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Euclid and Pāṇini

INTRODUCTION

THE PRIMARY AIM of this paper is to draw a comparison between the methods employed by the Greek mathematician Euclid (third century B.C.) and the Indian linguist Pāṇini (fourth century B.C.).[1] At the same time a comparison will be drawn between the influences which these two scientific methods have exerted within their respective philosophic traditions, i.e., the Western and the Indian. One thesis defended in this paper is that the mathematical method is characteristic of much of Western philosophy, whereas the grammatical method is characteristic of much of Indian philosophy.[2] Another thesis, partly implicit, is that the recent occupation of modern Western philosophy, not only with linguistic analysis, but also with the science of language,[3] can be expected to yield results of philosophic value. For this reason a comparison between linguistic and mathematical methods may be of interest to modern philosophers. Moreover, a consideration of linguistic method throws light on the relationship between the language of philosophy and philosophy itself. Some illustrations of this relationship will be considered.

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[1] This article is a free translation and revision of about two thirds of an Inaugural Lecture delivered on appointment to the Chair of General and Comparative Philosophy at the University of Amsterdam. I am grateful to Professor D. H. H. Ingalls and the editor of this Journal for reading an earlier version of the English translation and for making valuable suggestions concerning both contents and presentation.

For Pāṇini's date, see P. Thieme's review of T. Burrow, *The Sanskrit Language* (London: Faber and Faber, Ltd., 1955), in *Language*, 31 (1955), 428-448, which quotes further literature. The facts hardly justify Jaspers' theory of an *Achsenzeit* (in *Vom Ursprung und Ziel der Geschichte* (München: R. Piper & Co. Verlag, 1949), though a simultaneous flourishing of philosophy could to some extent explain a subsequent flourishing of scientific method.

Editor's note: Because of the highly technical and specialized nature of this paper and the notes—and the fear of errors and/or inconsistency due to editing—the author's style in the notes has been largely retained. Some minor inconsistencies of style and some apparently incomplete items are present, but clarity is not affected.

[2] It is probable that this was first pointed out by D. H. H. Ingalls in "The Comparison of Indian and Western Philosophy," *Journal of Oriental Research*, 22 (1954), 4. See also my review of D. S. Ruegg, *Contributions à l'histoire de la philosophie linguistique indienne* (Paris: E. de Boccard, 1959), in *Philosophy East and West*, 10 (1960), 53-57.

[3] See, e.g., J. A. Fodor and J. J. Katz, *The Structure of Language. Readings in the Philosophy of Language* (Englewood Cliffs: Prentice-Hall, Inc., 1964). → J. F. Staal, "Generative Syntax and Semantics," *Foundations of Language* 1 (1965), 133-154.

Though linguistic and mathematical methods will be described mainly in a historical setting, they exemplify differences which are of theoretical interest and which pose problems which remain, on the whole, unsolved. Some parts of contemporary Western philosophy and logic can be said to deal with the problems that arise from the confrontation of these two different approaches. Outside of philosophy and logic, the two approaches meet in the recently developed science of mathematical linguistics.[4] This paper may also present material to justify the expectation that mathematical linguistics will continue to be inspired by Indian grammatical methods.[5] It will be seen that these methods often possess a degree of systematization, formalization, and conceptualization which in the West is generally associated with mathematics and the mathematical sciences only. Lastly, it may not be superfluous to add that the conclusions to which these comparisons lead can help to dispel the naïve but persistent view that only the Western tradition has produced and employed rational and scientific views.

I

The classic example of mathematical method occurs in the thirteen books of the *Elements* (Στοιχεῖα) of Euclid. The classic example of linguistic method occurs in the eight chapters of the Sanskrit grammar (*aṣṭādhyāyī*) of Pāṇini. Both had precursors, whose results are largely lost,[6] but each constructed an almost complete system by means of a precise method, strictly maintained. Though there are, as we shall see, certain parallels between the two, the object material also has in one respect a similar structure. This similarity is indicated by the Greek term "*στοιχεῖον*," which denotes the elements as well as the letters of which words consist. Proclus declares in his commentary on the *Elements:* "Just as an expression in language consists of, first, most simple and undivided principles, which we call elements, and just as each word and each discourse is constructed from these, there are also certain theories, called elements, which precede the whole of geometry, function as principles for following theorems, extend over all theorems and provide proofs for many particular cases."[7] It is true that the linguist's activity is

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[4] In due course, this science may be called merely theoretical linguistics: Y. Bar-Hillel, *Language and Information* (Reading, Mass.: Addison-Wesley, 1964), p. 185.

[5] Bloomfield acknowledged indebtedness to Pāṇini. For a direct comparison with methods of mathematical linguistics, → J. F. Staal, "Context-sensitive Rules in Pāṇini," *Foundations of Language,* 1 (1965), 63-72.

[6] On the one hand, Hippocrates of Chios, Leon, Eudoxos, Theudios, etc.; on the other hand, Āpiśali, Kāśyapa, Gārgya, and the Northern and the Eastern school.

[7] See trans. P. Ver Eecke (Bruges: Desclée de Brouwer, 1948), 65; E. J. Dijksterhuis, *De elementen van Euclides* I-II (Groningen: P. Noordoff, 1929-1930), I, 107: ***hōs gàr tēs***

only very approximately described as the composition of words from letters. Moreover, linguists also deal with spoken languages, where there is no use for pencil and paper and where a tape recorder is required. Proclus and the ancient grammarians following Dionysius Thrax constructed words from letters instead of from phonemes, while neglecting morphemes altogether. In these respects Pāṇini and the Indian grammarians made a unique contribution.[8] But let us first confine our attention to Euclid's *Elements.*

Euclid starts the first book of his work with three groups of statements, and these are followed by numerous propositions or theorems. The other books of the *Elements* contain other theories, sometimes preceded by new definitions which circumscribe new topics. Propositions are derived from the initial statements by means of logical derivations, which sometimes refer to propositions that have been derived earlier. The main structure which holds the *Elements* together, therefore, is logical derivation. In general, each such derivation consists of several steps, of which the most important ones are construction (κατασκευή) and/or proof (ἀπόδειξις). The three groups of initial statements are definitions (ὅροι), postulates (αἰτήματα), and common insights (κοιναί ἔννοιαι). Since the time of Proclus the last have been called axioms (ἀξιώματα). The mutual relations between these three types of principles pose several problems, more so because there can be disagreement with regard to the status of some statements. For the present purpose they can be roughly characterized as follows: Definitions introduce concepts which are the object of geometry. Postulates either introduce constructions or assume the existence of constructions and their results. Axioms, according to Aristotle, are basic to all sciences. Hence the name "common insights."

Let us consider an example of each. The fifteenth definition runs as follows: "A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another."[9] This definition explains how the term "circle" will be used, without asserting that circles exist and without providing a construction for circles. This is done in the third postulate, which says: "To describe a circle

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eggramátou phonēs eisin archaì protai kaì haploústatai kaì adiairetoi, hais tò ónoma ton stoicheiōn epiphēmízomen, kaì pasa léxis ek toútōn hyphéstēken kaì pas lógos, hoútō de kaì tēs hólēs geōmetrías estí tina theōrēmata proēgoúmena kaì archēs lógon échonta pròs tà ephexes kaì diekonta dià pántōn kaì parechómena pollon apodeixeis symptōmátōn, hà de stoicheia prosagoreúousi: Proclus, *In primum Euclidis elementorum librum commentarii,* ed., G. Friedlein, Leipzig: B. G. Teubner, 1873, 72, 6. 13.

[8] See P. Thieme, in B. Shefts, *Grammatical Method in Pāṇini: His Treatment of Sanskrit Present Stems* (New Haven: American Oriental Society, 1961), p. ix.

[9] *Kúklos estì schēma epípedon hupò mias grammēs periechómenon, pròs hēn aph' henòs sēmeiou ton entòs tou schēmatos keiménōn pasai hai prospíptousai eutheiai ísai allelais eisín.*

with any center and distance."[10] This can be taken as postulating the construction or the existence of circles.[11]

The first axiom is quite different. It states: "Things which are equal to the same thing are also equal to one another."[12] Though this axiom can be interpreted as valid for line segments, it allows many other interpretations as well. For example, it can be expressed in terms of algebra by "if b = a, and c = a, then b = c."[13] One might feel inclined to consider such an apparently general axiom as belonging to logic, but the concept of equality (ἰσότης) has a mathematical character. Proclus asserts in this connection that the axioms are valid not only for magnitudes, but also for numbers, movements, and time intervals.[14]

Euclid proceeds to derive numerous propositions from these definitions, postulates, and axioms with the help (implicitly) of further axioms and logical rules and metarules. This is so similar to the familiar proofs of school geometry that it need not be illustrated. Moreover, a detailed examination of a complete proof would take up more space than is available here, especially on account of the characteristic references to results established earlier.

In how far do the five axioms of Euclid (and, correspondingly, the five postulates and first twenty-three definitions) meet the geometrician's need? In classical antiquity other axioms were current, too, e.g., "Things which are halves of the same thing are equal to one another."[15] Proclus notes that axioms should not be needlessly increased: such axioms "follow from preceding axioms and are justly omitted in most copies."[16] This principle is of fundamental importance for the entire Euclidean system. In modern terminology it could be expressed by stating that each axiom must be independent from the others. Proofs of independence are given much later by the use of models. The construction of such models has, in turn, led to the discovery of non-Euclidean, discontinuous, non-Archimedean and multi-dimensional geometrics.[17]

The requirement of independence is a simplicity criterion. In philosophy

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[10] *Kaì pantì kéntrōi kaì diastemati kúklon gráphesthai.*

[11] Dijksterhuis, *op. cit.*, I, 124. Compare T. L. Heath, *Euclid's Elements I-III* (reprint) (New York: Dover Publications, Inc., 1956), I, 195.

[12] *tà toi autoi ísa kaì allēlois estìn ísa:* Euclides, *Elementa I*, ed. I. L. Heiberg (Leipzig: B. G. Teubner, 1883), 10, 2.

[13] The commutativity of identity need not be presupposed here.

[14] Proclus, *op. cit.*, 195.25-196.I.

[15] Euclides, *op. cit.*, I.10.9. Compare Heath, *op. cit.*, I.223.

[16] *Tauta oun hépetai tois proeirēménois axiomasi kaì eikótōs en tois pleistois antigráphois paraleípetai:* ed. Friedlein, 198.2-4. Compare trans. P. Ver Eecke, 174.

[17] See, e.g., E. W. Beth, *The Foundations of Mathematics* (Amsterdam: North-Holland Publishing Co., 1959), 153-155; B. L. van der Waerden, *De logische Grondslagen der Euklidische Meetkunde* (Groningen: P. Noordhoff, 1937), 39; H. Weyl, *Philosophie der Mathematik und Naturwissenschaft* (München: Leibniz Verlag, 1928), 18 *et seq.*

it is known as Ockham's razor: Entities should not be multiplied beyond necessity (*entia non sunt multiplicanda praeter necessitatem*). In another version it runs: What can be done by fewer [principles] is done in vain by more (*frustra fit per plura quod potest fieri per pauciora*).[18] This principle is not a desideratum of philosophers alone and merely of aesthetic nature; it is a motive of consequence in the development of science. During the Middle Ages Ockham's razor led to the doctrine that terrestrial and celestial bodies consist of the same matter: the phenomena can be explained by assuming either one or two kinds of matter, and hence there is no reason to postulate more than one.[19] Such speculations paved the way for Newton's discovery of the law of gravity, which applies to planets as well as to apples. On the other hand, a complicated theory is less probable than a simple one. When astronomical observations became increasingly accurate, an increasing number of epicycles had to be constructed to explain the phenomena in terms of Ptolemy's theory. However, these laws could be derived from Newton's law, through the intermediary of Kepler's laws.[20] In modern physics, attempts at unification appear regularly, for example, in unified field theory or in derivations of elementary particles from other and fewer elementary particles.

In philosophy, the criterion of simplicity gave rise to the concept of monism, variously developed and exploited by metaphysicians. Another philosophic result provides a further example of Euclid's method. This is the doctrine of categories as it occurs, e.g., in Aristotle. Aristotle suggests that the categories of his list are independent by enumerating them as a disjunction in terms of "or."[21] In the *Topica*,[22] he notes that there are just as many categories as he has mentioned. In other contexts it appears that the independence of each category is presupposed. In the *Topica*, a relatively early work, two categories occur which are later omitted.[23] This may point to the fact that Aristotle had come to consider these as derivable. Throughout the history of philosophy lists of categories presuppose the independence of each category. In Kant, the categories constitute a system which is analogous to a structure of axioms and theorems. In the *Critique of Pure Reason* categories are called the true root-concepts of pure understanding (*die wahren Stammbegriffe des reinen Verstandes*). They possess, in turn, derived concepts (*reine abgeleitete*

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18 Weyl, *op. cit.*, 117.

19 E. Gilson, *La philosophie au moyen-âge* (Paris: Payot, 1925), 283.

20 Cf. Weyl, *op. cit.*, 117-118.

21 *Categoriae* 4, 1 b 25—2 a 4. However, see *ibid.*, 8, 11 a 37, quoted in W. and M. Kneale, *The Development of Logic* (Oxford: Clarendon Press, 1962), 30.

22 *Topica*, A 9, 103 b 38.

23 W. D. Ross, *Aristotle* (London: Methuen & Co., Ltd., 1953), 21-22; I. M. Bocheński, *Ancient Formal Logic* (Amsterdam: North-Holland Publishing Co., 1951), 33-34.

Begriffe), regarding which Kant says that they belong to transcendental philosophy but that he "can rest content merely to mention these" (*mit deren blosser Erwähnung . . . zufrieden sein kann*).[24] Kant, then, provides axioms without deriving any theorem and without asking how such derivation should be carried out.

In natural language, categories are not expressed by sentences but by words or terms. In mathematics, the corresponding structure is therefore a system of definitions. In such systems, too, independence is a meaningful concept. Alessandro Padoa derived a method for establishing the independence of a concept, i.e., its undefinability, in terms of earlier postulated concepts. It is natural to apply this also to systems of categories. As an exercise in traditional philosophy an attempt could be made to describe the world solely in terms of the first three categories of Aristotle. If any possible world can be described in terms of all the categories but one, the independence of this latter category has been thereby established.

In modern logic, corresponding investigations aim at the construction of formal systems without using particular principles or concepts. Intuitionistic logic thus dispenses with the principle of the excluded third; negationless logic dispenses with the concept of negation. The resulting systems appear more or less seriously curtailed from the point of view of ordinary propositional calculus. Other kinds of restrictions are brought about within propositional calculus, when it is shown that the connectors "and," "or," "not," "if . . . then" can to some extent be derived from each other or from "neither . . . nor . . . ," symbolized by Sheffer's stroke. Similarly, the number of axioms can be reduced. These transformations do not affect the results which can be obtained within the system. They merely reflect what Quine has called economy in grammar and vocabulary. This sort of economy leads to longer statements and derivations. On the other hand, brevity in statements and derivations calls for a wealth of basic idioms.[25] In general, within artificial languages, new categories are introduced by new notations and are therefore easily recognized. The categories presupposed by natural languages are not so easily recognized.

The analysis of traditional systems of categories assumes new importance when the language in which such systems were first formulated is taken into account. The dependence on natural language is generally implicit and goes unnoticed. Still, it has often been assumed that there is some connection between Aristotle's categories and grammatical distinctions. In the nineteenth

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[24] *Kritik der reinen Vernunft, 2. Auflage, Transcendentale Elementarlehre II, Transcendentale Logik* I, § 10, Akademie-Ausgabe. (Berlin: Georg Reimer, 1911) III, 94.

[25] W. Van O. Quine, *From a Logical Point of View* (Cambridge: Harvard University Press, 1961), 26-27.

century, Trendelenburg defended the view that Aristotle's logical categories can be derived from grammatical categories. However, his arguments were not cogent, and later scholars have rejected not only the reasoning but also the result. As late as 1958, Benveniste convincingly established that the categories of Aristotle can be understood against the background of Greek syntax.[26] For example, the two categories κεῖσθαι and ἔχειν, which were later deleted by Aristotle himself, reflect the verbal functions of the Greek middle voice and perfect, respectively. Faddegon established as early as 1918 that the categories of the Indian Vaiśeṣika system also were based upon grammatical distinctions;[27] this had been known to Indian scholars. Given the fact that Greek and Sanskrit are cognate languages, this explains the striking similarity between Western and Indian categories.

Discoveries of this kind open up new vistas. They emphasize the desirability that philosophers should take full account of linguistics. With the help of linguistics, philosophy is in a position to enter a fruitful area of research. In this respect, philosophers are in a more favorable position than mathematicians—for the latter investigate what they have first created, while the former face the richness and variety of natural languages, where reality surpasses the boldest imagination. Mathematicians can never enter other spaces than the one in which they were born, not even with the advancement of space travel. At most, they can propose to physicists that they should describe physical space with the help of another geometry. Philosophers, on the other hand, can learn a language and thereby enter a new world of experience: the linguistic categories of a newly learned language may not segment reality in the same way as do the categories Western philosophers are accustomed to. Philosophers obtain passports for non-Aristotelian worlds as soon as they begin to study the syntax of a language which is sufficiently different from Greek. Of course, the fruitfulness of such research increases if the language studied belongs to a civilization which has also produced philosophy, logic, and linguistics or related fields of study.

Important results of this kind can be expected from the study of the so-called exotic languages. But, since the language of philosophy is even more conservative than colloquial language, similar information can be obtained by studying the language of philosophy in this spirit. In Chinese, no verb corresponds to "to be," which both functions as copula and expresses existence. Kant's criticism of the view that existence is a predicate can therefore never

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[26] E. Benveniste, "Catégories de pensée et catégories de langue," *Les Études Philosophiques,* New Series, 13 (1958), 419-429.

[27] B. Faddegon, *The Vaiçeṣika System Described with the Help of the Oldest Texts* (Amsterdam: Johannes Müller, 1918).

be meaningfully translated into Chinese. A. C. Graham has shown that a Chinese translation of relevant passages of Kant's *Critique of Pure Reason* (itself based upon the English translation of the German original) is quite unintelligible to persons who know Chinese but no German.[28] Similarly, English translations of the categories κεῖσθαι and ἔχειν, for instance, as "position" and "possession," respectively, are hardly meaningful in English, since the relevant functions expressed by the Greek terms do not straightly correspond to similar structures in English. Even if ἔχειν is translated as "to have," and attention is paid to the fact that in English also a tense somewhat similar to the perfect tense can be expressed by means of "to have," it remains unintelligible what could have motivated Aristotle to assume this concept as a category. This provides a good illustration for Wittgenstein's slogan, that philosophic problems appear when language goes on a holiday—on a holiday in Greece, in this case.

Through the intermediary of the categories linguistic usage has exerted profound influence on thought. Philosophic innovation often springs from a reaction against traditional usage of speech and thought. A contemporary example of such innovation occurs in existentialism, as well as in analytical philosophy. According to Heidegger, the "essence" of the human being (*Dasein*) lies in its existence. It is possible to say that *Dasein* is, but not what it is. It is not a substance that can be further specified with the help of categories, for categories are especially adapted to another kind of being than human being (*"Seinbestimmungen des nichtdaseinsmässigen Seienden"*). Heidegger therefore introduces existentials, and thus avoids the error of substantialization. From a linguistic point of view this means that it is possible for a noun to refer to human being without functioning in the same way as other nouns which refer to things. In practice, Heidegger often replaces nouns by pronouns and more often by verbs. This might indicate a Semitic rather than a Greek inspiration. This impression is confirmed by other points of resemblance between existentialism and Biblical thought.

As mentioned earlier, Wittgenstein in a similar vein warns against substantialization of the words of a sentence. This may lead us astray, both in speech and in thought. Wittgenstein constructs a language where all words refer to things by imagining a system of communication between a builder and his assistant.[29] This language consists of the words "block," "pillar," "slab," and "beam." Each time the builder utters one of these words, the assistant hands him the relevant thing. Wittgenstein's artificial language is based upon

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[28] A. C. Graham, "'Being' in Western Philosophy Compared with *shih/fei* and *yu/wu* in Chinese Philosophy," *Asia Major,* New Series 7 (1959), 107-108.

[29] *Philosophical Investigations* (Oxford: Basil Blackwell, 1958) (I 2), p. 3.

a particular analysis of the use of language. Ryle, in *The Concept of Mind* and elsewhere, considers different types of category mistakes. They are basically of the question-answer type: "Where is Socrates?"—"Yesterday." Heidegger applies this by confining himself for methodological reasons (as he puts it) to *Dasein*. Strange as it may seem, *The Concept of Mind* and *Sein und Zeit* are similar in this respect. Indian thinkers of the Advaita Vedānta—in this respect comparable to Neo-Platonism, negative theology, and apophatic mysticism—specify the same in a different manner by maintaining that the Absolute is not a substance (*dravya*), because it has no qualities (*nirguṇa*). To think otherwise would be the result of a category mistake.[30]

The principles from which categories are derived and the simplicity or economy criterion play important roles in modern linguistics. Syntactic form-classes and morpheme-classes are defined by means of substitution in identical environments. Though morphemes, as was noted before, were not known in Western classical antiquity, these methods lead to results which are closely related to the parts of speech (μερή λόγου) put forward by Greek grammarians and logicians of the Stoa. These, again, are connected with Aristotle's categories. Pāṇini and the Indian grammarians use substitution for similar purposes. They draw explicit attention to its fundamental importance. The Indian method of substitution (*ādeśa*) corresponds to Aristotle's method for deriving categories. Characteristic for Aristotle is the fact that he starts from questions.[31] In modern linguistics, substitution is used for establishing a morpheme-class, for example, by studying the environment ()-(ly). In this environment, "slow," "near," and "quaint" fit, and these therefore belong to the same morpheme-class. Though "dead" can occur in this environment, too, it functions differently elsewhere; it does not, therefore, belong to the same class.[32] "Deadly" functions in a "deadly blow" in the same way as "terrific" does in a "terrific blow." But a "slowly blow" is never given.

It is hardly necessary to emphasize the importance of Euclid's methods for the history of Western science. According to Beth,[33] Euclid's geometry is the classical example for Aristotle's theory of science. In this theory, postulates, shown by history to be hardly compatible, have been combined. The postulate of reality (i.e., "each proposition refers to a particular area of reality") could

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[30] For the linguistic background of categories and category mistakes, see J. F. Staal, "Some Semantic Relations between Sentoids," *Foundations of Language*, 3. (In press.)

[31] For a modern development, see H. Hiz, "Questions and Answers," *The Journal of Philosophy*, 59 (1962), 253-265.

[32] R. S. Wells, "Immediate Constituents," *Language* 23 (1947), 81; B. Bloch and G. L. Trager, *Outline of Linguistic Analysis* (Baltimore: Linguistic Society of America, 1942), 60-61, 77-79.

[33] See, e.g., Beth, *The Foundations of Mathematics*, 31-51.

be maintained in the empirical sciences only by dispensing with the other postulates: the postulate of deduction, which incorporates the deductive structure referred to earlier; and the postulate of evidence, which requires that the meaning of the first principles and the validity of the axioms be self-evident.[34] The deductive sciences, on the other hand, have dispensed with the postulate of reality and adhere only to the postulate of deduction. Since the development of non-Euclidian geometry, the postulate of evidence has been rejected. In modern science, theories are established with the help of many criteria, but mainly with adequacy and simplicity in view.

Since Euclid, philosophy has made repeated attempts to become or present itself as a deductive science. In this development Proclus is of historical importance. He was a well-informed but not an original scholar who was not only commentator of Euclid's *Elements* but also systematizer of Neo-Platonic metaphysics. Though some of Proclus' comments on Euclid are astute, the applications of his learning to philosophy are less than illuminating. In his *Elements of Theology* (Στοιχείωσις Θεολογική) Proclus confines himself to enumerating and explaining propositions. He appears to assume that propositions can function as propositions without being derived from axioms. It is true that the explanation of a proposition may contain logical remarks. This is not surprising, since in this theology there are propositions such as number 147: "In any divine rank the highest term is assimilated to the last term of the supra-jacent rank."[35] But considerably stronger methods of proof would be required to establish such a proposition as number 187: "Every soul is indestructible and imperishable."[36] By the irony of history, Euclid's diligent scientific method entered metaphysics through the series of unproven propositions of his commentator. One has to agree with Martin when he says that Proclus attached value to the *Elements* of Euclid, mainly because they contradict neither the Chaldean oracles nor the speculations of the Pythagoreans.[37]

Endeavors to imitate Euclid's method are found in numerous medieval treatises.[38] Such attempts, which may simultaneously have undergone the

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[34] Formulated clearly by Proclus, ed. Friedlein, *op. cit.*, 195.17-19: *pánta axiómata hōs ámesa kaì autophanē paradotéon, gnorima aph' heauton ónta kaì pistá* (All axioms must be given as immediate and self-evident, being known from themselves and reliable).

[35] *Pántōn ton théiōn diakósmōn tà akrótata tois pérasin homoioutai ton huperkeiménōn.* E. R. Dodds, *Proclus: The Elements of Theology* (Oxford: Clarendon Press, 1933), 128.

[36] *Pasa psuchē anolethrós esti kaì áphthartos:* Dodds, *op. cit.*, 162.

[37] Quoted by Heath, *op. cit.* (above, note 11), I 30, note 2. See, e.g., Proclus' commentary on definitions 30-34.

[38] See H. A. Wolfson, *The Philosophy of Spinoza* (New York: Meridian Books, 1960), 40 *et seq.*

influence of Aristotle's syllogistic, occur in the works of Duns Scotus, Burgersdyck, Maimonides, Avicenna, Averroes, and many others. This tendency culminates in Spinoza's *Ethica more geometrico demonstrata.* One might argue that Spinoza's proofs are rarely convincing, or that the geometrical exterior has nothing to do with the metaphysical content, which closely follows medieval treatises. Wolfson has maintained the view that the real Spinoza appears only when an *Ethica more scholastico rabbinicoque demonstrata* is discovered behind the *Ethica more geometrico demonstrata.* Be this as it may, Spinoza intended to prove his propositions and paid greater attention (as appears, for example, from his letters) to the demonstrative value of his proofs than his modern readers, who are often disposed to irrationalism.

II

Let us now turn to Pāṇini's system of Sanskrit grammar. This grammar opens with a list of the sounds which, according to Pāṇini, occur in Sanskrit and which function as elements for later rules.[39] This list starts *a, i, u, e, o, ai, au,* etc. Consonants are always followed by a short *a,* for example, *ja, ba, ga, ḍa, da.* Then follow the *sūtras,* or rules, which can be divided into three types in a way roughly parallel to Euclid: (1) The *sūtras* proper, or theorems (*vidhi*), which describe linguistic facts while emphasizing word formation. (2) The defining *sūtras* (*saṁjñā-sūtra*), which introduce technical terms, e.g., "homogeneous" or "homorganic" (*savarṇa*) for sounds which "are pronounced in the same place and with the same tension of the mouth."[40] Examples of such sounds are *a* and *ā* or *pa, pha, ba, bha,* and *ma,* also called labials. Finally (3) the metatheorems (*paribhāṣā-sūtra*), which explain how rules have to be treated and applied in particular cases. Examples of these are given later.

Pāṇini's rules are exclusively derived from ordinary usage, and his principal problem is to give an adequate description of ordinary usage. Linguistics is not prescriptive, but descriptive. The description is based upon forms which everybody has at his disposal. He who wants pots goes to the shop of a potter, says Pāṇini's commentator Patañjali, but he who wants words does not go to the shop of a grammarian.[41] In practice, the problem of adequate description is a problem of correct formulation. An attempt is made to establish this aim by

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39 See J. F. Staal, "A Method of Linguistic Description: the Order of Consonants According to Pāṇini," *Language* 38 (1962), 1-10.

40 *tulyāsyaprayatnam savarṇam:* Pāṇini 1.1.9.

41 *yathā ghaṭena kāryaṃ kariṣyan kumbhakārakulaṃ gatvāha-kuru ghaṭaṃ kāryam anena kariṣyāmīti na tadvac chabdān prayuyukṣamāṇo vaiyākaraṇakulaṃ gatvāha-kuru śabdān prayokṣya iti:* Patañjali, *Mahābhāṣya,* Bhārgava Sāstrī, ed. (Bombay: Nirṇayasāgara, 1951) I, 64 b.

making predominant use of a principle of concision (*lāghava*), i.e., by a criterion of simplicity. This is illustrated by the famous saying that grammarians rejoice over the saving of the length of half a short vowel as over the birth of a son.[42] The first consequence of this simplicity criterion is that repetitions are eschewed. This requires the material to be ordered in a particular manner, which at first sight appears artificial to Westerners, who are used to the Latin "grammar of ideas," which, for example, completes the treatment of the noun before embarking upon the verb. But Pāṇini needs only one rule to explain the long *ā* in the second syllable of the nominal form *śivāya* (to Śiva), in the second syllable of the verbal form *pacāmi* (I am cooking), and other long *ā*'s.

One rule of this grammar will be dealt with in greater detail. This rule lays down that certain vowels are replaced by semi-vowels if a heterogeneous vowel follows—for example, *dadhi atra* (the milk here) is replaced in uninterrupted speech by *dadhyatra.* In customary transliteration, *i* is replaced by *y*. Similarly, we say, instead of *madhu atra* (the honey here), *madhvatra,* where *u* is replaced by *v*.

In formulating this theorem, Pāṇini makes use of several abbreviations which are basic to the structure of his grammar. Since the above-mentioned substitution also applies to the specifically Sanskrit vowels *ṛ* and *ḷ,* Pāṇini has to describe four linguistic facts which obtain if a vowel is followed by a heterogeneous vowel: *i* is replaced by *y, u* by *v, ṛ* by *r,* and *ḷ* by *l.* Pāṇini could have expressed this as follows: "*i, u, ṛ,* and *ḷ* are replaced by *y, v, r,* and *l,* respectively, in uninterrupted speech, if a heterogeneous vowel follows." But this is too verbose and has to be subjected to the economy criterion. In the beginning of the grammar, sounds are ordered in such a way that what has later to be combined is placed together and can be referred to by means of a particular device. In the initial series of sounds, the sequence *i u ṛ ḷ* is followed by an indicatory sound, *k,* and the sequences *ya va ra la* are followed by an indicatory sound, *ṇ*. The elements *k* and *ṇ* belong to the metalanguage. The convention is to abbreviate as follows: if in the list a sound, *A,* is followed by an indicatory sound, *B, CB* denotes *C* as well as the following sounds up to and including *A.* For example, *ik* denotes *i u ṛ ḷ; uk* denotes *u ṛ ḷ; yaṇ* denotes *ya va ra la; vaṇ* denotes *va ra la,* etc. Following this convention, Pāṇini could have formulated the rule as follows: "*ik* is replaced by *yaṇ* in uninterrupted speech, if a heterogeneous vowel follows." However, it is obvious that the phrase "in uninterrupted speech" occurs in other rules as well. Pāṇini groups all such rules to-

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42 *ardhamātrālāghavena putrotsavaṃ manyante vaiyākaraṇāḥ: Paribhāṣenduśekhara* 122, ed. F. Kielhorn (Bombay: Indu-Prakash Press, 1868), 115.

gether and places before the first rule a separate one, i.e., "in uninterrupted speech."[43] This applies to what follows, up to a particular rule, which was originally marked by an accent.

We are still handicapped by the clumsy expression "if a heterogeneous vowel follows." Here Pāṇini applies a simple argument. If there is a rule which states what happens if a particular condition is not fulfilled, there must also be a rule which tells us what happens if this condition is actually fulfilled. For example, we may wish to state: *dadhi atra→dadhyatra,* where the following vowel is not homogeneous. But what happens to *dadhi indra* when the following vowel is actually homogeneous? If the occurrence of a condition is explicitly formulated in one rule, its absence in another rule is thereby implied. Now, there actually is a rule which states that a certain phenomenon takes place "if a homogeneous vowel follows."[44] It is therefore sufficient to formulate in the rule under discussion: "if a vowel follows." Had this vowel been homogeneous, the other rule would be contradicted. Hence it is not homogeneous, and it is superfluous to state this.

This application of the simplicity criterion hardly differs from a type of conclusion we draw daily. If we see a notice, "No admittance for children and dogs," we do not need much self-knowledge to conclude that we are allowed to enter. We do not look for another notice which says, "Adults may enter." In later systems of Indian philosophy, similar arguments are systematically studied. A textbook example is the following. Fat Devadatta never eats by day. Since fatness is caused by eating, and eating by day is denied, it follows that Devadatta eats by night.[45]

In the preceding paragraphs we have encountered the principle of contradiction. In order to make his system consistent, Pāṇini applies in particular cases the following *paribhāṣā,* or metatheorem: "In case of contradiction [between two rules] the latter rule prevails."[46] Apparently, the rules of grammar are ordered in such a way that this metatheorem is valid. Only the final portion of Pāṇini's grammar begins with a *paribhāṣā* which states that the following *sūtras* are not ordered in this way.[47] Later grammarians have applied this

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[43] *saṃhitāyām:* Pāṇini 6.1.72. Cf. 8.2.108.

[44] *akaḥ savarṇe dīrghaḥ:* Pāṇini 6.i. 101.

[45] See J. F. Staal, "Contraposition in Indian Logic," in E. Nagel, P. Suppes, and A. Tarski, eds., *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress* (Stanford: Stanford University Press, 1962), p. 645.

[46] *vipratiṣedhe paraṃ kāryam:* Pāṇini 1.4.2. See J. F. Staal, "Negation and the Law of Contradiction in Indian Thought: A Comparative Study," *Bulletin of the School of Oriental and African Studies* 25 (1962) 52-71.

[47] *pūrvatrāsiddham:* Pāṇini 8.2.1. See H. E. Buiskool, *Pūrvatrāsiddham* (Amsterdam: H. J. Paris, 1934).

metatheorem of consistency to a greater number of cases than Pāṇini did himself.

The theorem under discussion can now be formulated as: "*ik* is replaced by *yaṇ* if a vowel follows." But the same method of abbreviation can be used to refer to vowels. Pāṇini avoids the introduction of special categories, such as vowels, since he needs only to group the required sounds together in his initial list and apply the above method of abbreviation. This is far more empirical, since simplicity of description is an objective criterion and does not commit grammarians to theories regarding different kinds of sounds, such as vowels and consonants. Pāṇini dispenses with such theories, since he constructs a descriptive grammar and not a "grammar of ideas." His initial list of sounds begins with what other grammarians call vowels (a distinction which was known in India before and after Pāṇini). The first vowel is *a,* and the indicator following the last vowel is *c.* Pāṇini can therefore denote vowels by *ac.*

The theorem can now be formulated as: "*ik* is replaced by *yaṇ* if *ac* follows." Still, we have not reached the most concise formulation. In grammar, expressions such as "x is replaced by y" and "if x follows" are obviously extremely common. These are, as it were, fundamental metalinguistic relations. Pāṇini expresses these relations by using the Sanskrit cases in a pregnant manner. Since cases are distinguished in Sanskrit by their terminations, Pāṇini says: "*ik* + genitive termination, *yaṇ* + nominative termination, *ac* + locative termination." In Sanskrit, this becomes: "*iko yaṇ aci.*" This is the rule as it actually occurs in Pāṇini's grammar.[48] Incidentally, one should not suppose that expressions like "*iko yaṇ aci*" are intelligible to Sanskrit speakers or scholars who have not studied Pāṇini—not any more than an algebraic formula in an English textbook is intelligible to English speakers who have no knowledge of algebra. *Sūtras* in Sanskrit scientific literature correspond to formulas in Western scientific literature.

The rule under discussion is primarily a rule for pronunciation and not for orthography. It is probable that Pāṇini constructed his grammar without using writing—an achievement hard to visualize anywhere but in India, where large bodies of knowledge are orally composed and transmitted. However, this rule could also be regarded as descriptive of a historical process, in the manner of the sound laws of comparative philology. Pāṇini, however, never says that *i* be-

[48] Pāṇini 6.1.77. Many of Pāṇini's rules are stated in accordance with this scheme:

a + *abl. term.* b + *gen. term.* c + *nom. term.* d + *loc. term.*

In modern terminology this can be expressed by substitution rules as:

$$a\ b\ d \rightarrow a\ c\ d \text{ or:}$$

$$b \rightarrow c \text{ in context } a\text{—}d.$$

In the latter expression and in Pāṇini's expressions, repetition of a and d is avoided. (See the article quoted above, note 5.)

comes y. He speaks of *replacing i by y,* in the manner indicated above. His commentators use in this connection the term "substitution" (*ādeśa*). It is of some importance to realize that laws of development such as "*i* becomes *y*," which are postulated without corresponding to perceptible change, are based upon purely descriptive statements of the type "*i* is replaced by *y*." As in the case of natural science, the most concise formulation yields the most probable theory. The scientific value of a diachronic hypothesis depends on the simplicity of a system of synchronic descriptions. This applies not only to linguistics but also to anthropology.[49] Morris Halle has shown this with regard to sound laws of Indo-European, such as the laws of Grimm and Verner: "The acceptance of these laws as historical fact is based wholly on considerations of simplicity."[50]

When comparing Pāṇini's system with Euclid's *Elements,* a characteristic of the latter, i.e., deduction, appears absent from the former. It is true that there is a kind of deduction in Pāṇini's grammar: *dadhyatra* is deduced from *dadhi atra,* and other forms are similarly deduced with the help of rules. But such deductions do not seem to attain the same degree of generality as Euclid's proofs. However, the difference reflects a distinction of object, not of structure. In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed *ad hoc* (i.e., *sūtras*). Both systems exhibit a structure of logical deduction with the help of rules, and both scholars attempted to arrive at a structural description of facts. In both systems, contradictions and unnecessary complications are avoided. In both cases, the aim is adequate and simple description.

So far, only one example of grammatical method in India has been given. More detailed investigations into the methods of Euclid and Pāṇini would throw light on points of difference as well. Another common characteristic is the above-mentioned desire to shorten principles (where Euclid pays attention to minimum number, Pāṇini to minimum length), while disregarding the length of derivations. In Kleene's mathematical logic, for example, the proposition $a = a$ is deduced in seventeen steps.[51] Pāṇini derives the aorist *ajāgariṣam* ("I woke up") from the stem *jāgṛ-* and the affix *-iṣam* by making use of nine

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[49] See J. F. Staal, "Sanskrit and Sanskritization," *The Journal of Asian Studies* 22 (1963), 261-275.

[50] M. Halle, "On the Role of Simplicity in Linguistic Descriptions," in R. Jakobson, ed., *Structure of Language and Its Mathematical Aspects* (Providence: American Mathematical Society, 1961), 89-94.

[51] S. C. Kleene, *Introduction to Metamathematics* (Amsterdam: North-Holland Publishing Co.; Groningen: P. Noordhoff, 1952), 84.

different *sūtras*.[52] In modern logic, the underlying problem has come to the forefront. Theoretical investigations on the length of proofs may also be relevant to questions regarding the time needed by electronic computors. Quine, who distinguishes, as we saw, two complementary types of simplicity, has obtained short derivations and avoided a complicated substitution rule by assuming an infinite list of axioms.

Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in Western thought.[53] Scientific developments have therefore taken different directions in India and in the West. Pāṇini's system produced at an early date such logical distinctions as those between language and metalanguage, theorem and metatheorem, use and mention, which were discovered much later in Europe.[54] In other Indian sciences, e.g., in mathematics and astronomy, as well as in later grammatical systems of Sanskrit, Prakrit, and Tamil, systematic abbreviations are used which not only are ingenuous but also constitute new adaptations of the same method.[55] In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. In the West, this corresponds to the belief that mathematics is the most perfect among the sciences. Just as Plato reserved admission to his Academy for geometricians, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics. In India, grammar was called the Veda of the Vedas, the science of sciences. Renou declares: "To adhere to Indian thought means first of all to think like a grammarian" (*Adhérer à la pensée indienne, c'est d'abord penser en grammairien*).[56] This has determined the form and method of a large part of Indian philosophy, an important feature which is generally lost when Sanskrit originals are translated into Western languages. It seems almost unavoidable that translations of an accurate original should therefore appear vague.

The simplicity criterion looms large in later Indian logic, where cumbersome solutions are replaced by more perspicuous and elegant formulations. Complicated arguments, theories, and technical concepts, as postulated, for example, by the ritualistic philosophers of the Mīmāṁsā, were rejected by

[52] Quoted in L. Renou-J. Filliozat, *L'Inde Classique II* (Paris: Payot, 1953), 88.

[53] See above, note 2.

[54] See J. Brough, "Theories of General Linguistics in the Sanskrit Grammarians," *Transactions of the Philological Society* (1951), 27-46; J. F. Staal, "The Theory of Definition in Indian Logic," *Journal of the American Oriental Society* 81 (1961), 122-126.

[55] See L. Renou, *Introduction générale* for J. Wackernagel, *Altindische Grammatik I* (Göttingen: Van den Hoeck & Ruprecht), 123 note 614. See also P. S. Subrahmanya Sastri, *History of Grammatical Theories in Tamil* (Madras: The Journal of Oriental Research, 1934), 108.

[56] *L'Inde classique II* 86.

logicians because they could be derived from more elementary expressions.[57] Such rejections are generally accompanied by the expression *"gauravāt"* (on account of heaviness). It is clear that this "Indian razor" could be fruitfully applied also to Western philosophy, where heavy examples are common. While simplicity of language is always a sign of good style, it has become a valued indication of clarity of thought, especially in British philosophy.

The algebraic style of the *sūtras* can assume forms which are hardly intelligible without a commentary.[58] What is clear to insiders becomes thereby abstruse to outsiders, as is often the case in modern science. This is one of the unavoidable implications of the use of an artificial language. A famous Indian example is provided by the definition of *Brahman,* the Absolute, in the *Vedānta-sūtra* as: "On account of which the origin, etc., of this."[59] In order to understand this it is necessary to know that the last word, "this," denotes this world, while "origin, etc." means origin, preservation, and dissolution. Hence, the *sūtra* defines *Brahman* as that on account of which the origin, preservation, and dissolution of this world exist. The term "etc.," at times ambiguous, which has also led to discussions in the West, was used by Pāṇini to indicate lists of words or roots by attaching it to the first member. Two appendices attached to Pāṇini's grammar, the *Dhātupāṭha* and the *Gaṇapāṭha,* enumerate such lists.[60] Elsewhere in the grammar, special cases are enumerated within a *sūtra,* provided these are not too many. In the following verse this is ridiculed, and a pun is made on the word *"sūtra"* which signifies both "rule" and "thread":

> No wonder that the girl strings together glass, gems,
> and gold on one thread.
>
> Even Pāṇini, who ought to know better, combined dog,
> youth, and king of the Gods in one rule.[61]

Finally, some *sūtras* will be quoted from the traditional manual of a system of Indian philosophy that more than any other has led in the West to un-

[57] For example, *arthāpatti, anupalabdhi.*

[58] The definition of *sūtra* by Madhva (13th cent.) is itself not in *sūtra* style: *svalpākṣanam asaṃdigdhaṃ sāravad viśvatomukham astobham anavadyaṃ ca sūtraṃ sūtravido viduḥ.* See also L. Renou, *Histoire de la langue sanskrite* (Paris: Editions IAC, 1956), 53-59, and especially L. Renou, "Sur le genre du sūtra dans la littérature sanskrite," *Journal Asiatique,* 251 (1963), 165-216.

[59] *janmādy asya yataḥ: Vedānta-sūtra* I.12.

[60] For a modern treatment of lists, see F. W. Householder, "Lists in Grammars," *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress,* 567-576.

[61] *kācaṃ maṇiṃ kāñcanam ekasūtre grathnāti bālā na hi tad vicitram viśeṣavit pāṇinir ekasūtre śvānaṃ yuvānaṃ maghavānam āha,* quoted in K. C. Chatterji, *Patañjali's Mahabhashya: Paspasahnika* (Calcutta: A. Mukherjee and Co., 1957), 126. This refers to Pāṇini 6.4.133.

reliable treatment and mystification: the Yoga system as dealt with in Patañjali's *Yoga-sūtra*. As indicated by the title, this treatise is composed of *sūtras*. In the first four *sūtras* the subject matter is mentioned, defined, and characterized. This is followed by a classification of some specific forms.

The *sūtras* run as follows: "1. Now follows a treatment of *yoga*. 2. *Yoga* is cessation of the functions of the mind. 3. Then the perceiver remains in his own form. 4. Else he assumes the form of the functions."[62] This is followed by an enumeration of these functions. The remaining part of the first chapter explains how the cessation of functions is brought about.

These four *sūtras* lay the foundation for the entire system, which is constructed logically and fulfills the requirements of Indian methodology. Having introduced and defined the subject matter, the *sūtras* continue to explain each technical term. The third *sūtra* states what is philosophically most important in the system. Whoever stops the functions of his mind, i.e., the *yogin*, is the real man, who abides in himself (i.e., his self). The fourth *sūtra* supplements this by declaring that those who act differently, lose themselves (i.e., their selves) and identify themselves with the functions of their mind.

In this system, the attempt at accurate description and construction is present to the same extent as, for instance, in the chapter on affections in Spinoza's *Ethics*. The study of Indian philosophy in general loses much of its importance if the underlying methods are neglected. These methods are not only related to the language in which they are expressed, but they are often directly inspired by studies of this language by Indian grammarians. In the West, the recognition that linguistic structures play a decisive role in philosophy is slowly gaining ground. In India, it has long been explicit. This recognition may be expected increasingly to affect, if not to undermine, our philosophic certainties.

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[62] (1) *atha yogānuśāsanam,* (2) *yogas cittavṛttinirodhaḥ,* (3) *tadā draṣṭuḥ svarūpe 'vasthānam,* (4) *vṛttisārūpyam itaratra.*