a similar intervallic structure as the initial segment. In particular, he made a note of all the successions of semitones. In a separate page in the sketches, the same procedure is found for the complement of the (37-element segment of) the sieve. He also made a note of the points that are common to both the sieve and its complement, past point 73. In both cases, it seems that the longer segments did not qualify the aesthetic criteria: their intervallic succession frequently includes strings of semitones longer than 2. In the inner symmetry chart, this formula would appear with 9 out of its 18 modules outside the red curve. In fact the modules that would appear inside the limits of inner symmetry would be the ones that are the same with the formula that the final algorithm suggests. Furthermore, the final algorithm suggests 16 modules, which imply a greater symmetry.

The range and density of the sieve are similar to those of SWF: D = 37/72 = 0.51, when SWF' has one more module in its formula (16). If we compare its inner symmetry chart (Figure 6.68) with that of SWF (Figure 6.66) we see that the more recent sieve is slightly more asymmetric. They both have a non-periodic module, but SWF' has 9 of its modules in the $R = 2$ region of the chart and only one module with $R = 4$, whereas in SWF there are 7 and 3 modules in these two regions, correspondingly.

The complement, labeled CSWF', is shown in Figure 6.69. Like CSWF it is inner-symmetric and is more dense than its original, but it has more modules (17). Compared with CSWF (with 14 modules) it is less symmetric by three modules. This is also shown in the inner symmetry chart. There are as many modules with $R = 2$ as there are with $R = 3$ and only two modules with $R = 4$. The connection between SWF and

SWF’ is shown in terms of inner symmetry: in both cases, the original sieve has one non-periodic module and the complement is produced only by periodic modules.

Another version of SWF’ was used in *Krinoïdi* (1991, for orchestra). Figures 6.70 and 6.71 show the sieve and its complement. We see that only two pitches at the bottom of the original sieve are different from SWF’, and the three lowest pitches of the complement are different from CSWF’. As with the previous sieves, the sieve of this work has one non-periodic module and its complement has only periodic modules. The density of the two is the almost same (D = 33/64 = 0.52 and D = 35/68 = 0.51 respectively) and in both cases there are 15 modules. In terms of inner symmetry, the sieve of *Krinoïdi* (Figure 6.70) is very similar to SWF’ (although with an extra module): it has most of its periodic modules in the two regions at the right, where R = 2 or 3, and only one with R = 4. The complement of the sieve of *Krinoïdi* (Figure 6.71) is slightly more symmetric than CSWF’. Since these two are essentially the same sieve, the inclusion of an extra module with R = 5 in the former, accounts for this increased symmetry.

In *Kyania* Xenakis also used a sieve that is similar to the ones analysed in this section, but also to some versions of the sieve of *Nekuïa*. It is shown in Figure 6.72. Only its upper half derives from CSWF, whereas the lower range differs significantly. In its inner symmetry chart, one module is non-periodic and only one module is more symmetric than the majority of R = 2.

6.14 ASK

For *Echange* (1989, for bass clarinet and ensemble) Xenakis used a sieve, which is unique to this work. Its range is 82 semitones, and differs from most of the sieves of the late period in that it includes stings of semitones longer than three. In the sketches he labelled it ASK, an obvious reference to the ASKO Ensemble for which the work was composed. Xenakis used the same version of his algorithm as he did for SWF'. Also, like SWF' he derived the formula of the initial 43-point segment and produced a longer sieve in order to check its properties. This time it was a segment of 200 points and he noted not only the long strings of semitones, but also intervals that exceed the major 3rd. Unlike the formula of the previous sieve, the formula in the sketches of *Echange* includes only one module where $M \geq I$. The inner symmetry chart for the formula found in the sketches is shown in Figure 6.73. The final algorithm suggests a formula that that differs precisely in the three non-periodic modules of the formula of the sketches. This formula, along with the sieve, is shown in Figure 6.74. It has 15 modules, all of which are periodic, with *R*-values reaching 6. Like with the previous sieves, the complement (shown in Figure 6.75) is less symmetric.[82] It has 19 modules, of which one is non-periodic, that produce 39 points. The non-periodic element is F#6, produced only by module (32, 30).

6.15 *Epicycle* (1989, for violoncello and ensemble)

The sieve of *Epicycle* was used by subsequent works, such as *Knephas* (1990, for choir), *Tuorakemsu* (1990, for orchestra), with alterations in *Tetora* (1990, for string quartet), and *Roaï* (1991, for orchestra). It is shown in Figure 6.76. Its range is 81 semitones, which is an expected one, but the number of points is somewhat larger than most of the

[82] The formula for the complement of ASK in the sketches produces the sieve up to B7.

sieve of the later period: 45 points produce a density D = 0.55. The inner symmetry chart shows that there is a large number of modules (22) with two of them being non-periodic [(29, 28) and (30, 28)]. The maximum starting points for $M = 29$ and $M = 30$ are $I = 23$ and $I = 21$ respectively; and the highest I for a sieve of $n = 81$ is $I = 26$. That is, there is no M-value for an inner-symmetric sieve with $n = 81$ that starts on point 28. The points that are not part of a periodic module are 28 itself and 58, which correspond to pitches E4 and A#5. We have to note that even thought E4 is covered by two modules, this does not mean that it belongs to an inner periodicity; this is because it is produced only by non-periodic modules (albeit more than one).

In *Tetora* Xenakis used a sieve that derives from that of *Epicycle* and its complement. Figure 6.77 shows the one that derives from the original sieve of *Epicycle*. From its 33 points only 24 are common with the original sieve. The inner symmetry chart shows a great degree of symmetry: 12 modules, all periodic with R-values greater than 2, except for one module. The range of the sieve is much smaller than that of the original, but its density is D = 0.57, which is slightly higher. On the contrary, the complement does not share the same degree of symmetry. Figure 6.78 shows the complement as it appears in the sketches; in fact it is only an exact complement after A2. The large number of non-periodic modules in this sieve show its highly asymmetric structure. Its range is 59 semitones, but its density is much lower, D = 0.46.

6.16 *Paille in the Wind* (1992, for violoncello and piano)

A segment of the sieve of *Paille in the Wind* was in fact used in a work of the previous year, *Dox-Orkh* (for violin and orchestra). It is shown Figure 6.79. Its 20 periodic modules produce 43 points in a range of 80 semitones. The symmetric structure is shown in its inner symmetry chart: it is a typical inner-symmetric structure, with the most of the modules in the $R = 3$ region of the chart and three with $R = 4$. As with several sieves of the same time, it has very few major 3rds; especially in the middle range, it has more frequently the interval of a tone. Its density (D = 0.54) is slightly lower than that of the sieve of *Epicycle*. When compared to the latter, the sieve of *Paille in the Wind* is more symmetric: these two sieves occupy the same range with roughly the same density, and the more recent sieve has 20 modules that are all periodic. Unlike the sieve of *Epicycle*, these modules are all concentrated, with no module appearing isolated in the left of the chart. This is natural for an inner-symmetric sieve with a large number of modules.

7 Inside-Time Analysis

7.1 Inside-Time Employment of Sieves

The term 'inside-time' employment of sieves, does not necessarily mean rhythmic sieves. The inside-time refers to the placing and treatment of sieves in the composition. In this chapter I will explore the most frequent ways that sieves are treated in the late music; I will provide examples of such treatment and I will analyse in more detail three works of the period: *Akea*, *À l'île de Gorée*, and *Tetora*. Appendix 3 provides the full scores of these works, as well as score excerpts of other works I will refer to. Xenakis used his sieves in various ways, but always their outside-time aspect remained more important for him. In almost all cases, sieves are used so that their intervallic structure is revealed in the most immediate manner. Inside time, sieves are treated in ways that can be either linear or vertical. The following quotation shows that sieves are not related to melodies and chords; rather they function by generating different timbres in the different ranges of the sieve:

> The structure of the melodic scale is very important, not only in melodic patterns – melodies – but also in producing chords of a different timbre. If you take a given range, and if the structure of the scale is rich enough, you can stay there without having to resort to melodic patterns – the interchange of the sounds themselves in a rather free rhythmic movement produces a melodic flow which is neither chords nor melodic patterns. […] They give a kind of overall timbre in a particular domain (Varga 1996: 145).

That is, it is not necessary to construct melodic or harmonic patterns. The issue is the free interchange (movement not to any specific direction) of sounds themselves – pitches or chords follow one another on the continuum of the sieve. In any case, whatever kind of movement (or stasis) we have, we do not expect the music to jump to and from distant pitches and chords:

> Tension is important for the melodic patterns, the chords, and for the flow of the music itself. In chromatic and well-tempered scales you can generate tension only through jumps, as in serial music. When the notes are closer to each other, as in the chromatic scale, you lose tension, unless you apply a kind of sieve locally – that is, you choose intervals that produce some tension (Varga 1996: 145).

The idea of tension (among others) motivated Xenakis to build sieves by juxtaposing smaller and larger intervals. On another level, the same aesthetic criterion can be found in the tension between symmetry and asymmetry, as a general principle in Xenakis's thought (cf. Xenakis 1996: 147). Having constructed a scale, a sieve in our case, in such a way that a certain succession of intervals is achieved, then its structure must be made perceptible merely allowing this intervallic succession to reveal its character. In other words, scale-construction is an essential part of the compositional process, located outside-time.

In general there are three means of producing timbres with sieves. The most obvious is allowing the sieve's intervallic structure to be perceived as a continuum (linear treatment). The notion of sieve-as-timbres is also strongly related to the construction of the chords, i.e. a type of cluster applied to the continuum of the sieve. I will refer to such chords as *sieve-clusters*. Finally, an extension of the idea of the sieve-cluster leads to the construction of chords. Chords in Xenakis's later music might not always derive directly from the sieve; but their intervallic structure is similarly irregular and consists of a combination of smaller and larger intervals.

The style of Xenakis's music in the later period is characterised by the abandonment of the earlier glissandi and microtonal structures. Gradually, he arrived at a sieves and chords that have the semitone as their unit distance; his compositions became increasingly based on sieves and the general aesthetics of sieve construction and employment. Once sieves were constructed they were used in less formalistic ways than earlier works, e.g. *Nomos Alpha* (1965-66). The transition took place in the mid 1980s, although it had started with the first employment of sieves in the late 1970s. In particular, although Xenakis used certain types of sieves as early as 1980, he abandoned his older style of writing some years later. As a further progress of Xenakis's compositional technique, Solomos (1996: 96) refers to the declamatory rhythms and standard instrument-group dialogues in the works after 1984. These elements replaced the style of previous periods. For example, in 1986 he composed six works that make minimal use of glissandi. However, with sieves of the later period linearity is in some way retained: it is precisely the free interchange of the notes, without being necessary to resort to melodic shapes or patterns. Similarly, the idea of mass sounds (which he had introduced from the initial stages of his compositional output) is also retained with sieves. In many cases his orchestral works exhibit large blocks of sounds that are based on simultaneous expressions of the same or different sieves. Before I go on to analyse some works of later period more deeply, I will explore some of the characteristic ways Xenakis used sieves.

Probably the most characteristic sieve-employment technique is the one I have already mentioned: 'halo sonority' (see Solomos 1996: 84-6). This is the characteristic opening of *Jonchaies*, but also of later works, such as *Echange* (1989) and *Tetora* (1990), or the final section of *Shaar* (1982).[83] In the latter cases, the technique originally applied to orchestral scores was used in smaller ensembles. In order to do this, Xenakis altered slightly the method of application. In *Echange* and in *Tetora*, the opening is similar: there is one line that stands out from the other instruments and expresses pitches from the work's sieve (see Appendix 3). In *Echange* this is the bass clarinet, and in *Tetora* the second violin. Five instruments from the ensemble in the former, and the three remaining strings in the latter, play the same notes with the main melodic line, but with softer dynamics. This results to the following effect: each new pitch is doubled by an additional instrument, which holds it until it has to double another pitch of the main line again, and so on. Thus, the first few pitches bring about a thick sonority that is complete when all the instruments in the passage have entered. It is important to note here that this is not intended to be perceived as an accompanied melody. The elements of the sieve are not used in a pattern; rather, they are neighbouring pitches (on the continuum of the sieve) that produce a certain timbre, enhanced by the 'artificial reverberation' produced by the other instruments.

Other inside-time techniques Xenakis used for the employment of sieves include the graphic approach of arborescence and the algorithmic process of cellular automata. The former is a technique first used in *Evryali* (1973, for solo piano) and it is a case of several simultaneous melodic lines that start from a common source and result in an expansion by branching out (see Matossian 1986: 228-38; Solomos 1996: 69-72; Varga1996: 88-91; Squibbs 1996: 116-22 & 2002: 100-3; Gibson 2003: 162-6; Harley

[83] Harley argues that '[t]he heterophonic bundling of the melody […] is a technique Xenakis adopted as far back as *Terretekhtorh* [1965-66]' (2001: 40).

2004: 79-88).[84] Although this technique is characteristic of an earlier period (and was not used only in the piano), it is still found less frequently in later works that involve the piano. Cellular automata is perhaps Xenakis's only formalised technique not developed in his theoretical writings, except in the preface to the revised edition of FM (xii). Cellular Automata are discrete dynamic systems, based on simple rules but exhibit complex self-organising behaviour. The earliest work where there is evidence of this technique is *Horos* (see Hoffmann 2002: 124-126; Gibson 2003: 166-8; Harley 2004: 176-180; Solomos 2005b).

In many works Xenakis used one sieve only, in combination with its complement and/or its (cyclic) transpositions. But there are also many compositions where he used several sieves, which as I have shown, might belong to previous works. In *Keqrops* Xenakis also used two transpositions of the sieve of *Nekuïa*. Additionally to these two transpositions, the piano part in the final section of *Keqrops* plays eight sieves that are based on segments of different transpositions of the sieve of *Nekuïa*. They are shown in Figure 7.1 in the order they appear in the composition. In bars 160-162 (see Appendix 3), the piano plays rapidly up and down on the sieves' continuum, expressing different versions of the same sieve in succession; but also, in some cases the two hand parts play different versions simultaneously. In Figure 7.1 (bar numbers as shown at the left) the segments of the intervallic succession that are underlined, do not derive from any other sieve, but are used under transposition among three different versions in the final section of *Keqrops*.

[84] In *Synaphaï* (1969, for piano and orchestra) Xenakis employed superimposed melodic lines, that can be thought of as pre-arborescences (see Solomos 2002: 13).

The case of a free interchange of sounds is also very frequent. Although Xenakis described this idea in the discussion of the ascending/descending movement on the sieve of *Nekuïa* in *Serment* (see Varga 1996: 144-5), a free interchange of sounds might also relate to a free, repetitive circulation of pitches that belong to a relatively short segment of the sieve. Figure 7.2 shows the sieve segments used in the strings in bars 47-50 of *Horos* (see Appendix 3). The sieve they are based on is actually the complement of one of the versions of the sieve of *Nekuïa*; it is the complement of the sieve of *Akea*, shown in Figure 6.47. Each string instrument plays in synchrony with the rest, on 8 pitches of the sieve that interchange freely. The pitches of each set are paired in tetrachords according to their position in the set. In another case of similar inside-time treatment, Xenakis paired the elements in a different way. In bars 68-70 of *Ata* (see Appendix 3) the strings play a set of 6 pitches each (Figure 7.3). The pitch content of these sets does not derive from a sieve, but their structure is still an irregular alternating of small and large intervals. Each pitch of each set is associated with a pitch of another set, so that the result is a succession pentachords. But the association is not according to their respective position in the set; it is rather arbitrary and the result is the pentachords of Figure 7.4. This way, Xenakis produced a differentiated structure from a set of pitches that in their outside-time arrangement would tend to chromatic saturation. In *Ata* Xenakis also used a technique that he employed in many subsequent works: that of chromatic clusters whose top pitch belongs to the sieve. This is the case with the opening of *Ata* (see Appendix 3). In the score the composer provides the limits of the cluster and with a vertical line he denotes that all the chromatic intervening pitches should be played as well. *Ata* in particular opens with a succession of such chromatic clusters in the strings, based on the sieve and its complement. Thus, sieves are used to produce local mass sounds that retain the original intervallic structure.

The sketches of *Keqrops* provide the way Xenakis produced a set of tetrachords used in the composition. In fact, these tetrachords are sieve-clusters, but from a sieve that is neither used in its entirety nor elsewhere in the work. Figure 7.5 shows these tetrachords along with Xenakis's labelling and the intervallic succession for each one; note that the lowest two tetrachords do not follow the descending order of the others. Six out of the eight tetrachords are used in bars 126-137 of *Keqrops* (see Appendix 3). They appear in the woodwinds and the strings, as a polyrhythmic, multilayered alternation of the pitches of each tetrachord. The arrangement of the tetrachords is shown in Figure 7.6 (note that the appearance of the tetrachord labels in this table does not account for the length tetrachords appear). In bars 133-137 each group is divided in two parts that play different rhythms. Each instrument group plays one tetrachord at a time (even when it is divided into to parts), apart from the two clarinet parts which play two tetrachords.

The elements of each tetrachord appear in random succession; this free interchange might occasionally move up and down the tetrachords or alternate, but does not repeat pitches. However, there is one layer more uniformly organised than the others. The flutes in bars 126-131 play permutations of the pitches of tetrachord **γ** (pitches F5, G5, B5, and C6). A 4-element set has 24 distinct permutations ($4! = 24$); in this passage Xenakis used 11 of them, in a total of 15 occurrences of the tetrachord. These permutations are shown, in the order they appear, in Figure 7.7. Each repetition of the tetrachord also comprises a permutation of 4 time-values. These are the values of 1, 2, 3, and 4 demisemiquaver (demisemiquaver, semiquaver, dotted semiquaver, and quaver);

they are shown under the pitch names in Figure 7.7. From the 24 permutations of these values too Xenakis used only 11.[85]

7.2 *Akea*

From the opening, *Akea* is characterised by a sense of contrast between and the piano and the strings. Mostly, the two parts express the sieve and its complement simultaneously (Figures 5.2 and 6.47 respectively). The original sieve is heard at the opening bars of the work, as the piano's wide broken chords, while the strings play pitches derived from the complement.[86]

The first section of *Akea* (bars 0-20) is a typical contrast between the parts; Figure 7.8 shows the sieve and the complement, as played by the five instruments in the section (where S = Sieve and C = Complement). Until bar 18 there is a gradual exchange between the two parts (piano and strings) from the sieve to the complement and vice versa. The strings gradually pass on to the original sieve, in bars 14-18, where the piano pauses in order to play again the complement, this time not with chords, but with an unfolding of a two-branch arborescence. This contrast between the parts is typical for all the sections of the work.

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[85] It does not seem that Xenakis used here one of the kinetic diagrams of the symmetric group S_4. Both on the pitch level and that of the time-values, the transformations that might correspond to a kinetic diagram stop at the 5th occurrence of the tetrachord (end of bar 127). Note that this is also where the time-values appear partially (1 1 4 4). Cf. Gibson's analysis of a similar case in *Epicycle* (2002: 48ff.; 2003: 152-4). For the application of Group Theory by Xenakis see also Vriend, 1981; FM 201-41; and Schaub, 2005.

[86] In many cases in the work, the sieve is not entirely identical to the one in the sketches. This happens at the extreme low and high ranges.

Bars 21-36 comprise a very characteristic section of *Akea*, that of a regular semiquaver rhythm carried out by the piano. In bars 21-29 it plays dyads (in the two hands) that are taken from the complement of the sieve. But the accents on this regular rhythm are irregular and different for the two hands. Also, there is an occasional two-demisemiquaver reiteration of a chord, that amplifies the effect of the accent. Figure 7.9 shows the accented dyads for both hands separately. Bold numbers stand for the time-points of an accent that follows a two-demisemiquaver reiteration; brackets stand for unaccented dyads that follow such a reiteration. We see that the arrangement of the intervals between the accents and the reiterations is an irregular one, consisting of 2- and 3-semiquaver intervals, with occasional single semiquavers. The same holds for the dyads, as they are picked up from the complement, and not used in any regular pattern.

The strings intervene with dense chords (employing double-stops) taken from the original sieve; again, a fairly typical case of contrast. The previous rhythm dissolves, and in bar 36 both parts conclude the section with a reiteration of a chord in homorhythm (shown in Figure 7.10). Here the contrast is not as clear: only three of the piano tetrachord's pitches are members of the sieve. The first violin and the viola play the sieve, the second violin the complement, and the cello plays a dyad made up of both. The effect of this situation is not unique in the work. In several cases the music departs from the original sieve (or the complement), which initially is established as the fixed tonal space of the work, to play either a transposition, or unrelated material. A similar process takes place at the coda.

Following bar 39, there is an absolute departure from the sieve. For two bars (40-42) all instruments (except the second violin) play a 'semi-connected' melodic line, as indicated in the score. In the context of *Akea*, melodic line in the piano actually means a two-branch arborescence, and in the case of the strings, simultaneous melodic lines in polyrhythm. The character of these melodic lines is distinctly different from the sieve: they tend toward chromaticism. This melodic flight is included between two instances of reiteration of the chord of Figure 7.10, which takes place again in bar 45 in the strings only, and which signifies the end of the section. From an analytical point of view, chromaticism is evidence for classifying bars 40-42 as not related to the sieve (at least not in a specific way). From the aesthetic point of view, this reflects Xenakis's tendency to chromaticism in his later compositions, especially the ones dating from the early 90's onwards.

In the middle section of *Akea* there is more than mere contrast between the instruments. We also observe an alternation between the sieve, its complement and their transposition; also, a partial departure from the sieve that takes place in each instrument at a time. The section starts at the end of bar 45, where (while the piano pauses) the second violin plays a melodic line based on the original sieve, followed by the viola and cello in bar 46 and the first violin in bar 47 (also with the original sieve).

In bar 48 the piano plays chords derived from a T_{-12} transposition of the sieve. This is the first instance of transposition in the work. The fixed tonal space is therefore differentiated from low to high, rendering a different timbral quality to any given region of the sieve. By transposing part of the sieve, Xenakis transfers a particular timbral quality to another, higher or lower register. The piano chords of bars 48-50 are typical examples of sieve-clusters (for each hand part). Sieve-clusters function as timbres that emerge from the intervallic structure of the sieve. Now, by transposing the sieve-clusters, Xenakis aims at transferring the various timbres of a certain range of the sieve (with apparently more interesting timbral quality) to another. Figure 7.11 shows the T_{-12} segment used by the piano sieve-clusters in bars 48-50. The actual range of the music is E2 to C5. The intervallic structure of this range in the original sieve (T_0) is

2 4 1 2 2 1 3 1 4 1 3 1 4 1

and the intervallic structure of the T_{-12} segment is

3 1 4 1 3 1 4 1 4 1 1 3 1 3 1.

The latter is favoured by Xenakis apparently for its intervallic structure: it is closer to the original idea of the interlocking 4ths. In other words, it is a clearer juxtaposition of small and large intervals. As I will show later, this is not the only case in the work where this specific range of the original sieve is being favoured. It is also the segment of the intervallic structure that derives from the original sieve of *Nekuïa* (and which remains in many of its versions).

The piano sieve-clusters are the only case of transposition in the piano for this middle section. The strings on the other hand, play downward octave transpositions of both of the sieve and of its complement. The versions of the sieve in the middle section of *Akea* is shown in Figure 7.12. Apart from the initial sieve-clusters in the piano and dyads in the strings, the middle section is mainly based on linear motion that changes between the different versions of the sieve at no specifically defined point. The table in Figure

7.12 is in fact an approximation; it shows only the bar numbers when the version of the sieve was introduced.[87] The new versions do not always start exactly at the same time, and occasionally the linear motion departs momentarily from the sieve. Question marks stand for longer such deviations from the sieve. In any case, *Akea* is characterised by the distinction in two parts (piano and strings) that are based on complementary sieves; but also it largely relies on octave equivalence.

In the fourth section (bars 71-78) the piano ends up in a complex polyrhythmic passage of alternating sieve-clusters. These sieve-clusters are indirectly related to the sieve's complement. From a certain range of the sieve's complement, the tetrachords of Figure 7.13 can be produced. In the same way that the elements of a sieve (unlike the major diatonic scale) do not have any specific function in relation to other elements in the sieve, nor do the sieve-clusters. They are rather perceived as timbres – elements of the sonority that is the sieve (cf. Solomos 1996: 90-3).

Figure 7.14 shows all the sieve-clusters of bars 71-78 and Figure 7.15 shows the pitch content of these clusters. Considering all the aesthetic characteristics of the sieves in Xenakis's later music, the analyst is confronted with an unexpected outcome: not only is there no apparent relation to the sieve of *Akea*, but there is a repetitive structure. The pattern 1 1 1 3 is repeated four times before the sieve dissolves into chromatic saturation. In fact, further attention shows that several chords in the passage appear to undergo transposition up or down one or two octaves. This transposition index is reflected in the sieve of Figure 7.15 as the distance between the repetitions of the aforementioned pattern:

[87] At the end of bar 53 in the second violin there is an indication to play at the higher octave. This causes a T_{+12} transposition of the sieve, which contradicts the general rationale of the passage.

it is repeated (transposed) four times at 6 semitones each time, which adds up to 24, i.e. two octaves.

The chords can be taken into account without considering their octave equivalent, wherever that might appear in the music (numbers in Figure 7.14 are labels that stand for octave-equivalent sieve-clusters). Of course, this might suggest a deviation from, if not a contradiction to the aesthetic rule that sieves are intended to obey (the rule of non-repetitiveness). However, we should not forget that some extent of repetition seems, in several occasions, to serve another criterion favourable by Xenakis; the music establishes a regularity in order afterwards to break it (regularity serves as a reference point for irregularity). This is also found in the complement of the sieve of *Akea*. From the middle register upwards, there is one short pattern that is repeated three times in the intervallic structure: 1 3 1 1 3. Now, if we disregard all octave repetitions of the sieve-clusters, in other words if we fit them in the shortest possible range, the results is the sieve-cluster collection of Figure 7.16, which in turn is based on the sieve of Figure 7.17: apart from B3, this is the T_{-6} transposition of the complement of the sieve of *Akea*. It is probably one of the rare cases when Xenakis used octave equivalence to such an extent.

7.3 *À l'île de Gorée*

À l'île de Gorée was composed in 1986, for amplified harpsichord and ensemble of 12 instruments. It is characterised by audibly clear transitions from one sieve to another. There are five sieves in the work, labelled here with the roman numerals from I to V and are shown in Figure 7.18. Sieves II and V appear more frequently (either as upward/downward movement on the sieve's continuum, or as a free interchange of the pitches); III and IV appear only once (bars 15-27 and 45-46 respectively), and sieve I appears partially.

In bar 25, the harpsichord plays sieve II; in the left hand part there is an interchange between two elements: F4 is replaced by F#4 and G#4 by G4. The same happens at the bassoon in bars 28-30 as well as in the harpsichord in bar 30. In this bar we hear for the first time a set of chords that will dominate the following section, until bar 59. This short statement of chords serves as a preparation for the following two simultaneous chord sequences that do not actually start before another statement of the sieve II in bars 31-32. In one of the two sequences, the central role belongs to the harpsichord, and the other chord sequence is played by the winds.

Figure 7.19 shows the chords for the intermitted chord sequence of the harpsichord in bars 30-59.[88] This sequence takes place at the same time as the one in the winds, but the two alternatively pause or join, so that occasionally one of the two takes over for a moment. The harpsichord sequence is based on two elements: an irregular, fast alternation between two tetrachords (either **c/d** or **e/f**), and a regular, semiquaver iteration of tetrachords **a** and **b**. The former element takes place in a rhythmically irregular way and at different length each time, while the latter occurs as an occasional (but random) punctuation, that takes place in a constant way. This pattern, i.e. the irregular alternation of two chords followed by a regular statement of another two, is where the (more concrete) final brass chord sequence is based on. Occasionally, the harpsichord is joined by the ensemble with the chords in Figure 7.19. Until bar 49 the irregularly alternating chords are always **c** and **d**, and after that, always **e** and **f**.

[88] Note that the labelling of the chords is only valid in the context of each sequence.

From bar 34 onwards there is a chord sequence in the brass (plus the bassoon) that exhibits three layers of activity. These layers are distinguished by three characteristics: the type of the chords, the instrumentation, and the dynamics. The chords for each layer are shown in Figure 7.20. The first layer relates to the trichords that are played by the bassoon, the horn and the trombone. They are played ***mf*** against the ***mp*** and ***p*** of the other instruments (these dynamics do not remain constant throughout the segment, but the respective relation of loud and soft chords does so). This alternates with the second layer, which is a dyad (either a semitone or a tone) and is played by any two brass instruments (horn, trumpet, or trombone). These chords alternate with another dyad, which forms the third layer; these are two semitone dyads, but unlike the second layer they are repetitive. In particular, they function in filling in the time-points that are not occupied by any other chord.

In terms of rhythm the chords are put in an irregular manner. There are four segments of this type of chord sequence: bars 34-39, 40-41, 47-49, and 51-53. These segments are distinguished from each other, either by a short statement of different character or by an interruption by other processes (which I will discuss later on). The first-layer trichords are scattered through the segments and are scarcer than the dyads of the other two layers. Second-layer dyads are played alternatively with the other layers. The first and second layers are shown in the table of Figure 7.20. The leftmost column of the table shows the bar numbers of the respective segment of the chord sequence. Bold characters refer to the first-layer trichords and normal typeface to second-layer dyads. The unit distance is the semiquaver and the (time-)intervallic structure is shown in grey numbers. The third-layer dyads would occupy the time-points that are not occupied by the other two layers;[89] for this reason it is not necessary to include them here. The table shows that the chords are arranged in an irregular way and with great asymmetry between the length of the segments. This is a general case in the *À l'île de Gorée*. There are only some chord sequences that are arranged in a less irregular way, and this is (as I will show) only when they are simultaneous.

In bar 57 we have the first appearance of a sieve that is very frequent and characteristic of *À l'île de Gorée*. This is sieve V shown in Figure 7.18.[90] It is played by the ensemble while the harpsichord pauses. The exploration of this sieve is achieved through an intricate way that takes place in three layers: the woodwind, the brass and the strings – each group having its individual rhythmic character. The two groups of winds move freely up and down the sieve's continuum, but each instrument starts on a different element. These elements are very close to each other and in most cases neighbouring pitches appear simultaneously. In other words, we hear (for the most part) successive sieve-clusters. This creates a thick texture that allows the sieve's local properties become manifest both vertically and horizontally. In the treatment of the strings we have the unique occasion in the work where Xenakis uses transposition. The first violin plays the sieve, while the second violin, the viola, and the violoncello play T_{-1}, T_{-2}, and T_{-4} transpositions of the sieve, respectively. This means that, in terms of intervallic succession, all the vertical clusters in the strings are of the type 2 1 1.

[89] Apart from three exceptions, that in the table of Figure 7.20 appear with intervals other than 2 or 3.

[90] The lowest pitches in this appearance of sieve V are G3, A3, B3, and C4. From D4 upwards the sieve is identical to the one in Figure 7.18.

Just after the fluctuation on the continuum, all the parts of the ensemble arrive at a large chord in the middle of bar 59. The structure of this chord consists of two parts attributed to the winds and the strings. The winds' part belongs to sieve V and the strings' part to its complement. Sieve V reappears immediately afterwards, in bars 59-62 (in bar 60 some exceptions cause a slight departure from the original sieve).

Bars 63-80 comprise a chord sequence (in two segments) similarly characteristic as the processes in bars 34-53. There is a chord sequence in the winds, that takes place again in three layers: two types of chords, and a superimposed repetitive gesture in the high register. The two types of chords are distinguished by the dynamics and the instrumentation. There are two instrumental groups: the brass (plus the bassoon) and the woodwinds (plus the trumpet). The former group (first layer) play ***f*** tetrachords in alternation with ***p*** tetrachords (second layer) played initially by the latter group and afterwards by the former (more detailed analysis will follow). Figure 7.21 shows the tetrachords of both layers. The first-layer chords do not exhibit any systematic mode of construction. The first one is the only trichord, and the following 7 are tetrachords. The first eight chords comprise the first layer in the first segment of the sequence (bars 63-74); the final large chord replaces all the eight first-layer chords in the second segment of the sequence (bars 76-80).

The second-layer tetrachords have an intervallic structure of 2 9 2 semitones (see Figure 7.21).[91] The six second-layer tetrachords appear in the first segment of bars 63-71 (played by the woodwinds) and in the second segment (bars 72-74) their one-octave downward transposition is played by the brass. Figure 7.21 also shows the intervals

[91] With three slight deviations in bars 77, 78, and 79.

between the six tetrachords (in semitones). We see that the intervallic structure is borrowed from the middle range of sieve II (in retrograde). The third-layer gesture is either an alternation or a combination of the high A5 and E6 by the woodwinds; this dyad also appears later held in the strings. In bar 75 the high tremolo A5 and E6 are, for a moment, taken by the harpsichord while the ensemble reiterates the final, large first-layer chord of Figure 7.21. This chord appears throughout the second segment of the sequence (in the place of the preceding eight), while the second-layer tetrachords remain the same. The third-layer tremolo high A and E are now played by the strings one octave higher.

In bar 99 all the parts of the ensemble stop on a trill, apart from the flute that is based on a sieve, that consists of the two main sieves of *À l'île de Gorée*: sieve V up to E3 and sieve II from G3 upwards – see Figure 7.22. A new section starts in bar 102. The harpsichord's sieve comprises another kind of composition of two different elements (Figure 7.23). As with many other cases in the work – already mentioned above – there are three layers of activity in the harpsichord. The section starts with a chord in the first layer, a nonachord, which is reiterated alternatively with random iterations of the chord's pitches (second layer). On the third layer, the harpsichord plays occasionally, short segments of the sieve, which consists of the conjunction of sieve V up to D#4 and of the ennead from F4 upwards.

The final section (before the coda) consists of four simultaneous chord sequences. They are played by the four parts: the harpsichord, the woodwinds, the brass, and the strings.[92] The chords of each part are shown in Figure 7.24. The harpsichord, the brass,

[92] Gibson has shown that these chord sequences are largely based on rhythmic motives appropriated from *Idmen B* (1985, for six percussionists), bars 29-30 (2003: 220-2 & 2005: 8-9).

and the strings play trichords (apart from a double trichord in the harpsichord). The woodwinds play the type of tetrachords that appeared in the second layer of the sequence in bars 63-80 (Figure 7.21). That is, their intervallic structure is 2 9 2 semitones. But in relation to the previous sequence, they also have an additional tetrachord that stands out as different; this is tetrachord **g** in the woodwinds' chords in Figure 7.24. Furthermore, there is a dyad, **f**, whose two missing elements are included in **g**; in turn, the high D# in **g** differentiates it from the others (in terms of intervallic structure).

Unlike the sequences analysed so far, the woodwinds' sequence is periodic. There is a periodicity of 14 semiquavers which is marked by a two-demisemiquaver reiteration of tetrachord **g**. This is also the chord that the woodwinds arrived at in the beginning of bar 126. The end of the previous process is based on sieve I. In bar 125 the music departs from the sieve with the addition of G#4 – the lowest pitch of tetrachord **g**. Examining the intervallic structure of the woodwinds' tetrachords, we could say that it can be constructed by transposing two elements of a four-note (chromatic) cluster, one octave upwards. For example, the chord G#4-B*b*4-G5-A5 could be said to derive from cluster G4-G#4-A4-B*b*4 whose first and third elements (G4 and A4) have been transposed one octave higher. The cluster tetrachord **g** would derive from, consists of the elements of sieve I with an additional G#4. Therefore, the addition of G#4 is intended to prepare the construction of tetrachord **g** in bar 126.

Each occurrence of a period of the woodwinds' sequence has a specific structure, both rhythmically and in terms of content. The iterations of tetrachord **g** alternate with accented iterations of any of the remaining 6 (**a** to **f**). Since there is an iteration on every semiquaver, in each period there are 13 iterations (the first is a two-demisemiquaver reiteration of **g**); 7 of them are unaccented iterations of **g** and 6 are accented iterations of some of the remaining 6. Figure 7.25 shows the *accented* chords in all the 12 periods of the woodwind chord sequence in bars 127-137; numbers stand for the distance, in semiquavers, between accented tetrachords. (Recall that in the beginning of each period there is a two-demisemiquaver iteration of **g**, which is also played on all unaccented semiquavers.) Since there is space for 6 (accented) chords in each period, one would expect that Xenakis would have applied here several of the permutations of the 6 tetrachords. However, the first accented tetrachord in each period is always the same (**a**); the rest are taken randomly and in every period there appear either 4 or 5 tetrachords.

In terms of rhythmic organisation the sequence can be examined in relation to the iterations of accented tetrachords only, in the sense that the unaccented iterations comprise the rhythmic complement. These accented iterations take place every 2 or 3 semiquavers. In every period there are three 2-semiquaver and two 3-semiquaver intervals (apart from one exception that I will mention later). These two intervals are arranged in two ways in the sequence: alternating (2 3 2 3 2) and consecutive (3 3 2 2 2). Xenakis generates the rhythmic structure of each period by selecting some of the cyclic permutations of these two ways of arrangement. Since each arrangement has 5 elements, there are 5 cyclic permutations of each, making a total of 10 permutations; from these Xenakis used only 7. In the sequence there are 4 permutations taken from the first arrangement (found at periods 4, 6, 7, 8, 9, 10, and 12) and 3 from the second (periods 1, 2, 5, and 11 – see Figure 7.25). The only exception is found in the 3rd period where there are four 2-semiquaver intervals and one 3-semiquaver, with an additional single semiquaver to complete the period.

The case with the rest of the sequences of the section is more straightforward. The strings share a similar rhythmic structure with the woodwinds. There is a periodicity here as well, but of 15 semiquavers. The sequence is shown in Figure 7.26;[93] unlike the previous example (which included only the accented tetrachords), this one includes all the trichords of the strings in Figure 7.24. There are 11 complete occurrences of a period, which are again marked by the two-demisemiquaver reiteration of a chord (the first chord in each row of the table in Figure 7.26 represents this double reiteration). However, this reiteration corresponds to any of the 6 in the sequence and not just to one, as was the case of the woodwinds. The accented chords occur at different points in each period and are shown by the underlined trichords. We see that there is a pattern for only part of the sequence: the 1st, 2nd, and 6th periods do not follow this pattern. As in the brass sequence, apart from the initial trichord there are 6 occurrences of a trichord in each period; and although there are 6 trichords in total, there is not a pattern in their selection (in each period there are between 3 and 5 of them). Similarly, the time-intervals range between 1 and 3 semiquavers, and Xenakis used 7 distinct intervallic successions.

The brass keep repeating 5:4 rhythmic cells where all the 4 trichords of Figure 7.24 appear each time. These cells are based on the irregular rhythm of the chord sequence of the harpsichord in bars 30-59. There are six different rhythmic cells, but all consist of two parts (see Figure 7.27). In the first part of the rhythmic cell there is an alternation between trichords **c** and **d** and in the second part there are two semiquavers with trichords **a** and **b**. The total sequence has 42 random occurrences of the 6 rhythmic cells, as shown in Figure 7.27. The harpsichord's sequence is the most randomly arranged

[93] Question marks in the table of Figure 7.26 denote trichords that are only used once.

among all the instruments. It is marked by irregular triplet and duplet reiterations in the right hand, which alternate with single chords in every semiquaver, whereas the left hand plays, at random and scarce time-points, the chords that appear in the lower staff of Figure 7.24.

7.4 *Tetora*

The opening of Xenakis's third string quartet (1990) is a clear articulation of its sieve (shown in Figure 6.77), played by the first violin and with an artificial reverberation (halo sonority) by the other three strings. The sieve is constantly heard up to bar 21, where a heptachord intervenes, but it is less obvious later, in bars 27-30. After this point, the sieve alternates with its complement, in short sections that occasionally overlap (e.g. in bars 34-36 the end of a section with the complement overlaps with the beginning of a section with the original sieve). In bars 68-69 there is a halo of the T_{+1} transposition of the complement; the original sieve is heard for the last time in bars 76-82; from this point onwards there are either linear passages not based on the sieve or chord sequences that dominate the end of the quartet.[94]

In bars 27-30, the sieve is treated in a less straightforward way. The actual pitch content of the passage tends to chromatic saturation in the middle range (see Figure 7.28[a]). Only the part of the collection above C#5 belongs to the sieve. This pitch is found half way through the passage in the first violin at the end of bar 28. The literal manifestation of the sieve takes place at the first violin's ascending passage up to the end of bar 30. The 'top voice' (i.e. the highest pitches) is shared between the two violins; this

[94] The sieve-segments that are not related to the main sieve of the work are shown in Appendix 2.

top voice plays the sieve and this is an evidence that this sieve still plays a structurally important role here, although it is not straightforwardly present. The violins and the viola play either single notes or dyads, while the cello plays dyads only; all dyads are a perfect 4th. There are however two exceptions: a) the first violin's C natural in the middle of bar 28 is the lower note of dyad C-F#, which is an augmented 4th instead of a perfect one. b) The F#3-B3 dyad in the viola at the beginning of bar 27 seems to be there in the place of dyad F3-B*b*3, which is heard throughout the passage. If we take these two exceptions into account, we can reconstruct the rationale of the whole passage, which seems to have been constructed to create perfect-4th dyads by attaching a pitch below the actual pitch of the sieve. This is clear if one excludes all the lower notes in the unfolding of the pitch content of the passage. The pitch collection of Figure 7.28(b) belongs to the original sieve (apart from G2, G#2 and F3). The F#3 that appears in Figure 7.28(b) is the one found in the viola at the middle of bar 29 (and not in the dyad of bar 27). As concerns the second exception I mentioned, we can confirm now that pitches F#3 and B3 do not belong to the sieve, whereas F3 and B*b*3 do so. All the lower pitches of the dyad in the passage, consist a subset of the T_{-5} transposition of the original: it is a matter of two transpositions of the same sieve that sound at the time.

Throughout the quartet there is a type of chord sequence that undergoes a series of transformations. Its rhythmic structure is part of a more general system that is applied from the first appearance of a chord sequence until the end of the work (cf. Harley 2004: 202-5). The first appearance of this rhythmic structure is found in bars 21-24; its intervallic succession is

3 1 1 3 1 1 2 3 1 2 1 1 3 2 2 2 1 1 1 1 2 3 2 2 2 2 1 1 1 2

where the unit distance is the semiquaver.

In this first appearance, the rhythmic structure belongs to a sieve-cluster sequence. In all the appearances of the chord sequence (apart from this first one) there is a straightforward layering of the quartet in the upper and lower strings. It is the initiation of a system of transformations that, as it appears in Xenakis's sketches, consists of eight chord sequences with their corresponding time-point sequences (rhythmic structures). For reasons that I will come back to later, I will compare the first three sequences. In Figure 7.29(a), the upper-case Latin characters stand for the four-note sieve-clusters, taken from the original sieve of the work (from C#5 to G6). '**X**' stands for the low heptachord of bars 21-24, shown in Figure 7.30; underlined characters refer to the lower strings. Lower case Greek characters stand for chords that belong to the chord set of Figure 7.31.[95] The intervallic structure is stated in grey numbers and under the bar numbers is stated the transposition index (this does not apply to sieve-clusters, but to the chords that appear later).

There is a process of gradual specification that takes place on two levels: a) the chords, first in the lower strings of the second sequence and then in all strings of the third

[95] The upper staff chords are actually sieve-clusters. (**β** is taken from the complement and **γ** – **ς** from the original sieve; **α** is of a similar structure but belongs neither to the sieve nor to the complement.) However, they are here termed as chords, because they belong to a set whose chords are used and transformed throughout the composition. The lower staff chords derive (mostly) from the complement, but are not sieve-clusters. Also, after bar 115 there is a new six-chord set whose members do not derive from the sieve or its complement, but they belong to the same system of metabolae as the chords of the former set.

sequence, belong in the chord set of Figure 7.31; b) the process of chord substitution from the second to the third sequence is consistent.[96] From now on, all the chords in the sequences will either belong to the set of Figure 7.31, or to that of Figure 7.33. During this process we have the first realisation in the work of the rhythmic structure of all the sequences in the work.

Metabolae (transformations) in *Tetora* are of three types: a) substitution of time-intervals, b) chord substitution, and c) chord transposition. (From these, only the second took place in the first phase of the process of metabolae – Figure 7.29). The second phase of metabolae appears in bars 59-85, as shown in Figure 7.32, which exhibits certain anomalies in its structure.[97] Part (a) consists of two sections: in the first (bars 59-68) there are three sequences in succession. In the second section (bars 69-76) there are certain anomalies in the transition from one sequence to the next. It is an incomplete realisation (through a process of metabolae) of a compositional system that I will here demonstrate. This imperfect realisation can be seen as follows: from the four sequences, the one of bars 73-74 appears partially. In the table, its role is shown by relocating it in relation to the other sequences (that is, it is shown at the rightmost, although it immediately follows the previous one). At this point I have to note that this examination ignores the lower strings, as there can be found no systematic application of any kind in the process of chord substitution (as I will show, this does exist though in the treatment

[96] In the chord sequence of bars 48-50 there is a reversal, where the lower strings take the role of the upper, and vice versa. See Figure 7.29.

[97] An additional chord, in the tenth column of the table, appears in brackets – **(α)**. The space between the third and fourth rows of the table suggests that other material intervenes between the corresponding chords sequences.

of the time-values). The criterion, therefore, is the systematic application of a design of chord substitution in the upper strings; this association is also aided by the actual structure of the sequence, in the way the upper strings are 'interrupted' by the lower.

Seven bars after the completion of the sequence in bars 74-76, there is another one which appears temporally isolated from the rest (bars 83-85), and whose relation to the process of metabolae is not straightforward (in the table it is found at the bottom of part [a]). This is due to two reasons: firstly, its isolation (actually, its displacement) and secondly, its very structure, which is the retrograde of the structure of the other sequences. It remains to discover its role in relation to the metabolae, first to the process of chord substitution and later to the one of substitutions of time-intervals – which I will discuss later on. Close inspection reveals that the incomplete sequence of bars 73-74 is actually found, in retrograde form, at the beginning of the one in bars 83-85. Therefore, if the retrograde form of the sequence in bars 83-85 is the reason to present it in its original form (with grey typeface in the table), its identification with the incomplete sequence of bars 73-74 is the reason to (re)locate it in the latter's position (in the table, the arrow that connects the sequences of bars 71-73 and 83-85). This will enable us to indicate the discrepancy that has been caused (with the displacement and inversion) in the process of transformations that does not appear on the surface. As a result, we can see that there is a chain of chord substitution that comprises 6 sequences, and a 7th sequence where all chords are replaced by chord **γ**.

Part (b) of the table shows the dual process of substitution – of chords and time-intervals. While part (a) could be characterised 'paradigmatic', part (b) is more 'syntagmatic': the columns of the former now appear as the rows of the latter. Its horizontal dimension now symbolizes time and can be read from left to right throughout, as a continuous process of substitutions. Examining, for now, the chord substitutions we can interpret the corresponding part of the table as a synoptic chart of this second phase of metabolae.

In Figure 7.32 the chains of substitution have 5 chords, apart from 4th row of part [b] which has 4. In bars 74-76 the whole procedure ends with the simple repetition of a single chord. Thus, the overall homogeneous nature of the process yields to a sequence's maximally non-differentiated structure, a sequence which also exhibits a non-differentiated rhythmic structure (the steady pulse of a semiquaver). It is a matter of convergence of two dimensions – of the horizontal, that is the internal structure of the chord sequence, and of the vertical, the process of chord substitutions itself. In the table this can be seen as the convergence of the 'paradigmatic' and 'syntagmatic' dimensions (the row of [a] and the column of [b] that correspond to bars 74-76). It is exactly this collapse of homogenous variety to the non-differentiated that has been prevented by the temporal displacement (and distortion) of a part of the process (bars 83-85). It is a matter of the very same aesthetic criterion of tension at a more distant level of focusing in time: a breakdown of regularity. It is not by mere chance that in the sequence of bars 83-85 the chords that belong to the set of Figure 7.31 sound for the last time, reminiscent of what has already concluded. From now on, we are transferred to the tonal universe of Figure 7.33, in bars 115-137, which I will examine straight away.

Figure 7.34 is constructed exactly as its preceding one and shows the chord substitutions and transpositions of bars 115-128. Now all the sequences follow one another in direct succession and the metabolae are straightforward. Comparing the chord substitutions in part (b) of the two tables, we notice the transition to an absolutely regular and homogeneous process. In bars 59-85 there was a more elaborate application of a compositional system, whose characteristics had already started to appear in bar 40. This system is eventually crystallised in bars 115-128.[98] Five sequences, each containing four chords, finally allow regularity be established. This a process concludes in absolute homogeneity, as if permitting the appearance of different sides of a symmetric object: there are 5 chord sequences and in each chain of substitutions there are 5 chords (in Figure 7.34[b] 5 rows have 5 chords each). However, the end of this process does not signify the end of the work; it ends with three more sequences (bars 128-138) where the lower and upper strings, in two-part counterpoint, present in irregular order the chords of Figure 7.33 (in T_0), based on the initial form of the rhythmic sequence. Figure 7.35 shows these three sequences; the final sequence (bars 134-137) is also shown in its normal form, in grey typeface.

The aforementioned time-point sequence undergoes transformations that are not parallel to the chord substitutions, but in certain places the two coincide. There is a clear sense of correspondence, for the greater part of the work, between the interval of the semitone and the time-interval of the semiquaver, as the common unit distance of both pitch- and rhythmic structures. The notion of tension, although far from being objective, is actually present in *Tetora*, as the juxtaposition of small and large intervals. In sieves tension is produced by the intervallic structure itself (without having to resort to melodic patterns). In other words, the succession of neighbouring elements is necessary in order

[98] This second instantiation of the system is not found as such in the sketches to *Tetora*. However, it clearly follows the same rationale.

for the structure of sieves to be revealed. Tension in the rhythmic structures of *Tetora* is found in the successive alternation of smaller and larger time-intervals.

Xenakis transforms the time-point sequence of *Tetora* selectively applying a permutation of (time-)values in order to either maintain tension, or allow a transition from tension to relaxation and vice versa. A 3-element set has 6 possible permutations (3! = 6). Whenever there is a permutation in *Tetora*, it corresponds to one of the three positions of the rotation of the triangle on a single axis. Note that permutation is a specific case of combination (combination does not always use all the elements in a set); and that rotation on a single axis is in turn a special case of permutation. This process is applied in certain parts of the system and each new position gives a new time-point sequence, which exhibits the same degree of differentiation in its structure. The process of metabolae in the time-point sequences throughout the quartet is shown in Figure 7.36. The structure of this table is based on that of the second part of tables 7.29, 7.32, 7.34, and 7.35; the indication 'position' stands for the position of the triangle as it corresponds to the three time values of the first sequence (bars 21-24); bar numbers refer to the bar that the sequence is initiated. Practically, permutation means to correspond each element of a set to another of its elements. Rotation, as a special case of permutation, is the symmetric procedure which is employed in order to help maintain tension in each new form (rotation) of the sequence. This tension is neutralised only when rotation ceases to happen. Two observations concerning the convergence of this process with that of chord substitution: the first, as I have already mentioned, refers to the convergence on the maximally non-differentiated structure of the chord sequence in bars 74-76 and its rhythmic structure – the persistent repetition (in the upper strings) of a single chord. The second concerns the sequence of bars 83-85 that appears (partially and transformed) before and after the one in bars 74-76. Its unique role (i.e. breakdown of regularity) is also apparent in the rhythmic structure itself: it is the only time in the work when the appearance of all three time-values has not resulted from rotation (neither from any other kind of permutation). With its maximally differentiated structure it arbitrarily intervenes between two totally regular sequences (one-semiquaver pulses).

From bar 128 to the end, all the chords in the set of Figure 7.33 appear in irregular order in both the upper and lower strings. The final sequence (from bar 134 to the end) negates apparent repetition, by presenting the time-point sequence in its retrograde form. This is the only time in the work, when this transformation of the sequence (1st position) appears in retrograde. (Recall that in the retrograde form of bars 83-85, the time-point sequence was not preceded by the original.) Xenakis transforms the time-point sequence for a last time, in a way that has not been encountered in the work so far. Xenakis employed the symmetric process of the rotation of the triangle, in order afterwards to negate it (Figure 7.36, bars 65-119). The retrograde form, as another symmetry, yields another form of the time-point sequence which (as in its initial appearance) manifests and maintains tension for the last time.

The last part of this thesis concerned the two approaches, inside- and outside-time, in the analysis of sieves and their employment in the compositions of the later period. The inside-time analysis was further extended to the compositional approach of distributing points on a straight line (either as pitch scales or as time-point sequences). The analytical methodology of Chapter 5, although preoccupied largely with pitch scales, implies a relation between outside- and inside-time structures (in the form of symmetries and periodicities). The relation between the two is shown in both Chapters 6 and 7; for example, in the construction of the sieves of *Shaar* and the treatment of the rhythmic structures in *Tetora*. These two examples are useful in the exploration of the notion of the Xenakian metabola, one of the main concerns of the following concluding remarks.

Concluding Remarks:

Symmetry and asymmetry, periodicity, and metatheory

The chord sequences of *Tetora* comprise an example of systematisation that does not derive specifically from any formalised 'Xenakian' theory. The connection between formalisation and composition has been researched and commented upon by several scholars (Vriend 1981; Squibbs 1996: 281-90; Gibson 2003: 279-81 and Solomos 2005b, have written extensively on the matter). Xenakis gave an insightful and striking description of his working methods in relation to formalisation. He stated that, all of his works (apart from the *ST* family) 'are mostly handiwork, in the biological sense: adjustments that cannot be controlled in their totality. If God existed He would be a handyman' (1987: 23). An engineer by training, Xenakis defined himself as a *bricoleur*.[99] The latter is differentiated from the former in their method of work. The engineer uses purpose-built, formalistic tools for the ideal realisation of a perfect design. On the other hand, *bricolage* refers to the putting-together of parts, materials, or tools, not as an ideal combination but simply as a possibility.

The level of formalisation in relation to Sieve Theory can be seen in the use Xenakis made of sieve formulae. In the 1980s Xenakis did not use the formula as the primary interface any longer (cf. Ariza 2005: 45). Rather, the formula is an important

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[99] Here he makes an obvious comment against the classical teleological arguments for the existence of God, which led to the recent, well-known 'Intelligent Design' Movement. It is this idea of the perfect design that bricolage is opposed to. For the notion of bricolage in the relation of music and biology see Bogue 2003: 62-72. Solomos in (2005b) uses the term bricolage in the sense that Claude-Lévi Strauss did.

tool, but primarily for the estimation of the sieve's inner-symmetry. As the sketches included in this thesis show, Xenakis constructed a sieve and derived a formula afterwards. It is an application of Sieve Theory both as an analytical and as a compositional tool (that led to the construction of his two computer programs). This is different from the early application, which was much more formalised, e.g. the case of the sieves of *Nomos Alpha* (see Vriend 1981; FM 201-41; Gibson 2003: 78-117). In this latter case (among others), Xenakis applied metabolae by altering the moduli, the residues, the unit distance, and the points of origin (the pitch that zero corresponds to). But in the more recent sieves, the theory is limited to analysing a sieve's inner periodicities. Cyclic transposition and other metabolae are applied informally, i.e. to the actual scale. This is what Xenakis did with his sieves of the later period (for example with the piano sieves at the final section of *Keqrops*), but also with the rhythmic sequences of *Tetora*.

In *Tetora* the rhythmic sequences were not calculated as sieves. However, a chord sequence is a case of distributing points on a line; therefore, it is indirectly related to Sieve Theory even though Xenakis did not apply sieve-theoretical processes. Also, it is a case of a transformational system, a system of metabolae.[100] The idea of metabola was developed in accordance with sieves and, in particular, it offers ways of transforming sieves on the level of their theoretical expression (the formula). However, transformations can be also applied in a non-formalistic way. In fact, metabolae refer not only to sieves, but to outside-time structures in general. The rhythmic sequence of *Tetora*

[100] The inspiration for idea of the metabola, Xenakis commented, came from the Greek poetess Sappho (see Emmerson 1976: 25).

is for Xenakis an outside-time structure in the sense that he showed in FM (264-5). Thus, metabolae do not relate only to sieves; nor do they refer generally to transformations (for example, Xenakis did not use the term 'metabola' for serial transformations). Metabolae are transformations that are applied to outside-time structures.

In 'Towards a Metamusic', Xenakis offered a summary of Byzantine scales, as well as his view of Byzantine music scholarship; this was part of his argument that Sieve Theory offers ways of incorporating all scales (see FM 186-192; cf. Turner, 2005). Xenakis refers to metabolae in relation to the outside-time structure of Byzantine music, and in particular to the Byzantine system-scales:

> [The] outside-time structure [of Byzantine music] could not be satisfied with a compartmentalized hierarchy. It was necessary to have free articulation between the notes and their subdivisions, between the kinds of tetrachords, between the genera, between the systems, and between the echoi – hence the need for a *sketch of the in-time structure* […]. There exist operative signs which allow alterations, transpositions, modulations, and other transformations (metabolae) (FM 190; italics added).

Metabolae then are related to outside-time structures by offering possibilities of arranging them inside time. In other words, metabolae offer ways of producing versions of the original outside-time structures and allow for further determination of the inside-time unfolding of these structures. The system of metabolae in *Tetora* is applied to the (outside-time) rhythmic structure and produces 19 rhythmic-sequences. The first three and the last three chord sequences of the system are based on the original rhythmic structure. Thus, the system of metabolae itself bears a certain symmetry. But this symmetry is not complete. Towards the centre of Figure 7.36 the symmetry disappears, as the sequence in bars 71-73 distorts it: were it not for this sequence, the sequences in bars

65-116 would comprise a palindromic structure with its centre being the sequence that starts in bar 74. We see that a certain ‘near symmetry’ characterises a large-scale process.

The notions of symmetry and periodicity are found very frequently in Xenakis’s writings. This is a general aesthetic principle. He expressed this idea in an interview of 1976, in the discussion of periodicity in the rhythmic structures of his percussion work *Psappha*: ‘You always have some form of periodicity if you do *not* have a stochastic distribution of points on a line, attacks in time. The contradiction of periodic and aperiodic is one of the fundamental games of systems’ (Emmerson 1976: 24). In another text twenty years later, also in the context of temporal structures, he equated periodicity with symmetry: ‘What repetition is in time, symmetry would be outside of time’ (Xenakis 1996: 144). This general principle is what seems to have inspired Xenakis in the construction of his outside-time structures, but also in the inside-time sketches of his compositions. The former is clearly evident by the analysis of his sieves (Chapter 6). Paraphrasing Xenakis, the contradiction between the symmetric and the asymmetric is extended to the idea of degrees of symmetry. Thus, although two sieves might be similarly non-repetitive, inner-symmetric analysis can reveal that the one might be more symmetric than the other. In his sieves, Xenakis achieved this contradiction not by progressing merely from symmetric to asymmetric sieves and vice versa. As a general characteristic, the sieves of the later period are constructed to include periodicities at the liminal levels of symmetry (frequently also including non-periodic elements). Each sieve bears itself the contradiction between symmetry and asymmetry, by being near-symmetric or near-asymmetric; given the irregular, non-repetitive intervallic structure of his sieves, ‘near symmetry’ is in fact a hidden symmetry (in terms of perception). So, another level of contradiction between symmetry and asymmetry is that between surface asymmetry and inner symmetry. Symmetry here is a point of reference for asymmetry. With the system of metabolae in *Tetora* Xenakis applied the same aesthetic principle, on another level. Whereas with the sieves the contradiction between symmetry and asymmetry refers to outside-time, in *Tetora* it is also found in the inside-time sketch of the chord-sequence metabolae.

The triangle rotations in the rhythmic sequences of *Tetora* are related to the group transformations Xenakis first applied in the mid 1960s. As Vriend pointed out (1981: 25-6), the notion of symmetry is strongly related to that of mathematical groups. But group transformations where offered by Xenakis as an inside-time organising principle (since they refer to permutations, to the *order* of elements). As with metabolae, group transformations could also be thought as a sketch of the inside-time structure, but Xenakis did not use the term 'metabolae' for group transformations. This is because group transformations are special cases of permutations, which are in turn special cases of metabolae. More importantly, permutation does not necessarily produce new (versions of the original) outside-time structures. For example, the pitch permutations of tetrachord γ in *Keqrops* (see Section 7.1 and Figure 7.7) did not produce a new sieve (nor a new sieve segment). The situation in *Tetora* is different. The chords are arranged in sequences, but each new sequence is not a new permutation of the same set of chords; it is instead a case of *substitution*, so that different chords (drawn from the same set) are included in each successive chord sequence. Thus Xenakis produced new chord sequences by using only some of the total number of chords and by applying chord-substitution instead of chord-permutation.

On the other hand, permutations of time-intervals do produce different structures. This is because there is a fundamental difference when applying permutations to the pitch-level and to the interval-level. Rearrangements of intervals produce new pitch structures, whereas rearrangements of pitches do not. Either in the case of a pitch scale or in that of a rhythmic sequence, permutations of (pitch- or time-)intervals produce new scales or rhythmic structures, new outside-time structures. Thus, the Xenakian metabola refers to transformations that are applied to outside-time structures in order to produce new ones. In practical terms, these metabolae produce new versions of the outside-time structures they are applied to, and offer the possibility of constructing the sketch of the whole or part of a composition. Metabolae are in this way situated between the outside- and the inside-time.

It follows from this that transformations that might produce new structures, are referred to as metabolae only if they are applied to an existing outside-time structure. Although permutations of intervals can serve for metabolae, this is not the only way of performing, nor does it imply, metabolae. An interesting case is that of sieves $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ of *Shaar* (Figure 6.52). There, as shown in Section 6.7, several permutations of three or four intervals, respectively, are placed in a certain succession; the result is a chain of permutations that renders the intervallic structure of the sieve. Unlike *Tetora*, the term 'metabola' would not be used for these sieves of *Shaar*, precisely because it is not a case of transforming an existing outside-time structure to produce a new one. It is rather a matter of transformations of the basic set of intervals that make up the intervallic structure of the sieve. This is a clear case where Xenakis uses an inside-time organising principle (succession of permutations) to construct an outside-time structure.

We see how permutations of intervals can produce new sieves, whereas permutations of pitches cannot. In other words, when intervals are permutated, they are treated as a *succession* of values that can be used to produce an *ordered set* of values (pitches). Xenakis used the idea of the *trace* to show that time-intervals are perceived outside of time (FM 264-5). Nothing prevents us from extending this idea to pitch-intervals. In that case, the trace would refer to pitches. In a sieve of the type of that of *Shaar*, once the intervals of the set are heard in its first occurrence, they are held in our memory and thus placed outside of time: each new permutation of the same intervals is another inside-time manifestation of the same outside-time structure (it becomes an ordered set thanks to the pitches' trace in our memory). Of course, the case of *Shaar* is unique among the later sieves. But the later sieves are also based on a finite number of intervals (usually 1, 2, 3, and 4 semitones). The only difference is that these intervals do not appear in chains of permutations, but rather in a succession of free *combinations* of all the intervals throughout the whole range of the sieve. Note that permutation is a special case of combination. Similarly then, these four intervals are first placed in *succession* and are then perceived in their outside-time aspect. (Obviously, the smaller the number of intervals the easier for our memory to compare and place in an ordered set.) It is important to note that this (inside-time) combination of intervals forms the sieve-construction principle. The opposite would not be true. That is, a succession of pitch permutations does not produce outside-time structures – it rather places the sieve inside time. Thus, sieve construction is preceded by permutations of intervals and is followed by its placing in the composition. In other words, the construction of an outside-time structure is both preceded and followed by inside-time processes.

Xenakis's metatheory suggests that removing the category of outside-time structures is impossible. But on many occasions we see that the two types of structures, inside- and outside-time, are so inter-dependent that it is impossible to distinguish the one from the other. In this sense, it is equally impossible to remove the category of inside-time structures. This can also be seen in the aforementioned comment by Xenakis, where 'you always have some form of periodicity' (unless there is a stochastic distribution). A similar case is that of the sieves of *Shaar* and by extension, sieve construction in general and, indeed, all outside-time construction. In the later period of sieves, which is more representative of sieve-based composition than the earlier (and for Solomos *the* period of sieves; see 1996: 86-102), Xenakis conceived sieves as multiplicities of inner periodicities. As I have shown, sieves cannot be conceived without including the notion of periodicity. Saying this does not imply that Xenakis's insightful views on the re-introduction of the scale do not hold. As he always insisted, outside-time structures do exist. An ordered structure is located at the foundations of any compositional style or language; this is the case of the (outside-time) chromatic scale and (inside-time) serialism. Furthermore, outside-time characteristics can be produced by inside-time structures (e.g. symmetric relations between different forms of the series). As outside-time structures can, and have been, produced in the scope of serial compositional principles, the (inside-time) inner-periodicities or intervallic permutations, render the structure of (outside-time) sieves. The distinction between what is inside and what is outside time is not impermeable.

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