



# UNIVERSALS



Frits Staal

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*Studies in  
Indian Logic  
and  
Linguistics*

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FRITS STAAL is professor of philosophy and South Asian languages at the University of California, Berkeley.

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# Preface

As a teenager in Amsterdam I would occasionally visit The Hague. It was not far but the natives struck me as decidedly odd, not to say weird. However, it didn't take long to find out that there also existed in The Hague people with whom it was easy and enjoyable to communicate. In the early fifties I spent three years in India where I had more exotic but similar experiences. Unlike many who came later, I did not go to India in search of drugs or religion. In Amsterdam, the first commodity had not yet arrived and the second was no longer in demand. What I encountered was a fresh experience that did not meet preconceived expectations, hopes, or fears. The experience taught me that those who want to feel good and also find truth should seek out both but never mix, let alone identify, them. Sanskrit says analogously that *preyas* "pleasant" is not the same as *śreyas* "good."

Outside the circles of Indian and Western experts, where everything has become familiar and unexciting, Indian thought is often used to provide various kinds of stimulation. This is not surprising but illustrates that most Americans and Europeans have not yet begun to understand Asia. The situation is not different from the common approach to Greek philosophy that existed a century ago. Nietzsche described it around 1880 in a passage of *Morgenröte* (§544) to which Philo Bregstein referred me:

I note that our philosophising youths, women and artists want philosophy to provide precisely *the opposite* of what the Greeks derived from it. A person who does not hear the constant joy which pervades every phrase and counterphrase of a Platonic dialogue, the joy which celebrates the new discovery of *rational* thought—what does he understand of Plato or of ancient philosophy? The Greeks were intoxicated with dialectics, concepts, ideas, arguments, generalizations and refutations, an intoxication that reminds us of what the great composers of classical counterpoint must have felt. . . . But our contemporaries do not wish to be looked upon as rational beings, they like to be exceptional souls, "intuitive natures" with "inner depth" and "intellectual vision." They want to be artistic, a genius in the head and a demon in the body, hence eligible for this world and the next, to which is added the divine prerogative of being obscure and unintelligible. And

*these* are the kind of people that now do philosophy! I am afraid they will notice one day that they have made a mistake, that what they are looking for is religion!

In India, I found answers to many questions that had never occurred to me. Everything demonstrated that Indian classical philosophers, grammarians, and ritualists—in Veda, Vedānta, Nyāya, Yoga, or Mīmāṃsā; in Buddhism, Jainism, or Tantrism—were rationalists and paid attention to arguments. They were in this respect like the ancient Chinese, Greeks, and many medieval European schoolmen, only more so. For their training comprised not only formal logic but also grammar, a discipline that until very recently was more highly developed in India than anywhere else. They were unlike contemporary irrationalists inspired by existentialism or hermeneutics, that is, in the end, religion. My conclusions were checked, expanded, and formulated in London and Amsterdam. The results were published in a series of articles, chiefly dealing with Indian logic and linguistics, which studied these subjects not from the viewpoint of the Orientalist but from a philosophic perspective.

Friends and colleagues have long urged me to republish these studies together in one volume as there has been a demand for them since their first publication in journals formidable and bizarre like *IJ* (*Indo-Iranian Journal*) or *BSOAS* (*Bulletin of the School of Oriental and African Studies*). This flattering suggestion also carried with it a whiff of mummification. Republication, however, offered an opportunity to explain their lasting relevance to a wider audience. I have always been convinced that the principal ideas and conclusions of these studies had remained inaccessible to what seemed to be their proper audience, namely, philosophers, logicians, linguists, and anthropologists. One would like to add other enlightened intellectuals, politicians, and businessmen who are not enamored of Asia but cannot fail to note its increasing importance—the kind of people whose children do martial arts but who remain themselves unimpressed by “Success through Musashi.” Is it more serious to argue that the ultimate source for Japanese management is Nāgārjuna? It could be done, but would not be simple, and would have to begin where I began, with Indian logic.

What is now rescued from self-imposed obscurity presents one more problem. Inaccessibility of the papers was not merely due to the originals having been tucked away in strange places; they also were addressed primarily to Sanskritists, who include eminent scientists but are interested in words, texts, and translations, perhaps history, not ideas. And so I have written a substantial, leisurely, mildly autobiographical Introduction that lifts these essays out

of their Sanskritic context and places them in the broader perspective of the search for universals, which was my original point of departure.

The book as it now appears does three things. First, it provides a new interpretation of Harvard philosopher Quine's "Indeterminacy of Translation" (now charitably called "Principle of Charity"), which had thrown doubt on the possibility of communication between civilizations. This interpretation provides access to a universe of analytic discourse outside Western analytical philosophy and shows how not to isolate philosophy from ongoing research in the human sciences, especially in Asian studies. Second, the book demonstrates that Indian *pandits* discovered the same universals that were discovered in the West. And third, it argues that the Western humanities and social sciences will have to learn from Asia before they can be of assistance in paving the way to the common future of mankind. The first and third argument derive their momentum and strength from the second. The second is based upon the studies in Indian logic and linguistics that are reprinted here in parts one through three.

Since writing a first draft of this Introduction, I have taught several seminars on comparative philosophy and logic at the University of California, Berkeley. Undergraduate seminars at Berkeley admit a limited number of only the brightest students, but they have no background in Indology, or in anything else for that matter. Graduate students are in general less open-minded than undergraduates, but none were averse to mixing Sanskrit and logic with linguistics, anthropology, and more than the customary minimum of ritual. The final text has accordingly benefited from comments by these students, of whom I should especially mention Risa Baumrind, Holly Dunseath-Cratty, Timothy Howe, and Richard Polt.

I have presented some of my ideas to other audiences, mostly in combination with other work, in various parts of the world. The most recent and memorable of these meetings took place in the winter of 1986 in New Delhi and in Amsterdam. The first was an international symposium where I joined with Indian savants in explicating Oriental concepts to foreign intellectuals, artists, architects, and town planners. In Amsterdam, students invited me to a discussion on the cultural relativism that had pervaded their entire education. In Amsterdam and Berkeley, the kind of rationalism I am espousing was until recently almost taboo. I am delighted to report that I could now establish in those places a countercult cult if only I tried. In New Delhi such a thing would not be feasible, nor would it be necessary.

I have decided, for the time being, to stay on the narrow path of academia. I made grateful use of comments on earlier drafts of the Introduction by George

Bealer, Noam Chomsky, Pamela MacFarland, Jeff Masson, and Bimal Matilal. I owe a special debt to Milton Singer for his unfailing interest in this work, and efforts to achieve its publication. The result is the handsome volume that the reader now has in hand. Mistakes and idiosyncracies are, I am sure, plentiful and remain, of course, my own.

# Introduction

## 1. Universals, Shadowy and Substantial

I use the term “universal” to refer to any feature, entity, or structure that is shared by all human beings but not by any other animals. That such universals exist is suggested by the fact that a man or woman is just as different from other animals as a fly is from a dinosaur or a bird from a fish. In other words, “universal,” in my usage, refers to characteristic or defining traits of the human species. “Universal” is also used as a noun: it refers, in that case, to something that is “universal.”

Let us try to apply the definition. “Heart” or “having a heart” is not a universal. It is found in all human beings but also in other animals (not in all, for amoebas have a nucleus but not a heart). A recent candidate for “universal” was bilateralization of the brain, until it was discovered in birds. Another, more persuasive universal is the capacity to use tools. The one I like best is language. All members of the human species possess language, even those that are in our eyes least developed. There are no simple languages; they are all equally complex, albeit in different ways. Experimenters who ascribed language to chimpanzees and dolphins, on the other hand, have been refuted; they demonstrated the existence in animals of something else, namely, the capacity for communication. But we knew already that animals communicate with each other. Human beings also seem to communicate with other animals (for example, farmers with their cattle or pet lovers with their pets). If they do, this would further support the idea that several animal species have the capacity for communication. Communication, however, is not the same as language, and it is not clear whether language is more or less than a system of communication.

Definitions must begin somewhere and then, as soon as possible, make way for well-organized speculation, that is, theory. I have tried to define “universal” in my own way because I wish to avoid a possibly ungainly neologism. I therefore had to pick an existing term and give it what is perhaps a partially new meaning. It must be admitted that it is difficult to distinguish my use of “universal” clearly from its numerous other uses. The notion originated in the Western tradition with Plato, who theorized about Forms or Ideas, such as *good*, *existence*, *sameness*, *difference*, *truth*, and *one*. These forerunners of our universals are related to particular, “sensible” things in the manner in

which science is related to sense perception. The soul, according to Plato, is an intermediate entity between ideas and particular things.

It took less than a thousand years for the Platonic ideas to turn into universals. During the Middle Ages their status was the topic of much discussion: Were they “prior to things” (*ante rem*), “derived from (or in other ways ‘posterior’ to) things” (*post rem*), or “in things” (*in rem*)? The third theory has always been the most intriguing. I still like to say of my universals that they reside “in” human beings, for example, “in” their minds. The “in” must have something to do with the organization of our brain but not in any direct or straightforward manner. We shall not find universals by looking very carefully inside and between neurons, axons, or synapses. I know that universals are unlikely to have anything to do with parts of the brain such as the brainstem, which evolved more than five hundred million years ago (“Because it resembles the entire brain of a reptile, it is often referred to as the reptilian brain.” [Ornstein and Thompson 1984, 4]). The rest lies in the future; we simply don’t know.

A definition of man (male and female), attributed to Aristotle, is ζῷον λόγον ἔχον, “the animal that possesses rationality.” This translates into Latin as *animal rationale* and into English as “rational animal.” Heidegger, who regarded man as *Dasein*, which he defined in terms of *Sein* or “Being,” proposed an attractive interpretation of the Aristotelian definition as “the being that speaks” (*Seiendes, das redet*; Heidegger 1953, 165). Or, in inimitable Heideggerian: *das Lebende, dessen Sein wesentlich durch das Redenkönnen bestimmt ist* “the living being, the being of which is essentially determined by the capacity to speak” (Heidegger 1953, 25). This definition is based on sound but irrelevant etymology (λόγος comes from λέγειν “to speak”), is Aristotelian in form but does not express what Aristotle had in mind, and should not displease such diverse contemporary thinkers as Quine, Grice, or Chomsky.

We know that language is a universal, but what about rationality? Isn’t the definition of man as a rational animal preposterous in the face of the colossal lack of reason that human beings display? In one sense, undoubtedly; but the concept of rationality inherent in the Aristotelian definition refers to the more specific domain inhabited by the principles of formal logic and other similar “ideas.” So the question arises: Do all people have logic? This question is the first that the present work seeks to answer.

In popular culture and among many natural and social scientists, doubts have arisen with respect to the universality of logical principles. It seems that we should be tolerant and allow other cultures their own kind of logic. However, respect for other cultures, which is not only ethical but also rational, is

not the same as cultural relativism, which is generally irrational. In Berkeley, many people over thirty are familiar with cultural relativism and are inclined to accept it. Scholars who have speculated on the history of science such as Thomas Kuhn, anarchist philosophers of science such as Paul Feyerabend, outdated polyglots such as Benjamin Whorf, physicists with a soft corner for Taoism—all are invoked in support of some sort of cultural relativism. Quine has a brilliant argument that addresses the issue and to which we shall devote much attention.

In the meantime, it remains undeniable that all human beings accept at least some principles of logic. Every human being who understands what is meant by “ $A = A$ ” will accept it as true. To most, “ $A = A$ ” will mean that for every thing it is correct to say (if one wishes to use one’s words in this manner): “this is itself,” “this apple is the same as this apple,” “this child is itself,” and so forth. Anyone who objects to such platitudes is being intentionally perverse—as some philosophers are—is a poet who uses metaphorical language; or is simply a person of unsound mind. A Platonist would say that “ $A = A$ ” comes from the idea of identity. This is correct, if one accepts that theory, and there is nothing deep or profound about it; its truth can be elucidated with the help of some philosophy of language and proven with a little bit of logic. Contemporary culture, however, does not favor logic. Tarski and Gödel are not mentioned as ceaselessly as Darwin, Marx, Freud, and Einstein, although their contributions were not less revolutionary. The names of logicians are vaguely remembered and fade away, even behind the current of intellectuals that continues to emerge from Paris: Merleau-Ponty, Lacan, Paul Ricoeur, Derrida. Our political leaders, with few exceptions, despise or ignore logic, and our citizens think it is an artificial and unnatural restriction invented by Western schoolmasters. Hence the phenomenal success of Oriental mysticism that is alleged to go beyond the laws of logic and embrace contradictions.

It must be admitted that the case for language as a universal remains more straightforward than is the case for logic. Language possesses features that are clearly universal, such as the distinction between consonant and vowel. The distinctions between noun and verb or subject and predicate are generally accepted as universal, though there have been scholars who have argued that they are not universal because there are exceptions (an example occurs below, page 19). Whatever the universals of language will eventually be, it has become clear since Chomsky that their study is the basic subject of linguistics. Universals of language are sometimes even referred to as linguistic universals. Here is a clear illustration of the Platonic idea that science deals with ideas, that is, universals.

Anything called a human science should deal with universals such as I have tried to define. Anthropology has discovered and discussed several promising candidates: kinship structures and other systems of classification, the prohibition of (some kind of) incest, and other features more closely related to the physical domain that characterizes man and his material civilization. But is man really defined and distinguished from other animals by such traits? As soon as we go beyond the level of other animals the experts disagree. According to Lévi-Strauss, the task of anthropology is to find universal structures that reside in the human mind. This idea was influenced by linguistics and stays close to its point of departure. In the United States, many anthropologists remain immersed in the study and interpretation of particular cultures and are inclined to accept some kind of relativism, generally tinged with irrationalism. If they take a hard line, they say, with Clifford Geertz, that “there may be no interesting general laws of culture, or of anthropology, to discover” (Liebersohn 1984, 45; cf. Geertz 1985 and Staal 1986b, 199–219); if they take a soft line, they say, also with Geertz, that we should study myths and symbols. The same holds, *a fortiori*, for scholars of religion.

The search for universals is a rational enterprise. If the universals include the principles of reason, we come full circle. Rationalism also depends on facts, but it is not therefore the same as empiricism. Nor is it complemented by irrationalism, as if “ $2 \times 2 = 4$ ” were “complemented” by “ $2 \times 2 = 5$ .” Nor is the irrationalism that inheres in many social sciences and humanities held in check by the statistical, quantitative, and behaviorist work that continues elsewhere in the same disciplines. Both kinds of anthropology, the “humanistic” and the “pseudoscientific,” tend to neglect the undeniable fact that anthropology must ultimately reside with the biological sciences. The biology that will be of use to anthropology may not yet exist. But the signs are encouraging, for biology is developing rapidly in many directions. Although some of its recent developments may be no more than fashionable flurries—socio-biology, for example—others, like ethology, are already well established. I found in India a science of ritual that deals with ritual universals and that fits more easily within the perspective of ethology than within any existing branch of the social sciences or humanities.

The term “anthropology” continues to be used in philosophy as well. Born during the Enlightenment, it was developed by Kant, retained by Hegel (who placed it between natural science and the science of spirit), and recurs in Heidegger, Sartre, Merleau-Ponty, and other phenomenologists already forgotten. Philosophical anthropology was later influenced by the movement of the *Geisteswissenschaften*, itself pervaded, through Wilhelm Dilthey, by irra-

tionalism and, in the final resort, religion. This holy mixture, which insists that the humanities are not a science, would never have crossed the English Channel and the Atlantic were it not for accidental circumstances: the fact that the English word “science” tends to be used mainly of natural science unlike French *science* or German *Wissenschaft*; the slogan of “the two cultures” created by C. P. Snow, who was a physicist and also a novelist; and most of all, the logical structure of the distinction between sciences and humanities which is such that it is difficult for it ever to be suspended. Researchers familiar with both sides are rare, and those who work on one side see “nothing” on the other. “Nothing” resembles a thick wall behind which the strangest things may happen; which explains, incidentally, why descriptions of what happens there are invariably caricatures. I have never found a single good reason for believing that there is a marked difference in nature or method between the natural sciences, the humanities, and the social sciences. All have to deal with data and produce hypotheses that may turn into full-fledged theories. But given its peculiar background, the myth of the two cultures is likely to stay.

The search for universals of logic and language is partly speculative and partly a matter of facts. The present study is therefore concerned with more than *two* questions: it addresses *four* that are closely related, like the four vertices of a tetrahedron: (1) Are there universals of logic? (2) Are there universals of language and/or linguistics? (3) What is the nature of Indian logic? (4) What is the nature of Indian linguistics?

The first question is of general philosophic significance: it asks whether there are logical universals, that is, principles of logic that are found in all human cultures and languages. When answering such a question, one cannot stay within the domain of Western logic without looking at facts that lie outside it. Quine and others have argued that the distinction between logic and facts, or theories and things, is hard to draw. But in the relevant literature we find that the question of logical universals is either discussed *a priori* and dealt with as an object of speculation with the result that it is left dangling in the air: in which case I shall call the universals “shadowy”; or else it is discussed in a more empirical fashion by instituting a comparison between Western logic and other logical traditions independent of it: in which case I shall call the universals “substantial.” The distinction between “shadowy” and “substantial” is reminiscent of Plato’s cave, but in reverse: for according to Plato, empirical investigations are mere shadows of substantial forms or ideas that exist in the world outside. Although few contemporary philosophers would accept this point of view, they continue to practice philosophy as if it were valid.

With regard to the second question, we observe an interesting difference. For although logic and language are likely to be closely related and may in the final resort be indistinguishable, universals of language are nowadays studied empirically as an accepted part of linguistics. How can it be done? The answer was formulated by Terence Langendoen at the end of a 1967 symposium on “Universals in Linguistic Theory” at the University of Texas, in Austin, when he was asked: What should we be teaching our students? He replied: “We should give them the ability to recognize an interesting linguistic problem when they see one, that is, one which throws some light—negative or positive—on our conceptions of what languages in general are like” (Bach and Harms 1968, vi). In the present studies, the empirical approach that is customary in linguistics is applied to the study of logical universals, more specifically to the logical tradition that is closest and most similar to the Western logical tradition without being historically related to it: namely, Indian logic.

There are several possible objections to this more empiricist approach and to the assumption that the Western and the Indian are the most similar independent logical traditions that humanity has evolved. The first is that comparisons between independent logical traditions are impossible. Here lies the importance of Quine’s argument, to which I have already referred. Let us suppose, says Quine, “that certain natives are said to accept as true certain sentences translatable in the form ‘ $p$  and not  $p$ ’” (1960, 58). But how do we know this translation is correct? If any evidence can count against translating certain expressions of the original language as “and” and “not,” certainly the natives’ acceptance of “ $p$  and not  $p$ ” as true counts overwhelmingly. Therefore, “the more absurd or exotic the beliefs imputed to a people, the more suspicious we are entitled to be of the translations” (Quine 1960, 69). Quine formulated this view later as the “principle of charity”: we should always interpret natives’ statements in such a manner that they make as much sense as possible. And since the distinction between language and logic is ephemeral, “a very alien conceptual scheme” is “a language awkward or baffling to translate” (Quine 1981, 41).

Within an empiricist perspective it is important to know what kind of natives Quine had in mind. Does his observation apply to Tasmanians, Bushmen, Neanderthals, Martians, Asians, or simply “non-Westerners”? With specific reference to Quine and Davidson, Ian Hacking (1984) remarks that “analytical philosophy” has “traditionally been doubtful about non-Western rationality.” Quine, as a matter of fact, has been less parochial: there are two references in *Word and Object* (1960) to articles by the Sanskritist John Brough on Indian general linguistics and theories of meaning. But *are* there natives who make nonrational claims, or anthropologists who still claim that

there are? True, Quine referred to Lévy-Bruhl's thesis of a "prelogical mentality," but such speculations have been long discarded, both by Lévy-Bruhl's successor Lévi-Strauss, who stresses that anthropologists study general features of the human mind, and by American anthropologists such as Clifford Geertz who reject such universal features. We must conclude that if Quine is right that our logical principles must be universal, then they are shadowy rather than substantial universals.

Quine's thesis of the "indeterminacy of translation" has been widely discussed by philosophers, and its reverberations continue to be with us. An example is Davidson's (1974) argument that was intended to show that it is difficult to make sense of Kuhn's "paradigm shifts," which are widely quoted in support of relativism. Yet neither Quine nor Kuhn give much support to relativism. As for Quine (to Kuhn we shall return later), his picture of the field-worker's translating activity is as idyllic and unrealistic as Jean-Jacques Rousseau's view of the natives' life in general. One reason for this lack of realism is that Western philosophers never look seriously at other languages or at the numerous translations from exotic languages that are available for study. The problems we face in the comparative study of logic, for example, involve translation but do not appear at the far end of the spectrum and are not as romantic or bizarre as Quine has depicted. The natives we encounter assert not that " $p$  and not  $p$ " is *true*, but that it is *false*. Moreover, even when they assert that it is true, our reactions are quite different from what Quine leads us to expect. Consider an example at only a short distance from English: German. If we confine ourselves to nonphilosophical German, there is overwhelming evidence to demonstrate that "und" should be translated as "and," and "nicht" as "not." Now suppose that a German asserts that " $p$  und nicht  $p$ " is true. Do we reject the translation of that assertion or revise our translations of "und" and "nicht"? Not at all. Assuming that our German informant is mad, a poet, or a philosopher, we will, depending on our mood, try to find out what he means or try to show him that what he says makes no sense.

This scenario does not refute Quine's point: we stick to logical principles such as the principle of noncontradiction "come what may." But Quine's point has little to do with the comparative study of logic. The odd native who asserts that " $p$  and not  $p$ " is *true* deserves separate treatment. The vast majority asserting that " $p$  and not  $p$ " is *false*, though less exotic, are more interesting and important: for their assertions provide empirical evidence for a logical universal that is substantial. This implies that rationality is not confined to the West, that mankind is more of one mind than has been suggested, and that we can communicate fruitfully within the species. Quine's argument is like an argument destined to show that if we can communicate with beings on other

planets we must assume that there is life on those planets. But it is obviously less interesting to speculate about there being life on a planet than to actually find it there. We on our planet are in the fortunate position of being able to communicate with the other human inhabitants. We find as a matter of fact and not of speculation that these other natives do not deny the principle of noncontradiction. And in the rare cases that we come across natives who *seriously* deny such a principle, there are interesting reasons. The following is an example.

In Indian logic we find that “ $p$  and not  $p$ ” is always treated as false. However, there occurs a different notice of negation for which it can be true: a logical connective that is in some respects similar to “not” and is in other respects different from it. If we translate it as “\*not\*,” the assertion “ $p$  and \*not\*  $p$ ” is sometimes true (see below, page 24). This situation does not refute Quine’s thesis; on the contrary, it confirms it. For it illustrates that if “ $p$  and not  $p$ ” is true, either “and” or “not” or both must be different from our customary “ands” and “nots.”

If it is true that independent logical traditions can be fruitfully compared, it stands to reason to begin with the two that are closest, namely, Western and Indian logic. But here a second objection may be made: it is useless to compare Indian and Western logic as if they were independent entities because, even if there are no historical connections between the two logical traditions, they are intrinsically related because both are expressed in related languages: the languages of the Indo-European family that comprises Greek, Latin, French, German, and English as well as Sanskrit.

This objection, formidable as it may sound, cannot be answered in isolation. We must know whether the features of language on which these logics presumably depend are features only of Indo-European, or whether they occur also in other, perhaps in *all*, human languages; in other words, whether they are universals of language. The principle of noncontradiction, for example, holds for every language that possesses negation because it expresses the fundamental property of negation. But the concept of negation is not limited to Indo-European languages. Plenty of empirical evidence, furthermore, demonstrates that this supposed dependence on Indo-European does not exist. For Greek logic was translated without loss into Syriac, Hebrew, and Arabic, and Sanskrit logic without loss into Chinese, Japanese, Korean, Mongolian, and Tibetan. All these languages are not only different from Indo-European, but many of them are not even related to each other. Although some of these translations were wrong or seriously misleading (for amusing examples, see Brough 1964), the majority have been good and even exceedingly good (see, e.g., Demiéville 1952 and van Gulik 1956). An illustration of a translation of

a Sanskrit original into Chinese that may be said to be *better* than the original, because *it disambiguates an ambiguity of the original*, occurs below on page 137. We conclude that the second objection, that of the apparent linguistically based relatedness of Indian and Western logic, is not clearly empty but at present futile: it is not an objection but a suggestion for a program of research. Some of this research is being done by linguists, and some is undertaken in the chapters that follow. Such research is not based upon the armchair method but upon translations of logical principles between unrelated languages. Though philosophers never take them into account, these translations provide copious substance for philosophic analysis.

Some readers may think of a third objection to the approach that is adopted in this book: namely, that the various logics we propose to study do not so much depend on their linguistic backgrounds as on certain metaphysical assumptions. I do not take this objection too seriously because it has been refuted long ago in the case of Western logic. As for Indian philosophy, although different schools developed different logical doctrines, most logical principles and techniques are shared between them. Metaphysicians with persuasions that are diametrically opposed (e.g., those Buddhist and Hindu stalwarts who respectively deny and affirm the existence of the soul and of substance) use the same logic because only in so doing will they be in a position to refute each other. In all systems of Indian philosophy, an opponent is held to be refuted when it has been shown that his position is inconsistent or leads to contradictions (for hosts of examples, see Granoff 1978 or Matilal 1986). The popular view that Indian philosophy embraces contradictions is totally unsupported.

A final objection that should be considered at least briefly states that it is of no interest to compare Indian and Western logics because they are too close to each other. This is like saying that the comparison of men and monkeys is not as interesting as that of men and insects. The answer is obvious: both must be done and we don't know beforehand what we will find. A third independent logical tradition is Chinese logic. It has not been studied as much as the Indian, but the later Mohist logic made accessible through Graham (1978) has already inspired a spate of technical work as well as more general books such as Hansen (1983). It remains undeniable, however, that the indigenous (i.e., non-Buddhist) Chinese works on logic, despite their sophistication, theoretical orientation, and taste for abstraction, have been relatively few and have spanned a relatively short period of time. There is promise of fresh discovery, but the harvest has been limited, so far, in comparison with the Indian and Western logical traditions, which have lasted for more than two millenia and produced thousands of original works.

I. M. Bocheński, to whom we owe the first history of logic not confined to the West, devoted roughly a tenth of his work to Indian logic but paid no attention to Chinese logic. Bocheński's work was published in 1956, five years before a series of articles on Chinese logic by Janusz Chmielewski began to appear, and twenty years before Graham's book was published. A contemporary history of logic could not omit China without peril. Yet it remains true that Chinese logic comes in importance after Indian logic, just as Indian logic comes (as we will see) after Western logic. It therefore stands to reason to first compare India with the West, and next China with both. As far as is presently known, any interesting piece of logical analysis found in the world is derived from the Chinese, the Indian, or the Western logical traditions. When we try to go beyond these three, we are confronted with a gap. This does not show that logic is elsewhere discarded, but that its principles are implicit. For whenever logical principles become explicit, it is the familiar principles of logic that unexotically but undeniably stare us in the face. If we tried to represent independent logical traditions by different means of transportation, it is as if only feet were freely available all over the world, to which China added chariots, India trains, and the West cars. Whatever the state of technology, the principles of locomotion are the same. They remain implicit and the users are unaware of them when locomotion is on foot. The analogy suggests that we are still waiting for submarines and airplanes and illustrates that the universality of logic does not imply that its principles must be always and everywhere explicit.

When referring to Chinese logic, I have excluded Buddhist logic because it came from India. But isn't logic precisely what is so conspicuously absent from the *koans* of (the Chinese) Ch'an and (the Japanese) Zen Buddhism that are fashionable because they seem so colorfully irrational? I studied this question elsewhere in some detail (see Staal 1975a, chap. 2) and shall revert to it below (pages 109, 125), but the general picture is clear. The two Buddhist schools that became popular in China are the Yogācāra-Vijñānavāda and the Mādhyamika. The first is a school of logic; it is illustrated in chapters 5 and 7. The second, which is the foundation of Ch'an Buddhism, was introduced by Kumārajīva's Chinese translations of Sanskrit works composed by or attributed to the famous Indian philosopher Nāgārjuna. It is characteristic of Nāgārjuna's work that he perfected the Indian method of defeating one's opponents by showing that their position leads to contradictions. This exercise makes sense only if the principle of noncontradiction is accepted. There is much unreliable literature on this topic, and the decisive studies are not easy to find (e.g., Bhattacharya 1971, 1980, 1981; and Tachikawa 1974). However, even more general books, as long as they are knowledgeable (such as Cheng

1984), acknowledge that Ch'an Buddhism is a practical application of Nāgārjuna's dialectic, which is strictly rational.

Before we get further entangled in Asian speculation, let us return once more to Quine. One reason for his universals being shadowy or empty was that he reacted to the Platonic tradition in Western philosophy, which culminated in a riddle of nonbeing—"Plato's beard" (Quine 1961, 2). Plato believed in universals, and located them outside his cave in a kind of heaven. Modern philosophers, with few exceptions (e.g., Gödel, see page 45 below), no longer accept such a heaven, but still postulate universals. Not being devoid of locus or substance, they now tend to be assigned, as we have already seen, to the mind. This assignment rests upon a method that is characteristic of science: the postulating of invisible structures to account for what is observed in reality. This method is based upon an idea about which some philosophers feel uneasy, even though it is both probable and fruitful: the theory that reality is different from the way it appears. This theory is widely accepted by natural scientists and ordinary people. In the human and social sciences it is rarer, but Lévi-Strauss, for example, adopted it from Marx, Freud, and geology, thereby creating a rift between himself and Sartre:

Le marxisme me semblait procéder de la même façon que la géologie et la psychanalyse entendue au sens que lui avait donné son fondateur: tous trois démontrent que comprendre consiste à réduire un type de réalité à un autre; que la réalité vraie n'est jamais la plus manifeste; et que la nature du vrai transparait déjà dans le soin qu'il met à se dérober. (Lévi-Strauss 1955, 61)

In order to interpret and explain human behavior, Lévi-Strauss therefore postulated structures that are invisible although they are "located" somewhere, namely, the mind.

Lévi-Strauss need not be right with respect to the particular structures he has postulated. We shall see, for example, that the distinctive features he introduced from Jakobsonian linguistics into anthropology do not fit there just as they do not fit syntax or semantics—their proper domain being phonology. In fact, he could have derived more benefit from the study of the mathematical and logical structures that influenced linguistics than from the study of linguistics itself (see Staal 1986b, 37–43; and 1988, chap. 22). What is of interest is Lévi-Strauss's method: the only scientific method that has so far been productive. Those who neglect it find mostly trivialities, as is amply demonstrated by such different philosophies as Wittgensteinian approaches, behaviorism, phenomenology, and hermeneutics.

Not only behaviorists and phenomenologists but also many logicians have

been suspicious of the mind as a seat for universals. But “seat” should not be taken in too literal a sense, and “mind” is not the same as “brain.” We have seen that it is unlikely that universals will be discovered in the brain. But that is not the only thing that worried logicians. The rise of modern logic coincided with the insight that logic is not part of psychology: this is what Frege showed in his refutation of the psychologism of Husserl. Although it is not clear whether Husserl understood the full force of this criticism, he subsequently adopted it and claimed it as his own (see Føllesdal 1958). Rejecting psychologism, however, does not imply that logical and mathematical structures cannot be located in the mind, but that their “location” is indirect and may be of an abstract nature. This became increasingly clear when Turing and others began to think about artificial intelligence.

I have referred to a problem area in the shape of a tetrahedron but have so far mainly reviewed it from its philosophically most interesting and speculative vertex: logical universals. We should now add substance to these speculations by turning to Indian logic itself and in its own right.

## **2. The Evidence from Indian Logic**

The argument of section 1 of this introduction is based upon the evidence that will now be presented in sections 2 and 3, itself being based upon the following chapters in this volume. Since the latter were originally published between 1960 and 1970 and are only now being offered to a wider public, the question arises: Why did I not continue these studies? Did I suddenly discover that this work was without promise or future, or—worse—worthless? But then why subject readers to their republication?

I shall answer these questions by reviewing the chapters one by one in their original context. In so doing I shall show how they are interrelated and provide answers to the problems introduced in the preceding section. From now on, the reader should begin to supplement this Introduction with plunges into the chapters and reviews themselves. This will facilitate the discussion and add substance to the argument. The reader will emerge from these diving sessions with doubts about the widely held view that Oriental thought goes beyond the straitjacket of Western logic and embraces contradictions. He will also become receptive, I hope, to other unfashionable truths. For I did not only establish conclusions that pertain to logic; I also used these studies as a foundation for other investigations that led to unexpected conclusions.

The organization of the remainder of the present Introduction is straightforward. Although the two areas are closely related, I shall in section 2 deal chiefly with the evidence from Indian logic, and in section 3 with that from Indian linguistics. Section 4 offers brief observations on seven reviews that

explain or illustrate different methodologies in comparative logic and linguistics. Section 5 will explain why this work developed in a direction that may at first sight seem unexpected but that in fact reflects a natural course of evolution.

## CORRELATIONS BETWEEN LANGUAGE AND LOGIC IN INDIAN THOUGHT

Chapter 1 touches upon many of the themes that will be developed in the other chapters. Though I have no quarrel with anything it says or claims, and would still have to use equally complex formal expressions to express its meaning and intentions, I would now use simpler English. For example, the chapter should merely be called: “Language and Logic in Indian Thought.” For what between language and logic is studied here if not *correlations* or, even more straightforwardly, *relations*?

Chapter 1 deals with three closely related issues; the first of which is indicated by the article’s title. In the fifties, Western philosophers were beginning to look at the traditional problem of language and thought in more specific terms. Throughout most of the history of Western philosophy it had been assumed without further question that “thought” and “thinking” were primary, profound, and basic human activities, for which “language” and “speaking” merely served as modes of expression. With the advent of the “ordinary language philosophy” variety of “analytical philosophy” it began to become clear that some of our thoughts depend on our language. J. L. Austin and Wittgenstein, each in his own way, discussed cases where such a dependence seemed probable. Now it began to dawn on at least some philosophers that “dependence on English,” or German or Greek for that matter, is not the same as “dependence on language.” As early as 1947 it had been argued that the “ordinary language” approach would require a philosophical study of languages other than English (Basson and O’Connor 1947). No one paid attention to this suggestion at that or any other time. Actually, not only does Sanskrit offer many opportunities to study how language is used to express a vast and complex conceptual system, but Indian philosophy itself offers interesting examples of “ordinary language analysis.” In the following three examples, a literal translation is possible because the argument from ordinary language is applicable to English as well as to Sanskrit:

1. In Bṛhad Āraṇyaka Upaniṣad 1.5.3 it is argued that we see and hear with our mind (*manasā*) because it is customary to say (that is, in spoken Sanskrit): *anyatramanā abhūvam nādarśam anyatramanā abhūvam nāśrauṣam iti*, “because my mind was elsewhere, I did not see; because my mind was elsewhere, I did not hear.”

2. Śāṅkara tries to refute the Buddhist śūnyavāda in Brahmasūtrabhāṣya

2.2.25 by arguing that the fact that we remember things can only be accounted for if we assume that we have a continuous personality. This is obvious because we say: *aham ado' drākṣam idam paśyāmīti*, "I have seen it and now I am seeing it (again)"; but we never say: *aham smarāmy adrākṣid anya iti*, "I remember because someone else has seen it."

3. The followers of the Mīmāṃsā school of Prabhākara Guru support the *anvitābhidhāna* view, according to which the meaning of a sentence arises directly from the collection of its words, so that words convey no meaning except in the context of a sentence, by arguing that children learn language in this manner: for a child who hears the sentences *gām ānaya*, "bring the cow," and *aśvam ānaya*, "bring the horse," understands in each case the meaning of the entire expression from the context or situation and arrives at the meaning of individual words only by distributional analysis (*anvayavyatireka*: see Kunjuni Raja 1963, 196–98; Staal 1969, 512–13). This argument sounds better in Sanskrit than in English because the flow of speech is more continuous in the former language on account of the so-called "rules of sandhi."

Although a systematic investigation has never been undertaken, Western philosophers have sometimes been less parochial and have taken non-Western languages and philosophies into account. The greatest proponent of linguistic universals was Wilhelm Von Humboldt. Ernst Cassirer continued the trend, which was revived in a different spirit by Leo Weisgerber who founded a study group called *Sprache und Gemeinschaft*. Cassirer continued to overrate Indo-European, and Von Humboldt was misinterpreted by the champions of hermeneutics, especially Gadamer (cf. Staal 1967b). In the twenties, Georg Misch in Germany and Paul Masson-Oursel in France initiated "comparative philosophy" and paid attention to Indian and Chinese philosophies. Karl Jaspers included the Buddha, Confucius, Lao-Tzu, and Nāgārjuna in his "Grossen Philosophen" of 1957. Unfortunately, many of these philosophers, Cassirer excepted, were poor philosophers. Jaspers, for example, though an honorable person, never arrived at a definite conclusion.

Bocheński, a clearer mind, included Indian logic in his history of logic of the same year (1956), as we have already seen. What is remarkable is that English and American philosophers and logicians, despite the facts that they often stressed empiricism and that the English-speaking countries occupied a more international position than the countries of the European continent, continued to ignore anything not written in English or other Western languages. In the field of logic, for example, the 1962 *Development of Logic* by William Kneale and Martha Kneale is a "Development of Western Logic." In order to counteract these nationalistic and isolationist tendencies and bring philosophy and linguistics closer together, an international group of scholars (Morris

Halle, Peter Hartmann, K. Kunjuni Raja, Benson Mates, Pieter A. Verburg, John W. M. Verhaar, and myself) founded the journal *Foundations of Language* in 1965, shortly followed by a supplementary series of monographs.

Around 1960, philosophers of language were interested in various brands of particularism, some of which survive today in popular culture and among scientists. In 1956, Benjamin Lee Whorf's papers, mostly written in the thirties, were published and enjoyed great popularity. Whorf claimed that Western thought depended on a group of Western languages, which he (with characteristic disregard of linguistics and philology) lumped together as "Standard Average European" (briefly *SAE*). Whorf maintained that a language like Hopi, on the other hand, made distinctions that led to a fundamentally different worldview. According to him, Hopi does not, for example, separate spatial and temporal categories, as is done in *SAE*, and so the theory of relativity, which was so new and startling to Westerners, would be obvious to a Hopi thinker.

Whorf was inspired by the American anthropologist Edward Sapir, who had been more sober in his views and formulations. The idea of analyzing language within an anthropological perspective occurred already in Malinowski, who used it not without condescension but with caution and sophistication (e.g., Malinowski 1948, 232). Other anthropologists went further than Whorf. According to Dorothy Lee, for example:

When the Wintu want to show that two or more individuals are in the same situation, they do not use the *and* to express connection. They will say, "Mary we gathered wood," not: "Mary and I . . ."; "Coyote they mutually-grandmothered dwelled," not: "Coyote and his grandmother dwelled." . . . [But,] when they are referring to individuals who do not participate in the same situation already, that is, when they bring two individuals together in their statement they do use the *and*; for example, "Shell *and* Fly (unrelated and living in different villages) each had a son." I find no instance, however, when the *and* is used to connect relatives, or individuals who are intimately connected, to put it in our terms. (Lee 1959, 81)

Lee's argument is striking but not conclusive. The question is, have the original Wintu expressions been properly translated? It is not open to doubt that Quine is right on this issue, that the different logics attributed to other peoples are due to bad translation: "Wanton translation can make natives sound as queer as one pleases. Better translation imposes our logic upon them, and would beg the question of prelogicality if there were a question to beg" (Quine 1960, 58).

Although Quine is right in general, specific solutions to these problems can only be provided by linguistics. Lee's problem about "and" admits of such a linguistic explanation. The "Shell and Fly each had a son" of her example is transformationally derived from "Shell had a son" and "Fly had a son." "Mary and I gathered wood," on the other hand, is not transformationally derived from "Mary gathered wood" and "I gathered wood." But this is not a peculiar fact about Wintu: we find similar structures in other languages, though they are expressed differently. For example, in Latin—which is "Standard European" if anything is—"and" is expressed by *que* when the two forms that are concatenated are closely related, for example, *Senatus Populus Que Romanus*, "the Senate and the People of Rome." But when two independent phrases are juxtaposed it is generally done with *et* and not with *que*. In English itself, "both" is used when the two members of a pair are closely related, for example, "I saw it with both eyes," not: "I saw it with my left and with my right eye." "He gave me both his ticket and a headache," on the other hand, sounds funny because the two underlying sentences—"he gave me his ticket" and "he gave me a headache" are not sufficiently similar ("gave," for example, implies "handing over" in the one case, but not in the other). Such a sentence may be used for special effect. We conclude that similar structures occur in Wintu, Latin, and English; but in English, there is only one "and" whereas in Wintu and Latin there are two. The Wintu case therefore shows something different from what Dorothy Lee suggested: the surface forms of one language are different from the surface forms of other languages although the underlying structures are the same.

Analyzing surface occurrences of "and" in a language—be it English, Latin, Hopi, or Wintu—is not going to solve problems of universals because there is no need for such "natural language and's" to be closely related. The English "and," for example, is different from the logical connective "&"; it carries with it the temporal notion of "and subsequently" and even the causal notion of "and in consequence." Hence, "she took arsenic and fell ill" is not the same as "she fell ill and took arsenic," as was pointed out by Ryle (1956, 118; for similar cases, see Traugott 1986). The ordinary uses of "and" and other particles *presuppose* their logical uses, which often remain implicit. We must accordingly study linguistic deep structures as well as the technical uses of connectives such as "&" in different logical systems, especially in Indian logic because it is the most developed system of logic that has arisen outside the Western logical tradition.

The idea that Indian logic is relevant to contemporary problems was the first and primary motivation for writing chapter 1. If one wants to see whether thought depends on language, one should study a specific case and ask whether *Indian* logical principles depend on *Sanskrit* linguistic structures. Vague

claims can be tested by Indian logic because it offers precise and explicit logical principles expressed by the linguistic structures of a precise and explicit language: Sanskrit. To carry out such an investigation, it would be necessary to use a precise language to represent both logical and linguistic structures. I therefore introduced the use of a mathematical language inspired by the language of contemporary Western logic. But now a familiar objection could be raised: Was I not importing *Western* ideas by using this language, and thus begging the question and determining beforehand the outcome of my analysis? How could I be certain that I was not forcing Indian structures into the straitjacket of Western preconceptions?

I am not preaching an absolute, unchanging truth; my attempt was a scientific inquiry, hence not final. I was careful not to introduce *concepts* of Western logic into my language but *symbols* that stand in one-to-one correspondence to the original expressions. This symbolism was introduced in such a manner that it became possible to automatically replace Sanskrit expressions by symbols and vice versa. I did not use Western propositional or predicate logic—not to mention Aristotelian categories of thought; I did not presuppose the universality of the subject-predicate relation or expressions based upon it such as  $P(S)$  or  $F(x)$ . I made use of a special notion, the *restricted variable*, introduced by Hailperin in 1957 in an article in *The Journal of Symbolic Logic*—one among numerous special notions that have been introduced by logicians and mathematicians to perform a variety of tasks—because I had found that this notion expressed a basic feature of the logical expressions of the Sanskrit originals. This expression is similar to Church’s  $\lambda$ -operator (see Church 1941), and similar notions are now used in linguistics (an example, taken almost at random, is Sag’s notation as quoted in Zohrab 1986, 427). The proof that I did not introduce extraneous ideas lies in the method, which permitted *Sanskrit* expressions to be translated into *mathematical* expressions and vice versa—barring only synonyms. Thus,  $\alpha xB(x, sādhyā)$  translates automatically into *sādhyavad*, in accordance with the principles I had laid down, and vice-versa; a computer could be programmed to produce these results.

The attempt in chapter 1 to test the idea that “thought” depends on “language” induced two other features: a formalization of Sanskrit expressions, such as nominal compounds, which made use of the analysis provided by the Sanskrit grammarians, thus pointing from language to linguistics; and a formalization of the formulas of Indian logic, especially of *navya-nyāya*, concentrating on the definitions of *vyāpti* that had already received a measure of formal analysis at the hands of Saileswar Sen, D. H. H. Ingalls, and I. M. Bocheński.

The result of chapter 1 in the area of logical universals was clear: there was

no evidence for supposing that Indian logicians “thought differently” from logicians in the Western tradition. Although some Indian notions look different, the difference does not stem from the use of a radically different logic. This result was in accordance with Quine’s *a priori* reasoning, but it went beyond Quine in establishing certain universals as substantial and not just shadowy. The result also accorded with the results of a variety of other empirical investigations on substantial universals that appeared at the time and were quite independent of Quinean arguments.

I shall give three examples of such investigations. The first is the most specific: in 1969, Brent Berlin and Paul Kay published the result of a careful linguistic and psychophysical investigation of color terms in ninety-eight languages of diverse language families. They concluded that the prevalent doctrine that each language and culture predisposes its bearers to see the world in terms different from those employed by other languages and cultures—a doctrine presumably obvious especially in the area of color perception—was false. They were able to show, quite to the contrary, that eleven psychophysically defined colors underlie all basic color words in all the languages of the world.

Two other and much more general theories, the first more speculative and the second more firmly established, accord with these ideas stressing the universality of thought and language. In anthropology, especially in France, the ideas of a “prelogical mentality,” espoused by Lévy-Bruhl in the twenties and challenged by Quine, were rejected and replaced by the views of Lévi-Strauss, according to whom the human mind is everywhere the same and operates with structures that are universal and in some sense logical. Although Lévi-Strauss formulated this view with brilliance, he attracted few serious followers, especially among anthropologists, and his conclusions are far from being universally accepted.

Lévi-Strauss was inspired by Roman Jakobson and by linguistics in general, and it is in this area—my third example—that one should look for solid, substantial universals. In the thirties, Jakobson, partly inspired by N. S. Trubetzkoy, introduced phonological universals, the so-called distinctive features (see Halle 1983). Though the list of features may be modified, the idea of such features has been generally accepted. In the sixties, Noam Chomsky went much further and established that there are linguistic universals of various kinds: phonetic, syntactic, and semantic. Chomsky called some of these *substantive* to distinguish them from others that are more abstract, which he referred to as *formal*. All these universals involve the notion of *rule* (see, e.g., Chomsky 1961 and Staal 1988) and are not shadowy but substantial. Chomsky’s proposals for linguistic universals have not been accepted by all

linguists, and the candidates proposed as linguistic universals are still changing; but the few linguists who opposed these ideas most virulently did not understand the general nature of such claims. Moreover, more recent work on universal grammar reflects a wider range of empirical materials than was used in most of the earlier investigations, particularly as linguists with widely varying language backgrounds have begun working on their own languages (see, e.g., Chomsky 1981). It is justified to claim (as I did in my first review of Chomsky's work: Staal 1966) that Chomsky turned a respectable branch of scholarship into a contemporary scientific discipline. To logicians, philosophers, anthropologists, or anyone else interested in universals of logic and language, the work of Chomsky and his followers retains fundamental importance.

Though Whorfian ideas are still occasionally met with among philosophers and anthropologists, and also in popular culture, most work of the seventies and eighties has tended toward universal structures. Chapter 1, first published in 1960, exhibited related ideas derived from the study of Indian logic. However, this article was rather technical and appeared in an Orientalist journal where it remained safely hidden: for Western logicians are for the most part uninterested in Indian logic, and Sanskritists are unwilling to read what look like mathematical formulas. Others who are interested in universals read neither Sanskrit nor formulas. The reason the article appeared at all is that John Brough, then Chairman of the Editorial Board of the *Bulletin of the School of Oriental and African Studies*, which published "Correlations," was a *rara avis*: a Sanskritist at home in linguistics as well as in logic.

## FORMAL STRUCTURES IN INDIAN LOGIC

Chapter 2, which originally appeared in a special issue of the journal *Synthese* devoted to "The Concept and the Role of the Model in Mathematics and Natural and Social Sciences," is reprinted here despite its overlap with chapter 1 because it addresses readers who don't know Sanskrit or Indian cultural history. We now know a little more than was known at the time the article was written. The structure of subject and predicate is not an adventitious feature of such Whorfian creations as "SAE," but is either a syntactic universal or a feature of the large group of languages that Ken Hale has called "configurational." These languages are distinguished from "nonconfigurational" languages, such as the Central Australian language Walbiri (see Hale 1978; Chomsky 1981, 94–95) and perhaps Japanese (Chomsky 1981, 127ff.; but see Kuroda 1984), where the distinction between subject and predicate does not occur. If this is a valid distinction, the subject-predicate structure is not

universal and the relevant universals of language must be found at a more abstract level.

### MEANS OF FORMALIZATION IN INDIAN AND WESTERN LOGIC

Read in 1958 in Florence to the history section of the International Congress of Philosophy, chapter 3 extends the methods of formalization adopted in chapter 1 to the analysis of a long passage in a well-known work of Indian logic, the *Siddhānta-muktāvalī*. The passage deals with four insertions that result in four improvements of the first definition of *vyāpti*. Here the formalization is more complete than in any other work with which I am familiar, with the possible exception of Goekoop (1967). Each expression of the Sanskrit original is represented by a formalized expression. Yet, the argument as a whole is not represented by anything resembling a formal demonstration in contemporary mathematical logic. My inability to provide such a full-fledged formalization is due to the fact that the four “insertions” are introduced in a nonsystematic manner; for although their implications are discussed in full, their emergence is nowhere explained.

Chapter 3 explains how the technical term *avacchinna* is used in the Sanskrit texts to express generalization. This formalization may be of future use, for the formulations of Navya-nyāya bristle with *avacchinna* and *avacchedaka*. The chapter also illustrates various uses of the principles of contradiction and the excluded third.

Timothy Howe has pointed out that the term “counterexample” at the top of page 84 is misleading; a better expression would be “complementary example.”

### THE THEORY OF DEFINITION IN INDIAN LOGIC

Readers unaccustomed to the formal expressions of Western logic will discover in chapter 4 that such expressions do not introduce anything extraneous or alien to Indian modes of thinking, but that even a small amount of formalization is helpful because it expresses precise and unambiguous distinctions in a precise and unambiguous manner. The analysis illustrates clearly that the use of so-called Western mathematical expressions does not add one jot or tittle to the Sanskrit originals—just as Sanskrit *dvir āvṛttā daśa viṃśatiḥ sampadyante* may be translated into English as “two times ten equals twenty,” but it can be translated even more clearly by using what is not a Western but a neutral mathematical expression:  $2 \times 10 = 20$ .

Chapter 4 illustrates that Indian logic is sometimes *extensional* and not mainly *intensional* as Bocheński (1956, 517) had claimed. It also illustrates

that Indian classifications exhibit the same “logical” features inherent in Western classifications. In this context we are again dealing with features of the human mind that seem to be universal. This throws light on the classifications used in preliterate societies to which anthropologists have drawn attention. That such classifications, especially in the area of genealogical systems, go back to the paleolithic age is now made plausible by Edmund Carpenter’s publication of the work of Carl Schuster (Schuster and Carpenter 1987).

The Indian systems of classification that underlie the Indian theory of definition expose the illusoriness of such classifications as Foucault has quoted in the Preface to his book *Les mots et les choses* (1966), following J. L. Borges who had attributed it to a Chinese encyclopedia:

- (1) Animals that belong to the emperor;
- (2) embalmed animals;
- (3) tame animals;
- (4) suckling-pigs;
- (5) mermaids;
- (6) mythical animals;
- (7) free running dogs;
- (8) animals classified here;
- (9) animals that carry on like mad;
- (10) animals that cannot be counted;
- (11) animals drawn with a fine brush of camel’s hair;
- (12) animals that just broke a jar;
- (13) animals that resemble flies from afar.

Since this classification is imaginary and not Chinese, it does not support widespread preconceptions about “the mysterious, topsy-turvy character of the Chinese mind” (criticized by Eliot 1954, 3:223). At any rate, even when genuine, such lists are products of literature, not philosophy. I do not know whether Chinese logicians have analyzed definitions with the same measure of precision that is employed in Indian logical texts such as *Tarka-saṃgraha*, *Tarka-dīpikā*, *Bhāṣā-pariccheda*, and *Kārikāvalī-muktāvalī*. But some Chinese logicians knew quite well that “*animals of four feet* form a broader group than that of *oxen and horses*, while the group of *things* is broader still” (Needham 1956, 2:174). The famous discussion of the statement “a white horse is a horse” exhibits logical problems of a similar nature and is much more sophisticated (and complex) than used to be thought (see Graham 1978, 481–94). Similar problems are discussed in Indian logic (see Matilal 1968, 48 and page 250, below).

The same holds for India, China, and the West: there are fairy tales as well as scientific investigations. The Indian theory of definition is not invalidated by the fact that the *Vāyu Purāṇa* declares that Dakṣa, a son of Brahmā, cre-

ated “sages, gods, heavenly musicians, men, serpents, demons, tree spirits, ghosts, devils, birds, domestic cattle, and wild beasts” (O’Flaherty 1975, 47). In the Western medieval *bestiarium* literature, one may similarly read that deer are fond of music but cannot get fever, that the goat breathes through its ears, and that the *wexelvia* is fertilized through the ears but gives birth through the mouth. The animal *catoblepas*, which lives on the banks of the river Nile, keeps its head on the ground because it is too heavy to lift—fortunately for the other animals since looking into its eyes would cause immediate death. Such fairy tales do not invalidate serious research in zoology or logic.

A special feature of chapter 4 is the discussion of metalinguistic concepts such as *paribhāṣā*, interpreted and translated as “metatheorem.” The use of this term as synonymous with *nyāya* indicates that certain features of Indian logic can be traced back to the earlier sciences of grammar and ritual. The concept of metalanguage was introduced into the study of Indian logic and grammar independently and in the same year by Scharfe (1961, 25). The terms “metalanguage” and “metatheorem” or “metarule” were adopted by Louis Renou (1963, 199; 1969, 495), Cardona (1967, 38; 1969, 4) and others. Scharfe (1971) devoted a monograph to Pāṇini’s metalanguage, and I reverted to the Indian background of the concept in Staal 1975b.

## CONTRAPOSITION IN INDIAN LOGIC

Chapter 5 establishes that a basic equivalence found in Western logic is also found in Indian logic: the equivalence between a statement of inference of the form:

$$(1) \quad (x)(A(h, x) \rightarrow A(s, x))$$

“for all  $x$ , if  $h$  occurs in  $x$  then  $s$  occurs in  $x$ ,” and its contraposition of the form:

$$(9) \quad (x)(\sim A(s, x) \rightarrow \sim A(h, x))$$

“for all  $x$ , if  $s$  does not occur in  $x$  then  $h$  does not occur in  $x$ ” (the formulas are numbered following their numbers in chapter 5).

A simple example of contraposition is:

“if the sun shines, it is warm”

is equivalent with:

“if it is not warm, the sun does not shine”

but not equivalent with:

“if the sun does not shine, it is not warm”

(for example, it may be warm because the fire is on although the sun does not shine). This illustrates, in passing, that the principle of contraposition is less

basic and more technical than, for example, the principle of noncontradiction (on which it depends). To say that it is found in Indian logic does not therefore imply that every Indian thinker is aware of it. The same holds in the West: even Plato struggled with a logical principle that is very similar (Bocheński 1951, 17).

Although a mistake in the derivation of formula (1) from formula (7) was noted by several reviewers, chapter 5 formalizes arguments and not only expressions. Emphasis is laid upon the logical use of *eva*, “only,” and more attention is paid to historical development than in any of the earlier chapters. This illustrates the continuity of Indian logic, from the brilliant investigations of the Buddhist logicians to the codifications of the Navya-nyāya which came almost a millenium later—a continuity in some respects similar to that from Aristotle to contemporary logic. The chapter also shows that Stcherbatsky failed to recognize the logical significance of contraposition, whereas Bocheński erred in claiming that Indian logicians remained convinced that examples possess demonstrative value. This study refers briefly to logical controversies in the philosophical systems of Vedānta and Mīmāṃsā, and traces the terms *pūrva-pakṣa* and *uttara-pakṣa* to their original, ritual context where similar terms refer to the two wings of the Agnicayana altar of the Vedic *śrauta* ritual (Staal 1983).

The reader will note that the difference between Buddhist and “Hindu” logic is concerned with degrees of formalization and other technical developments. It does not affect basic logical principles and has nothing to do with the differences between Buddhism and “Hinduism” (a label I place between quotes because there is no such religion, a “Hindu” is merely an Indian who is not a Buddhist, Jain, Sikh, Christian, Muslim, or adherent of a foreign religion; see Staal 1985b). Thus, “Hindu” logic is the same “Indian” logic to which both Buddhists and Jains contributed substantially. This independence from doctrine is precisely what makes logic into the interdisciplinary discipline it is. In the West, its status has not been different.

#### NEGATION AND THE LAW OF CONTRADICTION IN INDIAN THOUGHT: A COMPARATIVE STUDY

I wrote chapter 6 while I was still attached to the School of Oriental and African Studies, and it was published in its *Bulletin*. It was prophetic in anti-nomy, so to speak, for I had yet to sample the fashion of subsequent decades, widespread in the West and especially in California, that the Law of Contradiction or, rather, the Principle of Non-Contradiction is an artificial and unnatural restriction invented by Western schoolmasters, which inhibits free cre-

ativity and is absent from the profound and mysterious philosophies of the Orient, especially of India. Unperturbed by such qualms, then encountered only rarely, I set out to show that the principle of noncontradiction was accepted as basic in the ritual and grammatical sūtras and the later philosophical systems of India, especially Mīmāṃsā, (Advaita) Vedānta, and Nyāya.

In the Mīmāṃsā there arose an interesting ramification, based upon the ritual sūtras, because a distinction was made between *two* different kinds of negation or prohibition, of which *one* corresponds to the Western type of negation that goes back to Aristotle and continues to underlie contemporary logic. According to Aristotle, the negation of a proposition is the negation of its predicate. This is explicit in Russell and Whitehead's *Principia Mathematica*, which formulates the negation of a proposition in such a way that it is automatically equivalent to the negation of the predicate: for  $F(x)$  means  $(\sim F)(x)$ , and cannot mean  $F(\sim x)$ , which is not well formed. The other kind of negation or prohibition recognized and analyzed in the Mīmāṃsā is not governed by a principle of noncontradiction. However, there is nothing mysterious or mystical about this; on the contrary, it illustrates in an interesting way Quine's observation that the principle of noncontradiction exhibits the basic feature of negation and is necessarily true because its denial would indicate that we are not dealing with what is ordinarily conceived of as "negation" but with something else. Chapter 6, therefore, demonstrates again that logical thought, in spite of interesting ramifications and variations, is the same in India and in the West.

Richard Polt has pointed out that the last statement quoted on page 116 should read: "(The suffix *ghañ* is) not (applied) to the nominative (but is applied) to *other* case relationships."

I returned to the topics of this chapter in a wider context in *Exploring Mysticism* (1975a), a book that tries, among other things, to discover the origins of the Western desire to find irrational mysteries in the Orient. These origins turn out to lie in Christianity, which harbors an irrationalist trend that is absent from other religious traditions. It has reappeared in the West from time to time, most recently in existentialism, hermeneutics, and related theologies. The close association between Western religion and the irrational, and not the philosophies of India themselves, is responsible for the mistaken opinion that Oriental thought, which is believed to be exclusively religious, also goes beyond logic and reason. Not surprisingly, this opinion has been eagerly adopted by contemporary Oriental sages who seek success and followers in the West—a circumstance that has led to more confusion not only among credulous seekers of salvation but also among serious scholars.

The second chapter of *Exploring Mysticism* deals with the apparent counter-example of the Buddhist *catuṣkoṭi*: "Everything is such as it is, not such as it

is, both such as it is and not such as it is, and neither such as it is nor such as it is not.” The fourth footnote to chapter 6 states this principle that “indeed contradicts the law of contradiction” (below, page 109). Apart from the fact that this gets us involved in the difficulty mentioned by Aristotle that one cannot disagree with the principle of noncontradiction without in so doing accepting it (cf. Staal 1975, 37), the *catuṣkoṭi* raises a host of other difficulties. Whatever their solution, there is no doubt that even philosophers who made use of *catuṣkoṭi* use the principle of noncontradiction and invoke it explicitly. This conclusion was established in the second chapter of *Exploring Mysticism*. The entire discussion given there may be formalized along the lines of chapter 6 (see Staal 1976).

The inescapability of the principle of noncontradiction is of particular interest in connection with the Mādhyamika philosopher Nāgārjuna, who lived around 200 A.D. As the father of the Mādhyamika, grandfather of the Chinese Ch’an, and great-grandfather of the Japanese Zen, he is often regarded as a typical proponent of Buddhist irrationalism. However, the principle of noncontradiction is “invoked constantly” (Robinson 1957, 295) in his work and formulated explicitly. It underlies his favorite methodology, which involves the perfection of the common method of Indian philosophers of refuting an opponent by showing that he contradicts himself (see above, page 00, and Staal 1975a, 34–35). This is important for the understanding of logic not only in India but in Asia generally: for like many other Indian texts, Buddhist as well as non-Buddhist, the Mādhyamika texts were soon translated into Chinese, subsequently into Korean and Japanese, and finally into Tibetan.

We shall return to the importance of the study of Indian logic (and, as we shall see, linguistics) for the understanding of both Indian philosophy and Buddhist philosophies in other parts of Asia. Although the philosophies of Asia are not all similar, Indian logic and linguistics provided a general methodology which molded and pervaded so many of them that it becomes possible to refer in general terms to “Asian thought” and to characterize it as “Oriental rationalism.” Although such a generalization is, like all generalizations, to some extent misleading, it is far less misleading than the common characterization of Asian thought as “Oriental mysticism.” The latter *slogan* (from Sanskrit *śloka*, a verse in a popular meter) is largely based upon the prejudice that I try to expose in chapter 6.

The implications of such a shift in characterizations are plentiful. I shall illustrate some of them with reference to ideas expressed by Fritjof Capra in his bestselling book *The Tao of Physics* (1975). According to Capra, modern physics is similar to Oriental mysticism (in which he includes “Hinduism,” Buddhism, “Chinese Thought,” Taoism, and Zen) because both stress the interconnectedness of all things. Capra compares, for example, Aśvaghoṣa’s ex-

planation of *tathatā* (thusness)—“What is meant by the soul as suchness, is the oneness of the totality of all things, the great all-including whole”—with Heisenberg’s: “The world thus appears as a complicated tissue of events, in which connections of different kinds alternate or overlap or combine and thereby determine the texture of the whole” (Capra 1975, 139).

Such a parallel is by its very nature vague. But it is not so vague that we cannot show that it is also invalid. For, on the one hand, we have Heisenberg attempting to express *in ordinary language* (“the world thus appears . . .”) what is expressed *precisely* and *exactly* by means of the mathematical equations of quantum mechanics; on the other hand, we have Aśvaghōṣa’s characterization of *tathatā*, which also refers to something else but not to anything definite, and avowedly so. The comparison between something definite and something indefinite is bound to be indefinite, and therefore invalid. What, then, are the implications for the topics we are dealing with in the present context, namely, language and logic?

Capra is right when he stresses that both modern physics and Oriental mysticism go “beyond language,” as in the following quotes:

The contradiction so puzzling to the ordinary way of thinking comes from the fact that we have to use language to communicate our inner experience which in its very nature transcends linguistics. (D. T. Suzuki in Capra, 1975, 45)

The problems of language here are really serious. We wish to speak in some way about the structure of the atoms . . . But we cannot speak about atoms in ordinary language. (W. Heisenberg in Capra 1975, 45)

Such a parallel does not lead to any striking conclusions, however, because many things go beyond language. Music and ritual go beyond language. Much work in the sciences, not only in modern physics, requires the expressions of artificial languages derived from mathematics or logic that also go beyond natural languages. To express scientific results in ordinary language for the benefit of nonscientists or nonspecialists is always a hazardous undertaking, which yields at best approximations. In physics, the inapplicability of ordinary language means, for example, that macroscopic concepts such as “particle” and “wave,” which have shaped our imagination and language, are not applicable to the ultimate elements. The mathematical equations do not refer to such concepts. In linguistics, the notion of “deep structure” is shorthand for a technical concept and can be similarly misunderstood or misapplied. Clifford Geertz (1972) suggestively manipulated this notion when coining the expression “deep play,” which has nothing to do with “deep structure”. For in lin-

guistics, a “deep structure” is an abstract underlying syntactic structure, which has to meet conditions of well-formedness, and which is postulated to provide a starting point for derivations of “surface structures” with the help of rules that also meet conditions of well-formedness. In Geertz, “deep play” is simply an activity that seems innocuous and irrelevant but that in fact expresses important issues (such as, in the case of the Balinese cockfight, honor and status; see Geertz 1972; Staal 1986b, 210–11 and 1988, chap. 25).

Ordinary language is not the most adequate means to express scientific facts and relationships. The progress of physics is largely due to the fact that its problems were not discussed in ordinary language but in the language of mathematical equations. It is surprising that Capra has failed to recognize this because he is a physicist and not a phenomenologist or an ordinary language philosopher. Ordinary language statements by mystics, on the other hand, are not approximations of scientific expressions. We do not know whether mystical experiences may be adequately described with the help of an artificial language; it has not been done. But the reason that they go beyond language is probably due to the fact that the mystical state is similar to a state of mind that preceded the origin of language (cf. Staal 1979a, 20; 1980, 138–39). Music, songs, and ritual similarly precede language in the course of biological evolution (cf. Staal 1985a).

Regarding logic, Capra proceeds from the interconnectedness of all things to the irrationality of all things; but for this step there is no justification. By stressing that both Western science and Eastern wisdom go beyond “the world of opposites,” he suggests that both accept contradictions. Whether this is correct with reference to physics I doubt; with reference to Indian thought it is not.

#### THE CONCEPT OF *PAKṢA* IN INDIAN LOGIC

The study presented in chapter 7 attempts to unravel confusions that resulted from the uncritical projection of Western—Aristotelian as well as Kantian— notions onto Indian concepts. The principal culprits are two pioneering scholars of Indian logic: Th. Stcherbatsky and S. C. Vidyabhusana. Both may be compared to K. Prantl, the great nineteenth-century historian of Western logic who was more of a philologist and historian than a logician. Following these two pioneers, many scholars continue to refer to Indian concepts with the help of Western terms such as “syllogism,” “major,” “minor,” etc. These terms are misleading and their use has led to invalid conclusions. Chapter 7 also demonstrates that Indian logical entities were never considered in the abstract, as if hanging in the air; they always occur in a *locus*, their feet firmly planted on the ground. An additional result has already been mentioned: a confusing ambi-

guity of a logical expression in a Sanskrit text is disambiguated in its Chinese translation. This does not merely refute Arthur Waley's contention that, in many cases, the Chinese translators did not appreciate the subtler points of the Sanskrit originals, but merely replaced words by characters, the results being fit for recitation and perhaps meditation, but not susceptible to further rational analysis (below, page 138); but it also illustrates that the structures of Indian logic are not restricted to Indo-European.

That Chinese translations of these Sanskrit works exist is well known to Indologists and other Asian scholars, whom this chapter originally addressed. However, these matters raise issues that are often misunderstood by Western readers who tend to adopt a religious perspective. The mention of "Buddhist" seems to justify such an approach. However, it is a misnomer, just as "Christian logic" would be. The role of logic has to be understood in a different context in India and in Asia generally. For, like in the West, the value of logic as an instrument (Aristotle's *organon*) depends on its independence from religions, philosophies, and other conceptual systems. If its status were different, it could not be of use in other scientific disciplines. Logic is, in terms of chronology and importance, the first interdisciplinary discipline.

In India, logic was primarily used in the discussions between ritualists, grammarians, and philosophers. To illustrate these discussions, let us briefly consider the important philosophical problem of whether there exists a soul, which has been debated throughout the centuries. Most thinkers and scholars accepted that man has feelings, perceptions, knowledge, a will, an intellect, and other faculties that may be thought of as mental properties. These may include the "universals" we are studying in this volume (the "ontological status" of these has been discussed; see, e.g., Murti 1963; Chakrabarti 1975). It was generally agreed that these entities, which together resemble what in English is called "mind," do not continue to exist after death. For no Indian philosopher accepted the idea of an "immortality of the soul" of the Christian variety, which would include personal traits. Indian philosophers have similarly rejected as superstition the popular view of transmigration when it is believed to include such traits. The question about the existence of the soul, therefore, was the question of whether there existed something else, underneath and beyond not only our physical but also our mental faculties. If it could be demonstrated that such an entity existed, the next question was whether it survived beyond death.

The Upaniṣads had not been very clear on this issue, but they tended to favor the view that there is a universal entity called *brahman* which each living being regards as its "soul" or "self." It would not be incorrect to say that this entity "survives" after death, but this would be in a manner of speak-

ing and, therefore, misleading; for *brahman* is impersonal and never associates itself with any individual life.

The Buddha did not comment on the existence of a soul. He did not merely fail to do so; he explicitly stated that discussions of such problems are useless and are obstacles on the path that leads to nirvāṇa. That the question, “Who attains nirvāṇa?” was nevertheless related to this very problem was of course clear; but it was taken up seriously only by Buddhist philosophers in later centuries. Some of these Buddhist thinkers tried to prove the existence of a universal entity consisting of consciousness and discussed its status and properties in detail. These idealistic philosophers of the Vijñānavāda or Yogācāra produced many great logicians, among whom Dignāga and Dharmakīrti are the best known (they are referred to in chapters 5 and 7). Another school denied the existence of the soul: this is the Mādhyamika, whose most famous proponent was Nāgārjuna.

We have already seen that Nāgārjuna explicitly accepted the principle of noncontradiction and also used it in his favorite method of refutation. The same holds for the logicians and philosophers of the Vijñānavāda/Yogācāra. In this respect these scholars were not different from their predecessors: all of them used logic when debating among themselves and with their opponents, and they acknowledged their use of logic. The logical techniques were gradually developed and exported all over Asia when the Sanskrit texts of Buddhism were translated into Chinese, Korean, Japanese, Tibetan, and other languages. The translation of this so-called Buddhist canon, which comprises hundreds of volumes, should not be thought of in terms of “religion” or compared to the translation of the Bible into the languages of Europe. It rather resembles the large-scale translations from the Greek into Latin, Arabic, Syriac, Hebrew, and the modern languages of Europe (cf. Scharfstein et al. 1978, 35–47). The “Buddhist canon” contains Buddhist doctrine but also Indian works on literature, grammar, ritual, astronomy, medicine, and other topics (including logic) that are not confined to Buddhism.

### 3. The Evidence from Indian Linguistics

Logical relationships are often expressed through language. Their origins may be closely related to structures of language, or they may reflect principles of reasoning that are implicit in language and commonly used. But logic can also make use of an artificial language expressly constructed for logical purposes. Such an artificial language may follow features of natural languages or deviate from them. In the Western logical tradition, especially since the nineteenth century, logic has been inspired by mathematics. The situation might have

been different earlier had logic been created by Plato, who was a great mathematician, rather than by Aristotle, who used formalization but not much mathematics. The pioneer of mathematical logic was Leibniz, but the importance of his investigations was discovered two centuries after he made them.

In the Indian tradition, mathematics plays a less crucial role; its place is taken by grammar or linguistics. The Indians were good mathematicians, and Vedic and Greek geometry probably have a common origin (see Seidenberg 1983). Indian contributions in later centuries include important work on algebra. But whatever their eminence in mathematics, Indian scholars were extraordinarily creative as linguists and grammarians. In the West, linguistics developed relatively late. In India, it was the first science, the “science of sciences.” The Indian grammarians discovered universals of language long before any logic was developed. These universals were not shadowy but substantial, for they were based upon the empirical investigations by grammarians first of Sanskrit and later other languages, including non-Indo-European languages such as Tamil and Tibetan.

We have already noted that some of the principles that play an important part in Indian logic derive from the sister sciences of grammar and ritual. That ritual can be a science and a precursor of logic may surprise Western readers; it is a topic to which I shall return later. That Indian grammar was a precursor of logic is less surprising; it is connected with the fact that the first logicians were not the self-styled logicians of the Nyāya, but the grammarians themselves. Patañjali, author of the “Great Commentary” on Pāṇini’s Sanskrit grammar, was not only an earlier but also a better logician than most of these later thinkers. This also explains the early emergence of logico-linguistic principles. The distinction between *use* and *mention*, for example, was known to Indian grammarians such as Pāṇini and Patañjali more than a millenium before it was discovered in the West (Brough 1951). The same holds for the concept of metalanguage and other notions (see Staal 1975b). Even later philosophers, such as Nāgārjuna, were in their logical work influenced by grammar (see Bhattacharya 1980; 1981). In India, the search for universals should therefore begin not with logic but with the Sanskrit grammarians and scientists of ritual. I shall pursue these topics through a summary discussion of chapters 8 through 12.

## EUCLID AND PĀṆINI

Chapter 8 begins with a comparison between the mathematical method, which is characteristic of Western philosophy, and the linguistic or grammatical method, which is characteristic of Indian philosophy. This chapter may also

serve as a first introduction to the study of the Sanskrit grammarians. The 1965 version published here was excerpted from an Inaugural Lecture delivered in Dutch on the occasion of my appointment to the Chair of General and Comparative Philosophy at the University of Amsterdam in 1962. (The original has been reprinted in a collection of general essays entitled *Over zin en onzin in filosofie, religie en wetenschap*, “On Sense and Nonsense in Philosophy, Religion, and Science,” Staal 1986a, 77–115.)

The central place occupied by mathematics and linguistics in Western and Indian culture, respectively, can be explained to some extent by the fact that both disciplines exhibit marked similarities in logical and methodological structure. Chapter 8 illustrates this by comparing a mathematical proof in Euclid with a grammatical derivation in Pāṇini. Though geometry may seem to be universal, and in that sense different from the grammar of a particular language, in the case of Sanskrit that difference is only apparent: for the methods of grammatical derivation used by Pāṇini can be used with respect to a variety of languages and linguistic systems, something which indicates that we are dealing not merely with a particular grammatical system but with linguistics and with universals of language.

Mathematics and linguistics share certain features that also occur in other sciences but which are especially well-developed in these two: the expressions are fully explicit, and the system as a whole is as simple as possible. This results in the postulation of a small number of elements which are combined in accordance with rules of various types. In India, these requirements led to the development of the *sūtra* style, which emphasized *rules*, and which, having originated in the ritual manuals, culminated in Pāṇini’s grammar. The importance of overall simplicity in science has been recognized in the West at least since Ockham. Its importance to linguistics was emphasized by Morris Halle (1961), and its Indian counterpart is the insistence on *lāghava* on the part of the Sanskrit grammarians. But this “brevity” is more than mere *mātrālāghava*, or “brevity of syllables;” it is functional and not merely a matter of elegance.

The importance of linguistics as a characteristic feature of Indian civilization has been stressed by several scholars (e.g., Ingalls 1954, Ruegg 1978). I would now formulate it differently, as follows. Indian science is characterized by the fact that it is concerned with rule-governed activities, a concern that is predominant both in linguistics and in the science of ritual. However, similar approaches can also be fruitfully used in the scientific analysis of other domains of activity in humans and other animals. This has become apparent, for example, in the development of ethology. It is likely that this is the area in which the Western humanities and social sciences, and perhaps ethology it-

self, can learn most from India. The reason is twofold: the Indian approach is rational, and the science of ritual can be developed into a general science of activity that is not confined to “behavior” and is applicable to other rule-governed activities. The Indian approach to the scientific analysis of ritual has been more productive than any contemporary work on ritual because it was not handicapped by the irrational belief in “two cultures”—the natural sciences and the humanities—which are nowadays often believed to be incompatible because one is regarded as rational and the other as irrational. A different approach to the study of rule-governed activity may therefore assist in transforming the Western humanities into human sciences (as happened in the transformation of philology into modern linguistics) and in bridging the gap between the biological sciences and these human sciences, including a new science of anthropology. The entire topic is speculative, but I have touched upon it before and shall return to it later in this Introduction. It is also the topic of a separate publication (Staal 1988).

#### A METHOD OF LINGUISTIC DESCRIPTION: THE ORDER OF CONSONANTS ACCORDING TO PĀṆINI

Chapter 9 deals with an apparently elementary topic and shows how it raises numerous questions of formulation involving simplicity and functionality. The grammatical problems that are treated here can only be solved if a large part of Pāṇini’s grammar is taken into account. This is partly due to functional abbreviations, partly to the order of the rules: for the metarule that a later rule prevails over an earlier rule is effective only because the rules have been arranged carefully and in accordance with a specific methodology. An important feature of the Indian grammarians’ method is rule order, a feature of linguistics that was unknown in the West prior to the development of contemporary generative grammar. Its independent origination in both manifestations of the science shows again that we are dealing with features of universal grammar.

#### CONTEXT-SENSITIVE RULES IN PĀṆINI

The grammatical rules examined in chapters 8 and 9 exhibit a specific type of rule which Chomsky referred to as “context-sensitive” (e.g., Chomsky 1965, 112, 120–22). Our interest in these rules lies in the fact that they are stronger in explanatory power than the “context-free” type of rule that is common in linguistics as well as in logic, but they are less powerful than the transformational rules introduced by Chomsky. The context-sensitive rules themselves



What does all of this have to do with “universals of language”? The answer is: everything. It is obvious from even a brief glance at some of the languages on the planet that they are extremely different. They sound different, they look different when written, and it is difficult to learn more than a few of them. Such facts impressed Whorf and others and led them to the idea of linguistic relativity. But all such impressions are superficial in the literal sense of that term: they take into account no more than the surface phenomena of linguistic forms. If one studies the deeper structures that underlie phonological as well as syntactical relationships in a variety of languages, one finds that the same kinds of rules, often the same rules, have to be postulated in order to account for these relationships. Paul Postal, a student of Mohawk, referred to this universality in the following terms: “I know no one who has attempted to investigate the deeper grammatical structures of languages who has not been impressed with the underlying similarities revealed in even the most diverse surface sentence organizations” (Postal 1969, 425, n. 20). Postal refers here to the same features that we encountered in the analysis of Dorothy Lee’s example from Wintu (above, page 15): the forms of “and” differ greatly in different languages, but the logic of “and” and the deep structure of concatenation and coordination are always the same.

The study of the Sanskrit grammarians contributes to the discovery of universals of language because it demonstrates that grammarians in a different tradition had to postulate the same kind of rules as were being postulated independently by Western linguists. One type of rule, the context-sensitive rule, was used, consciously and consistently, by Pāṇini in India about 2,500 years before it was rediscovered by Chomsky. Such linguistic universals display universals of language that are on a par with universals of logic, such as the principle of noncontradiction, for that principle expresses a feature of negation that is also a necessary feature of negation in the semantics of natural languages.

Although chapter 10 can be read by Sanskritists and linguists as a technical exposition of a feature of Pāṇini’s grammar, it has wider, philosophic implications. It also throws light on the structure of scientific development and on the history of science, topics to which we shall return.

#### PĀṆINI TESTED BY FOWLER’S AUTOMATION

Chapter 11 is a brief response to the suggestion made by Murray Fowler to test the consistency of Pāṇini’s grammar by means of an automaton. The issue is fundamental: a fully explicit and formalized system of rules, whether in logic or in linguistics, should under the right circumstances be amenable to

being programmed and computerized. Fowler's article, however, did not distinguish the various types of rules that have to be distinguished in order for such a program to be effectively written. My note attempts to introduce the required distinctions, and it briefly touches upon the extremely complicated problems connected with the order of Pāṇini's rules—problems discussed by Kielhorn in the nineteenth century and reexamined recently by S. D. Joshi and Paul Kiparsky (1979).

In sixteenth-century Europe, Petrus Ramus wrote a textbook of geometry in which he changed the order of Euclid's theorems. In seventeenth-century India, Bhaṭṭojī Dikṣita wrote a grammar of Sanskrit in which he changed the order of Pāṇini's rules. Both tried to simplify and improve the structure—and in so doing both destroyed the system.

#### SYNTACTIC AND SEMANTIC RELATIONS IN PĀṆINI

Jointly written with Paul Kiparsky, chapter 12 studies Pāṇini's solutions to the kind of problems that led Chomsky to postulate transformations. Pāṇini did not use transformational rules like he used context-sensitive rules. But he clearly distinguished between "surface" properties of linguistic forms and "underlying" or "deep" syntactic and semantic relations. These distinctions are similar to the traditional distinction between "grammatical subject" and "logical subject," but they are more sophisticated and range over all of syntax. That most of Pāṇini's syntax is contained in his *kāraka* theory and presupposes the distinction between surface and deep structures had been shown in my book *Word Order in Sanskrit and Universal Grammar* (1967a). Kiparsky helped me discover a large domain of linguistic universals and universals of language that is only beginning to be explored. Why have these universals been hidden for so long? Because they are much more abstract than anyone had imagined, and they can only be found by digging deep, that is, by analyzing data that become accessible only when, in terms of Plato's metaphor, we leave the cave and come out into the open.

Kiparsky has continued and extended this discussion in the first lecture of his book *Some Theoretical Problems in Pāṇini's Grammar* (1982).

#### 4. Seven Reviews

The reviews that appear in part THREE of this volume speak for themselves, but a few additional observations may be helpful. Most—especially the reviews of Ingalls, Sreekrishna Sarma and Matilal—try to show how formal methods can be used to an even greater extent than the authors under review

had done themselves. The review of Scharfe's book on the *Mahābhāṣya* demonstrates that Patañjali was not a clumsy logician, as the author had claimed, but occupies a pivotal place in the history of Indian logic, comparable to that of Aristotle in the West (a statement endorsed in Renou 1969, 498). The review of Cardona's monograph on the *Śivasūtras* is of methodological interest because it establishes that philology and linguistics presuppose each other and that no early scientific investigations can be adequately understood unless we address the problems they addressed, that is, unless they are evaluated within a contemporary perspective.

## 5. Conclusions

What conclusions can be drawn from our Indian journey which started with Quine's unassailable but rather barren skepticism and arrived at the notion of abstract but substantial universals of logic and language that underlie a variety of surface forms?

We have already noted that Quine's universals are shadowy because the scope of his observation is undetermined. Quine's observation is consistent, for example, with Whorf's theories: it could imply that we are unable to go beyond the conceptual scheme of "Standard Average European." This would throw fresh doubt on Whorf's claims regarding Hopi, but these were implausible in any case. (I once paid a visit to the Hopi village of Oraibi and asked to see a Hopi villager to whom I had an introduction; as it happened, he was busy and an interpreter conveyed the message: "Could I return to the same place in the afternoon?" It would be interesting to know how the distinction was expressed in the original Hopi which according to Whorf does not distinguish between place and time.)

In light of the evidence from linguistics provided by studies not only of English, German, Sanskrit, Chinese, etc., but also of American Indian and Australian languages, such as Mohawk (studied by Paul Postal), Walbiri (studied by Kenneth Hale), and numerous others, the Whorfian hypothesis loses all plausibility: There are in fact substantial universals of language when we come out of Plato's cave. Accordingly, Quine's observation does not mean that we are confined to the conceptual scheme of a particular language or language family, but of all human languages; in brief, of "Language." This implies in turn that our "charity" is not wanted: Whatever philosophers or anthropologists may tell us, we never come across natives who themselves seriously claim that "*p* and not *p*" is true in their language and culture.

These conclusions could be made plausible without visiting India. But the evidence from Indian logic and linguistics is relevant to the specific nature of

many of these universals, for they include not only familiar logical principles, such as noncontradiction and contraposition, but they also include the notion of *rule* (which corresponds to the Indian concept of *sūtra*), the concept of rule order, metarules, and other metalinguistic notions. Abstract conditions that rule structures must meet occur in both India and the West. In India, most of these rules derive from the conceptual schemes of the sister sciences of ritual and grammar. This is partly explained by the fact that Vedic ritual was a fully explicit formal system from which contradictions and ambiguities had been excluded. This system became the object of scientific analysis, which in turn paved the way for linguistics. The historical development of these Indian sciences, however, has been strikingly different from the development of their Western counterparts. The Western use of rules in logic goes back to Aristotle, but these rules have become important topics of research only in contemporary logic and linguistics. In India, on the other hand, rules were studied systematically by the earliest ritualists. Baudhāyana, author of a ritual manual that is generally assigned to approximately 700 B.C., invented the notions both of “rule” (*sūtra*) and “metarule” (*paribhāṣā*) (Renou 1963, 175–79, referring to Willem Caland). These notions were perfected by Pāṇini (fifth century B.C.?) and other early grammarians and were retained, developed, or otherwise modified by the later logicians.

It is difficult to understand these differences adequately without evaluation and some kind of quantification. Though doubtlessly premature, I shall make such an attempt. To pave the way, let us briefly review the histories of the sciences of logic, linguistics, and ritual in the West and in India by means of thumbnail sketches.

*Western Logic* Created by Aristotle (384–322 B.C.) almost out of nothing (as he himself asserted), Greek logic flourished among his own followers in the school of Megara and among the Stoics (especially Chrysippus, 279–206 B.C.). The last creative logician was Galen (A.D. 129–199). Medieval logic, both in Arabic and in Latin, was cultivated between the ninth and fourteenth centuries. This was followed by a large gap with a few bright exceptions (e.g., Leibniz, 1646–1716) until modern “mathematical” logic began to develop in the nineteenth century. It expanded rapidly in the twentieth century along with other mathematical sciences.

*Indian Logic* The beginnings with Kaṇāda and Gautama are difficult to date (first century A.D.?) and also meager. With Vātsyāyana (sixth century) logic begins to flourish: this continues through the eleventh century with Praśastapāda, Uddyotakara, Vācaspatimiśra, Udayana, and others. (Udayana was

probably the first to argue that universals with the same members are the same; see Chakrabarti 1975, 375). In Buddhist logic we have Nāgārjuna (second century) and the “three D’s” of the Yogācāra: Dignāga, Dharmakīrti, and Dharmottara (sixth–ninth centuries). The Navya-nyāya begins with Gaṅgeśo-pādhyāya (thirteenth century) and continues uninterruptedly to the seventeenth century with a few more recent offshoots.

*Western Linguistics* In antiquity, the Alexandrian school of Dionysius Thrax (around 100 B.C.), extended by Apollonius Dyscolus, “marked a conception of grammar that endured with little real change for more than ten centuries” (Robins 1976, 18). Thrax was influenced not so much by Aristotelian logic as by Aristotelian principles of classification. His own influence was channeled through the Latin grammar of Priscian (around A.D. 500). There is scattered information on a school of Byzantine grammarians. During the Western Middle Ages, linguistics prospered between the twelfth and fifteenth centuries and inspired theories and philosophies of grammar during a few more centuries. Important innovations were due to the Modistae, who taught in the later thirteenth and early fourteenth centuries and constructed their grammars in line with a theory concerning the mode of signifying (*modus significandi*), that is, in line with semantics (see, e.g., Bursill-Hall 1971). Though phonology began to flourish in the nineteenth century in close collaboration with historical grammar, scientific synchronistic linguistics emerged only in the twentieth century with de Saussure and reached its first peak with Chomsky.

*Indian Linguistics* The construction of the Padapāṭha of the Ṛgveda (1000–800 B.C.?) evinces keen linguistic activity (cf. Jha 1973, 1975a, 1975b, 1976), and so does the construction of the Sāmaveda (cf. Staal 1986b); but the most impressive work was done by the three founding fathers—Pāṇini, Kātyāyana, and Patañjali—between the fifth and second centuries B.C. The Pāṇinian tradition continued until the eighteenth century (Nāgojībhaṭṭa), with modern offshoots, like the Navya-nyāya with which it later became connected. Non-Pāṇinian schools proliferated from the first century A.D. onward, beginning with Kātantra, Candra (sixth century), Śākaṭāyana (ninth), Hemacandra (twelfth), Vopadeva (thirteenth), and Saupadma (fourteenth).

*Western Science of Ritual* There is little to report: nothing from antiquity (cf. Hillebrandt 1897, 1: “Die Sanskritphilologie hat gegenüber der klassischen Altertumskunde den Vorzug, dass sie ihre Kenntnis aller sakraler Gebräuche nicht aus einzelnen, zufälligen Angaben zusammenstellen muss, sondern die alten Lehrbücher besitzt, welche die Grundlage für die Opferkenntnis der

Priester bildeten.”), scarce medieval rumblings touching upon liturgy from the Carolingian Renaissance, and contemporary investigations by anthropologists and scholars of religion on a woefully inadequate level in comparison either with the Indian science of ritual or with the Western sciences of logic and linguistics (cf. Staal 1984, 1986b, 1988).

*Indian Science of Ritual* The Vedic Śrautasūtras of Baudhāyana, Vādhūla, Śāṅkhāyana, and Jaiminīya seem to belong to the pre-Pāṇinian period, whereas Āśvalāyana, Bhāradvāja, Āpastamba, Hiranyakeśin, Kāṭhaka, Mānava, Vārāha, Lātyāyana, and Drāhyāyana belong to the period of Pāṇini, Kātyāyana, and Patañjali. Vaikhānasa is much later, as are the commentaries, compendia and handbooks, and philosophico-ritual developments, such as the Mimāṃsā (from the third through the seventeenth centuries). (On the scientific nature of the work of the Śrautasūtras, see Staal 1982, especially pp. 2–3, 19–38; 1983.

History is not an exact science and the Indian dates are notoriously volatile. Moreover, for a comparative evaluation we need qualitative as well as quantitative evaluations. Though I am well aware of these and other restrictions, risks, and dangers, I shall summarize our findings by means of two graphs (figures 1 and 2) that picture the development of the three sciences we have considered in the West and in India by plotting time against relative excellence.

Even allowing for a goodly percentage of error, no one who gazes upon these graphs can fail to be impressed by their whimsical appearance and the extraordinary differences they seem to portray. We shall return to their interpretation, but the first conclusion that is relevant in the present context must be that the apparently inexplicable contrast between: (a) the historical asymmetry between the Western and Indian developments, and (b) the conceptual similarity between both, can only be explained if we assume that (b) is due to the fact that the universals of logic and language that appear in both are substantial and not merely shadowy universals. They are not vacuously universal on account of a principle of charity; they are really universal because they arose and exist independently. The situation is similar to that in mathematics: whatever the differences of emphasis and development, different cultures have discovered identical truths in arithmetic, geometry, and algebra. This conclusion is also valid for logic, linguistics, and the science of ritual, although the latter poses problems because it has been studied so little.

Logic and linguistics are closely connected. Logical principles must be assumed to underlie some of the semantic universals of language. For example, no language with an expression for negation can fail to incorporate the principle of noncontradiction in some form or other. This may be effected by a

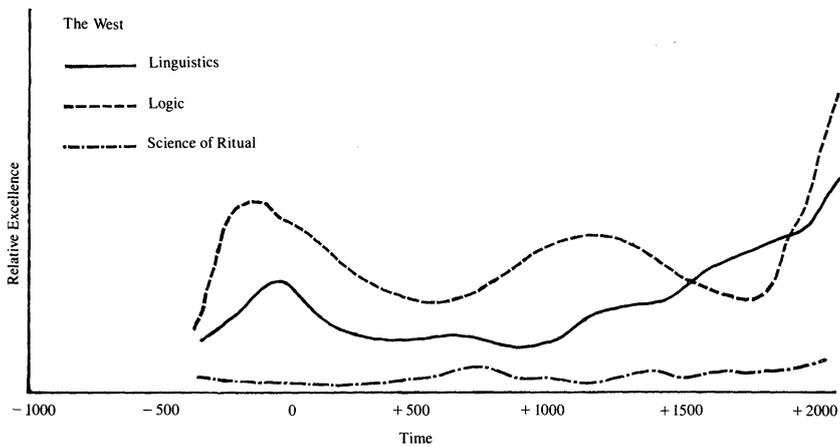


FIGURE 1

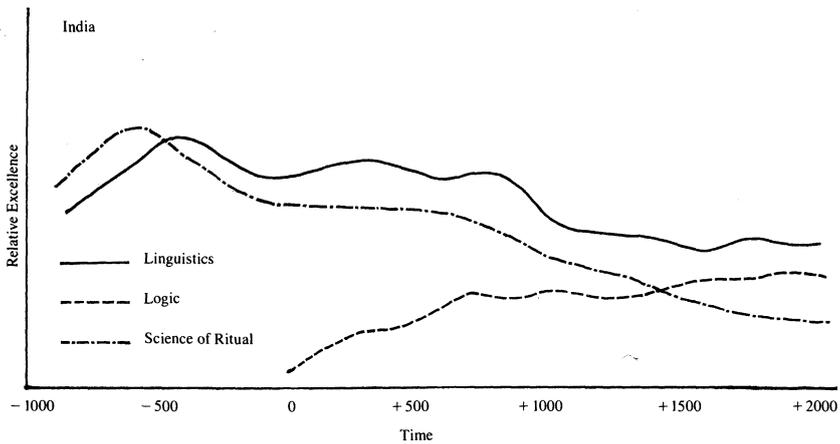


FIGURE 2

“meaning rule” (Carnap’s concept) or other mechanisms that linguists have to produce. Although the links between logic and linguistics remain close, there are also differences, and the differences become more pronounced when we add ritual to logic and linguistics as a third rational system. We have already noted that universals of language are universal in the sense that they seem to appear among all people, whereas universals of logic become explicit only where logic appears, namely, in India, China, and the West. Ritual seems to be universal, but a science of ritual is almost exclusively Indian. The question arises: To what extent are we entitled to speak of universals at all?

The only concept of universality that makes sense in these contexts is the concept of *conditional universality*. This expresses the idea that logical universals, for example, are conditioned by other universals already contained in a language. I am not referring to the fact that many universals of logic are themselves “conditionals” or are true “on condition” that others are. I am referring to the fact that logical principles such as noncontradiction and contraposition are valid only if the language in which they are expressed contains notions such as negation. These principles are universal, therefore, in spite of the fact that they do not appear in all languages: they could not appear, for example, in a language without negation. Since all natural languages contain negation, they are universal even if they remain implicit. Other properties of language, however, appear only in some languages. The situation in linguistics is therefore not different from that in logic and the science of ritual. In all three cases the same problems face researchers: when universals are hidden and not explicit, it takes work and imagination to bring them to the surface.

The most extensive, detailed, and reliable study of universals have been undertaken in contemporary linguistics. Linguists recognize universals that are not manifest in all languages of the world; they are found in some, hidden in others, and there are gaps, systematic as well as unsystematic. There are general studies on “gaps” in grammar and culture and on apparently “sporadic” universals that turn out to be real and substantial (see, e.g., Hale 1975). Not all languages, for example, have retroflexes, but that does not imply that there are no universal principles governing their occurrence and use. There are hundreds of examples of such conditional universals in Chomsky and Halle’s *Sound Pattern of English*. For example, in addition to the two kinds of boundaries between linguistic segments that all languages possess, there is a third that presupposes the others and is found in some. In English it exists between certain prefixes and stems, for example, the prefixes *per-*, *con-*, *com-*, . . . and the stems *-mit*, *-cur*, *-pel*, . . . The rules that are postulated to account for the accentuation of *permít*, *concúr*, *compél*, *detér*, are universal even though they come to the surface only in some languages (Chomsky

and Halle 1968, 94–96, 371). A similar condition applies to the color universals discovered by Berlin and Kay (1969): if a language has two color words, they are black and white; if three, black, white, and red; if four, either the same three and green, or the same three and yellow; etc.

Although universals have been studied more thoroughly in linguistics than in logic, the history of logic is better known than the history of linguistics. While there exist only three highly developed logics—the Western, the Indian, and the Chinese—the situation in linguistics is less clear. It seems probable that after early beginnings (1600 B.C. in Mesopotamia), the only two highly developed forms are the ancient Indian and the contemporary Western. With regard to the science of ritual, the situation is more puzzling. There is hardly anything in the West, a great deal in India, and striking parallels between India and China with regard to ritual structure that suggest that the Chinese were also conversant with a tradition of systematic and rational ritual analysis. If there are universals of the science of ritual, they are doubtless conditional; and, for all we know, confined to India and China (see Schipper and Staal, forthcoming).

The answer to the question that I raised earlier—“Why did I not continue these studies?”—has already been indicated. I continued the search for universals, but concentrated on ritual at the expense of logic and linguistics. The rest of the answer is to some extent autobiographical. I was no longer fascinated by Indian logic. Perhaps I did not go far enough into it; but the obstacles that remained were in the domain of *navya-nyāya* and seemed to be largely technical. I am not afraid of technicalities; in fact, I have a strange liking for such things. (They seem to weed out the amateurs.) But it did not seem likely that new or interesting logical truths would be found in the formidable bastions of technicality of these later works. I had come upon what I regarded as an important result: that Indian logic, whatever the differences of emphasis and development, was not basically different from Western logic. I have always regarded this conclusion as solid. It continued to form a foundation for some of my later work because it provided a method for the study of Asian civilization. At the same time, it separated me from Western colleagues and contemporaries who continued to maintain divergent views without having looked into the matter.

The dossier is not closed, for no scientific result is final. Not that rationality will in the last resort be found to be only Western: rationality is universal. If it does not always seem so, this is surely because reason, at all times, has been a rare commodity, not least in the West (see Staal 1975a, part 1). But I may be wrong in my comparatively low evaluation of Indian logic. Bimal Krishna Matilal or Kisor Kumar Chakrabarti, for example, will disagree with me, and

so may others. Some issues of the *Journal of Indian Philosophy* challenge me to reconsider the matter. It is furthermore clear that the application of *navya-nyāya* to philosophy is not only of interest but will yield new insights, as is shown, for example, by Phyllis Granoff (1978) and Matilal (1986). It is also likely that logico-grammatical works, such as Jagadīśa's *Śabdaśaktiprakāśikā*, contain important new ideas that no one has yet evaluated within a broader perspective.

Whatever the promise of these speculations, another glance at figures 1 and 2 will explain why I did not pursue my researches in the direction of these late scholastic niceties but concentrated instead on their origins in the science of ritual, which seemed to stand at the beginning of Indian science. Here was a science not merely neglected in the West; it did not even exist there. Here was a domain that had been made accessible and had been intensively studied for at least a century by some of the greatest Western Sanskritists—scholars such as Albrecht Weber, Willem Caland, and Louis Renou. I could reach the sky by simply standing on the shoulders of giants. Renou had gone furthest in detecting the close relationships between the Indian sciences of ritual and grammar in an important article that appeared in the *Journal asiatique* of 1941–1942 (reprinted in Staal 1972). He had already concluded that the Indian science of ritual probably predated the study of grammar because of the undeniable priority of the Vedas themselves. But none of these savants had grasped the important theoretical, comparative, cross-cultural and, indeed, universal significance of this science, which was not recognized for what it was merely because such a science did not exist in the West. No one had realized that what we were witnessing in ancient India was not merely an early form of independent rationality, but one of the birthplaces of science—a science not constructed in conformity with Western expectations and therefore all the more interesting.

At this point I made a discovery that any anthropologist or scholar of religion could have made if only they had been less attached to their preconceptions. Ritual is not only human; it occurs among animals. Biologists have long been familiar with features of animal behavior that ethologists now finally refer to by their proper name: ritualization. Ethologists have been cautious; they have not claimed that this ritualization is the same as the formation of ritual among humans. But caution is nothing compared to the obstacles created by anthropologists and scholars of religion; they, who should have started a new science, played blind, deaf, and dumb. I was saved from this debacle. My eyes had been opened by Vedic ritual, which led to two discoveries that already had shattered my Western prejudice: Vedic ritual is a ritual without religion and was the object of a separate science.

These discoveries combined lead to unexpected conclusions in the context of our present undertaking. If rituals occur among animals, there are no ritual universals in the sense of “universal” I have adopted. This makes the science of ritual all the more intriguing, for it is not only immune to the kind of scientific analysis that deals with hearts, lungs, and other physical organs; it also refuses to resort under the humanities or social sciences. The comparative picture had already been striking: three Indian sciences—one (logic) generally lagging some distance behind its Western counterpart; a second (linguistics) generally ahead (in the past) and sometimes behind (at present), but on the whole occupying a comparable level; and a third (the science of ritual) continuously soaring above comparable developments elsewhere. From this comparison it was obvious that the third of these sciences was the most interesting and promising. Its rational analysis elucidated an area that had always defied analysis in the West. In so doing, it refuted age-old preconceptions, for progress had been hampered not by lack of data, but by lack of understanding. The view that ritual expressed religious values turned out to be a Western bias.

At the same time it also became clear that the science of ritual should take its position in the space between the Western sciences and humanities. That space has been empty and well on its way to developing into a bottomless chasm. Although I had been studying Indian logic and grammar partly because I was interested in universals, an inkling of what seemed to be ritual universals began to dawn on me only gradually and in inverse proportion to the dwindling of my Western preconceptions. I had always assumed that there was nothing in that empty space, last of all a science of ritual. As for the circumstances surrounding its birth in India, I began to detect them only much later (see, e.g., Staal 1987). As for universality, it turned out that rituals were, in a manner of speaking, more than universal. Being so unabashedly overweight, they burst the seams of the system.

Let me return once more from the vista of history to the idiosyncrasies of autobiography. Contacts in Kerala going back to the middle fifties, along with chance and good fortune, combined to give me direct access to the most flourishing, authentic, and long-lived tradition of ritual specialists that has survived in India, or anywhere in the world for that matter. The Nambudiri brahmins continued, in strict seclusion, to preserve and practice the ancient science of ritual, just like the nineteenth-century Sanskrit paṇḍits who embodied Indian linguistics and without whom Pāṇini would not have been understood by Kielhorn or anyone else outside the tradition. I became involved in the 1975 Nambudiri performance of the Agnicayana, a famous Yajurveda ritual in which an altar is built in the shape of a bird—the same bird to whose wings the logical terms *pūrva-pakṣa* and *uttara-pakṣa* happen to owe their

origin (see above, page 23). As for me, I was through with the *prima facie* view and its refutation and was heading for a *siddhānta*. The account of this ritual performance together with studies and discussions of its background, structure, and significance was published in two volumes (Staal 1983). Theoretical implications are the subject of a forthcoming book (Staal 1988). Empirical support from different parts of Asia is set forth and discussed in Schipper and Staal (1987, forthcoming).

\* \* \*

That Plato's heaven continues to appeal to even the greatest logicians leaps to the eye from the following passage in Bertrand Russell's *Autobiography*:

The last part of our time was spent at Princeton, where we had a little house on the shores of the lake. While in Princeton, I came to know Einstein fairly well. I used to go to his house once a week to discuss with him and Gödel and Pauli. These discussions were in some ways disappointing, for, although all three of them were Jews and exiles and, in intention, cosmopolitans, I found that they all had a German bias towards metaphysics, and in spite of our utmost endeavours we never arrived at common premises from which to argue. Gödel turned out to be an unadulterated Platonist, and apparently believed that an eternal "not" was laid up in heaven, where virtuous logicians might hope to meet it hereafter. (1951, 1:341)

The investigations of the present volume support the alternative explanation, that universals are attributes not of heaven but of the human mind. It is tempting to claim that they are among the defining characteristics of the human animal. This would confirm the Aristotelian definition of man to which I referred at the beginning of this introduction and would fit in with other recent findings. That logic constitutes a system has always been obvious. That language constitutes a system was discovered by Pāṇini (who believed Sanskrit to be eternal) and rediscovered by de Saussure (who introduced a synchronistic approach in reaction to the earlier investigations of linguists that had been almost exclusively historical). That ritual was a system was discovered by the ancient Indian ritualists. Ritual is found not only in humans but also in animals. If language and logic are among the characteristics that distinguish the human animal (and not merely its Western sub-species) from other animals, ritual fits in a wider biological perspective—the most natural context in which humans must be studied—and demonstrates in passing how misguided is the contention of existentialists and religious thinkers that man is an undefinable creature. A preliminary definition of the human animal along these lines supports and is supported by the present investigations. It supports and

is supported by contemporary linguistics. In anthropology, it is consistent with Lévi-Strauss's general outlook (though not with his more specific speculations), and it throws doubt on the views of anthropologists who believe that "there may be no interesting general laws of culture, or of anthropology, to discover" (above, page 4).

It is obvious that nothing that is confined to the West, and is not also found in other civilizations such as those of Asia, can be a universal characteristic of humanity. While most Western philosophers continued in their parochial ways, Bertrand Russell had already spoken out clearly in his *History of Western Philosophy*:

I think that if we are to feel at home in the world . . . we shall have to admit Asia to equality in our thoughts, not only politically but culturally. What changes this will bring about I do not know, but I am convinced that they will be profound and of the greatest importance. (Russell 1946, 420)

Russell's openmindedness has two sides: he expected Asia to make important contributions; but he was undecided as to whether those contributions would turn out to be similar or dissimilar to the familiar contributions made by the West. What can we say with regard to this second question after almost half a century?

The chapters of the present volume clearly establish the conclusion that, in the general area of thought, Asian contributions are similar and not dissimilar to Western contributions. So it becomes desirable to have a final look at the relativism that is often attributed to Thomas Kuhn's *Structure of Scientific Revolutions* (1970), although he himself has always repudiated it. The only scholar who seriously defended the view that "each culture has its own science" on the grounds of at least a semblance of empirical evidence was Oswald Spengler in his *Decline of the West* of 1918. Spengler regarded even mathematics as dependent on culture since, according to him, "number" means entirely different things to "Apollinians" and to "Faustians." His conclusion was what we would logically expect on these premises, namely, that we can never understand any alien culture. Although Kuhn had defended no such view, protagonists of some of the human and social sciences assumed that he did, and others extended his presumed conclusions to areas that he himself had emphatically excluded from the domain of their applicability. An example is the German theologian Hans Küng who tried to apply "paradigm shifts" to the development of religion—an example that would merely be colorful and entertaining had it not been professed with such zealous intolerance (cf. Staal 1985b).

What Kuhn did show is that the sciences rarely progress gradually or in linear fashion and that their development is not simply due to the gradually increasing accumulation of data and information. Sometimes the entire field is looked at from a fresh and different perspective; this is, in simplified form, an instance of “paradigm shift.” We have seen that it is not easy to account for such events in philosophic terms, even with the help of clear concepts (above, page 7); even so, that shifts of this kind occur is an indisputable fact of which Kuhn has discussed many instances. If we turn to figures 1 and 2, we find that the line of development of some sciences is even more more haphazard than the notion of “paradigm shift” suggests. The “measure of excellence” of linguistics in India at around 500 B.C., for example, is greater than that of the nineteenth-century West. That measure is not based upon subjective impressions: the notion of “context-sensitive rule,” for example, though sometimes used, was not recognized as such in Western linguistics until the twentieth century, whereas it had been discovered in India before 500 B.C. This explains why Franz Kielhorn, the leading nineteenth-century Western expert on the Sanskrit grammarians, did not see that Pāṇini used context-sensitive rules: he did not know what they were. We can now assert, with the power of hindsight, that Indian linguists in the fifth century B.C. knew and understood more than Western linguists in the nineteenth century A.D. Can one not extend this conclusion and claim that it is probable that Indian linguists are still ahead of their Western colleagues and may continue to be so in the next century? Quite possibly; all we can say is that it is difficult to detect something that we have not already discovered ourselves.

Kuhn’s ideas have been widely discussed and always with reference to Western science. But once we acknowledge the importance of non-Western science, it becomes crucial to test Kuhn’s ideas with recourse to non-Western data. For such an undertaking we now have available a repository of facts that can serve as a point of departure: Joseph Needham’s monumental *Science and Civilization in China*, of which so far five volumes, the latter in three parts, have appeared (1954–1976). It would be no exaggeration to say that these volumes contain some of the most important contributions to scholarship of the present century. They provide, in addition, massive and devastating refutations of the idea of a “relativism of the sciences.” The evidence collected by Needham and his associates establishes that the sciences of the Chinese are logical, rational, and empirical and in every respect comparable to those of the West. The earlier quotation from Russell’s *History of Western Philosophy* appears, not surprisingly, as an epigraph in Needham’s second volume.

My own investigations have been much more modest and limited. But in one sense they claim to lead to a discovery of comparable significance: the

ancient Indians also developed sciences that are logical, rational, and empirical like Western sciences, but their strength did lie in the human sciences, an area that has always lagged behind in the West. For we have seen that Western civilization, with its growing emphasis on *nonhuman* science, never produced a science of ritual and produced a science of language only belatedly, after being influenced by the Sanskrit grammar of Pāṇini. As for anthropology, one cannot help but feel that such basic disagreements about its task, scope, and principles as are found among its most respected practitioners (e.g., Lévi-Strauss and Geertz with regard to “universals”) suggest that this discipline has not yet gotten off the ground. It seems safe to conclude from these observations that the West, as far as the human and social sciences are concerned, has much to learn from India and from Asia generally.

The likelihood of such future influences need not surprise us because they have never been rare in the past. We have seen that Greek and Indian mathematics probably rest on a common foundation. As for astronomy, the Indians were influenced by the West. The mathematical zero, on the other hand, comes from India, where it originated in the context of grammar. The relations between India and China in some areas (e.g., medicine—connected with Taoism on the one hand and Yoga on the other) remain unresolved, but they are well established in the extended domain of Buddhism. I shall conclude this Introduction with a brief sketch of some of the influences that provide a concrete background to the similarities between India and China and that add some flesh to the dry bones of logic and universals. Such a sketch could be extended in many directions and include references to other Asian civilizations. More importantly, it illustrates the only manner in which the West could effectively learn from Asia.

We have seen that the Chinese were mainly interested in two schools of Buddhism: the logical school of Yogācāra-Vijñānavāda and the “middle way” of the Mādhyamika. The greatest translator of Mādhyamika texts and the founder of the Chinese Mādhyamika was Kumārajīva (see, e.g., Robinson 1967, 71–95). He was born in 344 A.D. at Kucha in Central Asia on the northern branch of the silk route in what is now called Sinkiang. His father was an emigrant Indian who had been a monk, and his mother was what the texts call a Kūchean princess—let’s say, the daughter of a country baronet—who became a nun when Kumārajīva was seven years old. When he was nine, she took him to India where he studied the Vedas, Sanskrit grammar, astronomy, and a variety of Buddhist works. Eventually, his predilection went to the Mādhyamika. He was ordained, instructed many others, and vanquished famous teachers in debate by showing their position to be self-contradictory.

In 383, when Kumārajīva was back in Kucha, a Chinese expedition seized the town and carried him away as a captive. During the next decade and a half he learned Chinese. He became a favorite of the king of Ch'in and spent the rest of his life (from 401 to 413) at Ch'ang-an in Shensi where the Yellow River meets the silk route. There he had many disciples and set up a translation bureau to render a large number of works from Sanskrit into Chinese. He paid much attention to correct methods of translation: to what extent the substance of the originals should be adapted to the style and rhetoric of the Chinese, whether repetitions in the original should be translated, whether summaries, abbreviations, and notes should be inserted, and so forth. In later centuries, translation colleges with international faculties consisting of Indians, Central Asians, and Chinese were established in several parts of China. The work was done by a team of reciters, recorders, style correctors, style polishers, and style deleters together with numerous assistants (see van Gulik 1956, 25–31; Staal 1979b, 7).

Several works translated by Kumārajīva have been attributed to Nāgārjuna. There were, however, many Nāgārjunas. The author of the *Mūlamādhyamika-kārikā*, or “Basic Middle Way Stanzas,” a South Indian Brahmin educated in grammar, logic, ritual, and Vedic scholarship, came in due course to be regarded as the founder of the Mādhyamika, which started as a logical and metaphysical system, then absorbed trends of the day including magic and alchemy, and finally developed into the Tantrism of the Vajrayāna or “Diamond Vehicle.” There were accordingly not only several scholars but also Yogins, Siddhas or “Perfected Beings,” alchemists, magicians, a snake (*nāga*) king, a Bodhisattva, and other human and superhuman personages who were known by the name Nāgārjuna. Nāgārjuna was also regarded as the author of an encyclopedic work consisting of more than three million words, which Kumārajīva translated in Ch'ang-an for two years beginning in the summer of 404 A.D., producing a Chinese text comprising a thousand rolls: the celebrated *Mahāprajñāpāramitāśāstra* (in Sanskrit) or *Ta-chih-tu-lun* (in Chinese)—the “Treatise on the Great Perfection of Wisdom.” Its Sanskrit original being lost, the most important portion of the text, more than a third of it, was translated from the Chinese into French by Etienne Lamotte in five volumes comprising more than 2400 pages and published between 1944 and 1980—another prodigious achievement of Oriental scholarship. It is largely through this text that the Chinese began to understand and assimilate the Indian Mādhyamika.

The work as we have it must have been composed by an unknown Indian author, situated “on the middle way” (as Hubert Durt [1985, 22], Lamotte’s *exécuteur testamentaire*, happily put it)—between “Nāgārjuna, father of the

Indian Mādhyamika” and “Kumārajīva, father of the Chinese Mādhyamika,” between the second and the fourth century A.D., between South India and Central Asia; probably, therefore, in Kashmir, a part of northwest India that juts out beyond the Himalayas until it reaches the same latitude as Japan. “The cool air of Kashmir,” writes Sir Aurel Stein, “the northern aspect of its scenery and products, have at all times exercised a powerful charm over those visitors who, themselves born in colder climes, have come to the Valley from the heat and dust of the Indian plains” (Stein, 1900, 2:356). The nature, contents, and pedigree of the monumental “Treatise” that was probably composed in this valley and translated in Ch’ang-an, two thousand miles to the east, destined it to become one of the important links between India and China.

Links between civilizations are not based upon argument or speculation; they require solid information and concrete bridges. Only such bridges will pave the way for a world that is one or at least devoid of small minds and prejudice. Such bridges add substance to Bertrand Russell’s prediction that Asia’s share in the future will be important, and they enable us to demonstrate, beyond that conclusion, that what Asia has to offer is intelligible and rational. They also establish that Asia’s contributions are not only compatible with what has been produced in the West but are a necessary ingredient and essential component of any future science of man.

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*Part One*

# Indian Logic



CORRELATIONS BETWEEN LANGUAGE AND LOGIC IN  
INDIAN THOUGHT

## I

IT may be possible to study special cases of the general philosophical problem, how language and thought are correlated, by considering definite thought structures and definite languages. The difficulty, that thought seems to be accessible only or at least primarily through language, can be partly avoided by concentrating upon formal expressions of thought structures which are considerably different from ordinary language. In the following an attempt will be made to show, with the help of symbolic logic, how certain general structures are expressed in classical Sanskrit and, subsequently, how certain logical structures are expressed in the technical Sanskrit of Indian logic. The results do not prove that some logical principles depend on linguistic structures; for, evidently, the linguistic structures themselves may reflect a deeper-lying structure of thinking or 'being'. On the other hand, if it were possible to show that some expressions could occur only in languages with a special structure—e.g. some Indo-European languages—this kind of research might throw some light on the problem of the universality of logical principles.

It might be objected against this method that symbolic logic may not be a sufficiently universal tool for the investigation of problems of the above kind. It is true that this modern discipline itself may well reflect the structure of some modern European languages. Here symbolic logic will be utilized as a tool, the status of which may be questioned whenever it seems necessary. Thus, it is only through comparison with other systems of logic that its own relation to linguistic expression can be studied.

In the present study, use will be made of variables, such as  $x$ ; predicates (functions), such as  $F(x)$ , '  $x$  is  $F$  '; and relations (binary predicates), such as  $R(x, y)$ , ' between  $x$  and  $y$  the relation  $R$  holds '. The identity relation will be written in the usual way as  $x = y$ , its negation as  $x \neq y$ . Predicates and relations combine, according to well-known rules, to form well-formed formulas by means of the connectives:  $\sim$  (negation),  $\wedge$  (conjunction),  $\vee$  (disjunction),  $\rightarrow$  (implication),  $\leftrightarrow$  (the bi-conditional, where  $a \leftrightarrow b$  is defined by:  $(a \rightarrow b) \wedge (b \rightarrow a)$ ), and by means of  $(x)$  and  $(Ex)$ , expressing universal and existential quantification respectively. Special use is made of the expression  $\alpha xF(x)$  for a *restricted-variable* denoting the idea '  $x$  such that  $F(x)$  ' <sup>1</sup>; e.g. when applied to relations, if  $C(x, y)$  stands for '  $x$  is the capital of  $y$  ',  $\alpha xC(x, \text{France})$  denotes Paris and  $\alpha yC(\text{Paris}, y)$  denotes France. From this definition of restricted-variable we infer:  $(Ey)[y = \alpha xF(x)] \leftrightarrow (Ey)F(y)$ .

<sup>1</sup> See T. Hailperin, ' A theory of restricted quantification ', *Journal of Symbolic Logic*, xxii, 1957, 19–35; 113–29. For another symbolism cf. J. B. Rosser, *Logic for mathematicians*, New York, 1953, 140–50.

The fact that restricted-variables are, as will be shown, particularly useful in the representation of Indian logical structures may need some justification, as up to the present these symbols have been used relatively sparingly in Western logic itself. But here we are faced with one of those circumstances, where the structure of symbolic logic seems to be limited by the relation to its own background. Variables seem to have become an essential part of the vocabulary of modern logic mainly for two reasons: firstly because of the indispensibility of variables in mathematics; and secondly because they can represent nouns preceded by indefinite articles in Greek and in the modern European languages.<sup>1</sup>

It may be expected that wherever such variables occur bound (e.g. in quantification), other means of expression may render the same service; this is actually seen to be the case in Indian logic. Restricted-variables, on the other hand, can be adopted whenever we want to represent a noun of which it cannot be said that it should be preceded by a definite or by an indefinite article—for instance because there is no article at all, as in Latin or in Sanskrit.<sup>2</sup> Restricted-variables can also be used when the number of cases fulfilling a given condition is not known. For example, if  $P(x, y)$  denotes 'x is the father of y',  $\alpha yP(A, y)$  may denote any of the sons of A. If there is only one son—i.e. if the condition:  $(\exists y)P(A, y) \wedge (x)[P(A, x) \rightarrow (x = y)]$  holds—this would be equivalent to  $\iota yP(A, y)$ , 'the y such that  $P(A, y)$ '. This expression, which corresponds in our modern languages to a noun preceded by a definite article, is definable in terms of classes with the help of ordinary variables.

## II

Sanskrit is noted for its long nominal compounds. The possibilities provided by these means of expression were widely utilized (sometimes, it might be said, exploited) by Indian logicians. It seems therefore advisable to see first exactly what is the scope of expression of some of these compounds, as laid down by the grammarians in accordance with general usage.

Of the three kinds of nominal compounds which will occupy us here the *dvandva* is the least interesting. As easily as Pāṇini could define it in terms of *ca* 'and',<sup>3</sup> modern logic can define it in terms of conjunction.

More interesting is the *tatpuruṣa* compound. According to Patañjali it is characterized by the predominance of its last member, *uttarapada*.<sup>4</sup> The first

<sup>1</sup> It is possibly connected herewith that Alexander of Aphrodisias, the Greek commentator on Aristotle, seems to have been the first to state explicitly what a variable is. See I. M. Bocheński, *Formale Logik*, Freiburg/München, 1956, 157.

<sup>2</sup> Less often and in a more complicated way these languages do express what the article expresses. It might be asked whether restricted-variables might be useful in the study of medieval logic too. B. Boehner (*Medieval logic*, Manchester, 1952) uses individual and predicate variables only.

<sup>3</sup> 2.2.29.

<sup>4</sup> *Mahābhāṣya*, 2.1.6. This and some of the following references have been traced back with the help of L. Renou, *Terminologie grammaticale du Sanskrit*, Paris, 1957.

member is called *upasarjana* 'subordinate', 'le mot visé par une teneur au nominatif' (Renou).<sup>1</sup> We could describe the *tatpuruṣa* as a compound denoting the second member as qualified by the first member. The later logicians said that implied sense or *lakṣanā* lies in the first member,<sup>2</sup> as the second member is qualified by what the first member implies. The qualification which occurs here is based on a *kāraka*-relation or on a *ṣaṣṭhī*-relation, i.e. on an oblique case relation. In other words, a relation C may be defined as follows:  $C_k(t, p)$  denotes some relation between t and p in a sentence in which these occur as 't<sub>k</sub> p', where k denotes any of the oblique case-endings enumerated in Pāṇini's order (we shall continue to denote case-endings in this way by subscripts). Hence the *tatpuruṣa* can be defined as:  $apC_k(t, p)$  ( $k = 2, \dots, 7$ ), where t is *upasarjana* and p is the *uttarapāda*. Example: in *tatpuruṣa*, 'the man of him',  $k = 6$  and the analysis yields: t<sub>6</sub> p, or: *tasya puruṣa*.

The later Indian logicians have themselves considered the logical status of grammatical categories and have put forward their own definitions. Many problems of this kind were studied in the branch of Navya-nyāya called Śaktivāda (*śakti* denotes here the relation between a word and its meaning,<sup>3</sup> i.e. *śaktivāda* stands approximately for semantics). The *Nyāyakośa*<sup>4</sup> quotes a definition of *tatpuruṣa* given by Jagadīśa Bhaṭṭācārya in his *Śabdāsaktiprakāśikā* which runs as follows: *yadarthagatena subarthena viśiṣṭasya yadarthasyānva-yabodham prati yaḥ samāsaḥ svarūpayogyah sa tadarthasya tadarthe tatpuruṣaḥ*.

This definition provides a good example of a sentence, containing three subordinate appositional clauses, which is hard to translate adequately into a modern European language, but which can be analysed with the help of modern logic. We may first eliminate the term *svarūpayogyah* 'applicable to its own form', which expresses the fact that the term *tatpuruṣa* is itself a case of a *tatpuruṣa* (i.e. is autonymous).<sup>5</sup> We are then left with three relative pronouns, which can be reduced to two by eliding *ya*. This is justified, as . . . *yaḥ samāsaḥ . . . sa . . . tatpuruṣaḥ* can be expressed by introducing the definition below as follows: 'a *tatpuruṣa* compound denotes the following:'. If we

<sup>1</sup> Pāṇini, 1.2.43. The chief member is called *pradhāna*. It is possible that the *tatpuruṣa* originated from a combination of words where the *upasarjana* word kept its case-ending (see, however, T. Burrow, *The Sanskrit language*, London, 1955, 208). In the R̥gveda it sometimes preserves (part of) its case-ending (see L. Renou, *Études védiques et pāṇinéennes*, I, Paris, 1955, 51); in the Atharvaveda especially the locative termination (see L. Renou, 'Linguistic remarks on the Paippalāda version of the Atharvaveda', *Felicitation volume presented to S. K. Belvalkar*, Banaras, 1957, 70). The logicians argue that the *lakṣanā* is not understood because of the recollection of such an elided case-ending (*tuṣṭa-vibhakteḥ smaraṇam*) for the meaning is grasped also by somebody who does not remember this case-ending: *Siddhānta-muktāvalī*, ed. Hari Rāma Śukla, Banaras, 1951 (to be abbreviated as *SM*), 294. Cf. also the translation of Svāmi Mādhavānanda, Calcutta, 1954, 162.

<sup>2</sup> *tatpuruṣe tu pūrvapade lakṣanā*: *SM*, 293, ad *Bhāṣā-pariccheda* (to be abbreviated as *BP*), 82.

<sup>3</sup> *śaktiḥ padena saha padārthasya sambandhaḥ* (*SM*, 265).

<sup>4</sup> By Mm. Bhīmācārya Jhalakikar, Poona, 1928, 311, s.v. *tatpuruṣa*.

<sup>5</sup> cf. Pāṇini, 1.1.68, and J. Brough, 'Theories of general linguistics in the Sanskrit grammarians', *Transactions of the Philological Society*, 1951, 28-31.

denote the other pronouns in the order in which they occur as *yad'*, *yad''*, *tad'*, and *tad''*, and write these below the corresponding variables, the definition can be written as :

$$\text{apS}(\text{atC}_{jk}(t), \text{aq}\{\{q = p\} \wedge V[q, \text{atC}_{jk}(t)]\})$$

Here  $S(x, y)$  denotes *samāsa*  $x_6 y_7$ , 'compound (relating)  $x$  to  $y$ ';  $V(x, y)$  denotes  $y_3$ -*viśiṣṭa*  $x_1$ , ' $x$  is qualified by  $y$ '; and  $C_{jk}(x)$  denotes  $x_1$  *subartha*, ' $x$  has the meaning of  $x_{jk}$ ', where  $x_{jk}$  ( $j = 1, \dots, 3$ ;  $k = 1, \dots, 7$ ) denotes  $x$  followed by *sup*, the technical term used by Pāṇini to denote any of the seven case-endings in any of the three numbers.<sup>1</sup>

This formula is in one respect simpler and in another respect less simple than the Sanskrit expressions. It is less simple because sequences of more than one  $a$  preceding the same variable are avoided by introducing another variable  $q$  which is identical with  $p$ . The result makes a somewhat artificial impression. It is more simple, on the other hand, because it avoids the clause *ya . . . sa . . .*

The symbolical representation of the refined definition of the Naiyāyika requires much more powerful means of expression than the grammarian's definition. The original definition can be made to apply to several of the sub-kinds of *tatpuruṣa*-compounds by slight modifications or by additional conditions. Of these we shall only consider the addition which results in the definition of the logically most important of these compounds, namely the *karmadhāraya*. Pāṇini defined it as *tatpuruṣaḥ samānādhikaraṇaḥ* 'a *tatpuruṣa* (with both members) possessing the same *adhikaraṇa*'.<sup>2</sup> In order to formalize this we introduce a relation  $B(x, y)$  denoting  $y_6$  *adhikaraṇa*  $x_1$ , ' $x$  is the locus of  $y$ ',  $y_1 x_7$ , ' $y$  is in  $x$ ', or  $x_1 y$ -*vat* (*-mat*), ' $x$  has  $y$ '. From the equivalence of the first two of these denotations we see that the concept of *ādhikaraṇya* serves to define the locative. In fact, *adhikaraṇa* is the term used by the grammarians to denote the *kāraka*-relation of the locative. In order to express the condition that  $a$  and  $b$  have the same locus we need merely write:  $\alpha x B(x, a) = \alpha x B(x, b)$ . Hence Pāṇini's definition of the *karmadhāraya*,  $K(p)$ , may be written as :

$$K(p) = {}_a\{C_k(t, p) \wedge [\alpha x B(x, t) = \alpha x B(x, p)]\}.$$

Pāṇini's analysis of the *karmadhāraya* by means of the sameness of locus differs from the analysis of Western grammarians, who characterize this compound generally by some kind of statement, that 'the first and the second

<sup>1</sup> Pāṇini, 1.4.14, 103. An example, also from the *Śabdaśaktiprakāśikā*, quoted by P. B. Chakravarti (*The philosophy of Sanskrit grammar*, Calcutta, 1930, 300, n. 1) states how the *śaṣṭhī* 'genitive' is implied by *lakṣaṇā*: *rājapurūṣa ityādau pūrvapade śaṣṭhyartha-saṃbandhe lakṣaṇeti maṅgīrḍuktam api saṃgucchate* 'in cases such as *rājapurūṣa*, etc., the meaning of the genitive is implied in the first word, as also laid down by the author of the *Tattva-cintāmaṇi*'.

<sup>2</sup> 1.2.42.

member are related to each other as predicate and subject are.<sup>1</sup> Pāṇini uses *guṇa* with the meaning of 'predicate' relatively rarely,<sup>2</sup> though it is not at all a rare concept in Indian philosophy and logic. We shall see that there are other instances where Western thought is inclined to analyse a close relationship in terms of subject and attribute, whereas Indian thought considers the relation to the *adhikarāna*.

It may lastly be remarked that formally it is very simple to give a definition of *karmadhāraya* on the basis of the distinction of subject and predicate, which evidently plays an important part in symbolic logic:  $\alpha xF(x)$  or  $\alpha pK(p)$  itself serves the purpose. Identification of the case-endings, *ekavibhaktitva*—e.g.  $\alpha p[C_k(t, p) \wedge (p = p_k)]$ —is not a possible definition, as a *tatpuruṣa* in the same case as the case denoted by its *upasarjana* (e.g. *tatpuruṣasya*) does not become a *karmadhāraya*, unless  $k = 1$ .

The last kind of compound we shall consider is the *bahuvrīhi*. Pāṇini defined it as *anekam anyapadārthe* ' (consisting of) several (words) in the meaning of another word '.<sup>3</sup> If we denote the several words together by *b*, the relation *B* introduced earlier can be used and the definition can be formalized as:  $\alpha pB(p, b)$ . All the members denoted by *b* can be considered as *upasarjana*.<sup>4</sup> The restricted-variable *p* is freshly introduced as it were, and this constitutes a good example of *lakṣaṇā* 'implication'. For instance in *citraḡu* 'owner of brindled cows', the word *gu* or *go*, 'cow', implies *goman* 'owner of cows'.<sup>5</sup> In the symbolism of the definition, *b* when intended as a *bahuvrīhi* implies a *p* such that *B(p, b)*.

The definition of the *bahuvrīhi* compound as  $\alpha pB(p, b)$  follows closely the Sanskrit usage, as may be seen from the following instance taken from the *Mahābhāṣya*. Patañjali asks, when commenting upon Pāṇini's rule *nīpāta ekāḡ anān*,<sup>6</sup> whether *ekāc* in this *sūtra* is to be understood as a *bahuvrīhi* or as a *tatpuruṣa samānādhikarānaḡ*. In the latter case it should be analysed as: *eko 'c*, in the former case as: *eko 'j yasmin so 'yam*.<sup>7</sup> The latter expression corresponds exactly to  $\alpha pB(p, b)$ , if *b* denotes *eko 'c*,  $\alpha p$  denotes *so 'yam*, and *B(p, b)*, defined as  $b_1 p_7$ , denotes *eko 'j yasmin*.

### III

After the formalization of some of the nominal compounds the role which such expressions may play in Indian logic will be considered and attempts will be made at a formalization of some of the expressions of the technical Sanskrit of Indian logic. As an example of the possibilities of formalization first some of

<sup>1</sup> It is interesting to observe in B. Faddegoñ's discussion of the subdivisions of the compounds (*Studies on Pāṇini's Grammar*, Amsterdam, 1936, 61-2) how Wackernagel differed mainly from Pāṇini by utilizing the categories of noun and adjective.

<sup>2</sup> Böhlingk's index mentions three *sūtras*, and six *sūtras* using *guṇavacana* 'Eigenschaftswort'.

<sup>3</sup> 2.2.24.

<sup>4</sup> As the *Kāśikā* says (Renou, *Terminologie*, 108).

<sup>5</sup> *gopadasya gomati lakṣaṇā* (*SM*, 292).

<sup>6</sup> 1.1.14.

<sup>7</sup> *Mahābhāṣya*, 1.1.5, ed. Kielhorn, 1, 70.

the implications will be considered of the introduction of the relation  $B(x, y)$  into our—relatively vaguely delineated—formal system. This example will also provide a formalization of a very important device of Indian logic, i.e. abstraction.

A well-known rule of Navya-nyāya runs as follows: *tattvatat tad eva* 'that which possesses thatness is (just) that'.<sup>1</sup> For instance in the inference *dravyam sattvāt*, the word *dravyam* is to be understood as *dravyatvavān* and the *sādhyā* is *dravyatva*. If we denote *tat* by  $t$  and *tattva* by  $\hat{t}$  (Sen and Ingalls used subscripts, which are here used for case-endings), this may be expressed as:  $\alpha xB(x, \hat{t}) = t$ . Hence abstraction, as a counterpart of the expression which provides the locus, can be written as:  $\hat{a} = \alpha xB(a, x)$ . Or we may start with the relation  $B(a, \hat{a})$  and derive both by making use of the notion of restricted-variable<sup>2</sup>:

$$\begin{aligned} \hat{a} &= \alpha xB(a, x) && \text{(abstraction)} && \text{and:} \\ a &= \alpha xB(x, \hat{a}) && \text{(formation of the locus).} \end{aligned}$$

As an application of this abstraction the formalization of the concept *pratiyogitā* may be considered. *Pratiyogī* in Navya-nyāya<sup>3</sup> is a term which denotes the thing which is negated. It will here be defined for *atyantābhāva*, though it is also used in connexion with *anyonyābhāva*.<sup>4</sup> While *anyonyābhāva* denotes the negation of identity (e.g.  $x \neq y$ ), *atyantābhāva* denotes the negation of a presence or of a relation. If  $A(x, y)$  denotes *y-vrttitva*  $x_6$ , 'presence of  $x$  in  $y$ ', or *sambandha*  $x_6y_7$ , 'relation of  $x$  to  $y$ ', the *atyantābhāva* of  $x$  in  $y$  may be written as:  $\sim A(x, y)$ . Hence the *atyantābhāvīya-pratiyogī* is:  $\alpha x[\sim A(x, y)]$ . By means of the above formalization of abstraction the *pratiyogitā* becomes:  $\alpha zB\{\alpha x[\sim A(x, y)], z\}$ .

Next we shall consider one of the main problems of Navya-nyāya, i.e. the definition of *vyāpti* 'pervasion'. The prima facie definition, *pūrvapakṣavyāptinirūpanam*, is given in several forms (cf. appendix), out of which we shall here consider two only. The *Vyāpti-pañcaka* section of the famous *Tattva-cintāmaṇi* of Gaṅgeśopādhyāya defines *vyāpti* in the first instance as: *sādhyābhāvavadavrttitvam* (sc. *hetoh*) (def. I). The *Bhāṣā-pariccheda* (BP) of Viśvanātha Pañcānana Bhaṭṭācārya, following the fifth definition of the *Vyāpti-pañcaka*, defines it as: *sādhyavadanyasmīn asambandha* (sc. *hetoh*) (def. II). These expressions contain several *tatpuruṣa* compounds (*sādhyābhāva*, *sādhyavadanya*) which could be formalized in accordance with the above definition. But all these compounds express negation and can hence be formalized with the help of a simpler notation.

If we denote *hetu* by  $h$ , *sādhyā* by  $s$ , and the *vyāpti*-relation which holds between  $h$  and  $s$  by  $V(h, s)$ , the *tatpuruṣa* compound *sādhyābhāva* can in accordance with the definition be written in the form:  $\alpha aC_k(s, a)$ . This relation

<sup>1</sup> D. H. H. Ingalls, *Materials for the study of Navya-nyāya logic*, Cambridge, Mass., 1951, 36.

<sup>2</sup> See above, p. 109.

<sup>3</sup> That is, the *abhāvīya-pratiyogī* (cf. Ingalls, 55).

<sup>4</sup> cf. Ingalls, 54-8. See also the present author's review of Ingalls' work, to be published in the *Indo-Iranian Journal*.

$C_k$  holds between a term  $s$  and its negation and can hence be written as :  $x \neq s$ . Thus *sādhyābhāva* becomes :  $\alpha x(x \neq s)$ . *Sādhyābhāvavad* may be written as  $\alpha yB[y, \alpha x(x \neq s)]$  and the first definition can be finally formalized as :

$$V(h, s) = {}_d \sim A \{h, \alpha yB[y, \alpha x(x \neq s)]\} \quad (\text{def. I}).$$

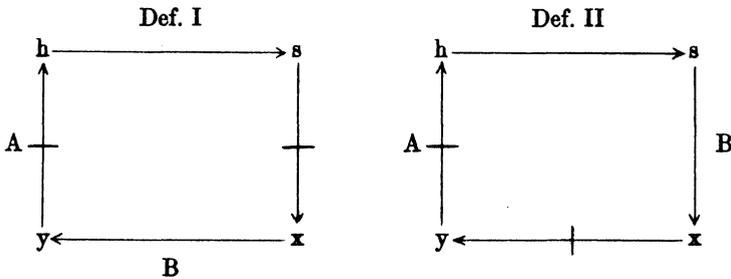
For the formalization of the second definition we may write for *sādhyavad* :  $\alpha xB(x, s)$ . *Sādhyavadanya* is a *tatpuruṣa* which can be expressed as :  $\alpha yC_s[\alpha xB(x, s), y]$ , in accordance with the definition. This can again be further simplified as :  $\alpha y[y \neq \alpha xB(x, s)]$ . For *sādhyavadanyasmint sambandha* or *sādhyavadanyavṛttitva* (sc. *hetoh*) we arrive at :

$$A \{h, \alpha y[y \neq \alpha xB(x, s)]\} \text{ and hence the second definition is : } \\ V(h, s) = {}_d \sim A \{h, \alpha y[y \neq \alpha xB(x, s)]\} \quad (\text{def. II}).$$

We have discussed definition II elsewhere and given a formal analysis of the argument of the *Siddhānta-muktāvalī* (*SM*), the author's own commentary on the *BP*, where four insertions are given to this definition which result in a final definition, where the generalization of the restricted-variables is expressed with the help of abstraction and of the concept of *avacchinna*.<sup>1</sup>

A simple scheme may be utilized to express the structure of both definitions, which is very useful in representing examples and special cases discussed in the text (it may be asked whether the Naiyāyikas themselves made use of similar aids). Thereby an *isomorphism*<sup>2</sup> can be established between the scheme, the formalization, and the original Sanskrit expression. At the same time the scheme provides a new interpretation for the restricted-variables  $x$  and  $y$ .

The two schemes may be sketched first :



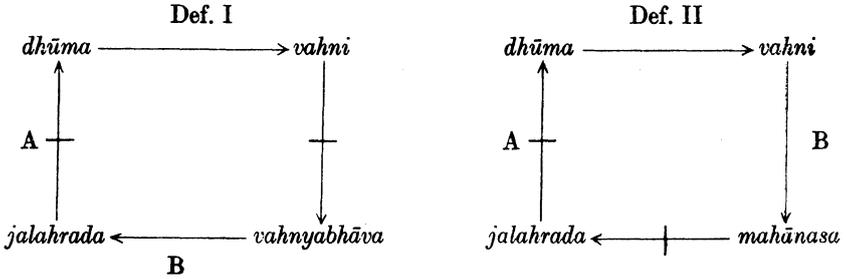
In terms of these schemes a proof of an inference in accordance with the definition consists in providing a connexion which links the *sādhyā* with the *hetu* by following the sides of the rectangle in the clockwise direction (*prada-kṣiṇa*, as the arrows indicate). For instance for the stock example of Indian logic, *vahnimān dhūmāt* 'it possesses fire because of smoke', we write *dhūma* 'smoke', the *hetu*, in the left upper corner and *vahni* 'fire', the *sādhyā*, in the right upper corner. Then, following definition I, we write below the *sādhyā* the *sādhyābhāva* : *vahnyabhāva* 'absence of fire'. Then a  $y$  has to be found such that

<sup>1</sup> 'Means of formalisation in Indian and Western logic', in *Proceedings of the XIIIth International Congress of Philosophy (Venice, 1958)*. (Chapter 3 in this volume.)

<sup>2</sup> A one-to-one(-to-one) correspondence where it is possible to pair off the elements in such a manner that the structure is not affected.

$B[y, \alpha x(x \neq s)]$ , e.g. *jalahrada* 'lake', which we write down in the left lower corner. Lastly we have to verify that  $h$  does not occur in this  $y$ , which closes the chain.

In terms of definition II, we have to find a locus  $x$  such that  $B(x, s)$ , e.g. *mahānasa* 'kitchen', which we record in the right lower corner. Thereupon a  $y$  different from that, e.g. *jalahrada*, has to be supplied in the left lower corner. Then again, with reference to this  $y$ , it is verified that  $A(h, y)$ . Carrying this out in the schemes we have:



If we make the corresponding substitutions in the definitions the results are isomorphic:

Def. I	Def. II
$h : dhūma$	$h : dhūma$
$s : vahni$	$s : vahni$
$\alpha x(x \neq s) : vahnyabhāva$	$\alpha xB(x, s) : mahānasa$
$\alpha yB(y, vahnyabhāva) : jalahrada$	$\alpha y(y \neq mahānasa) : jalahrada$
$\sim A(dhūma, jalahrada)$	$\sim A(dhūma, jalahrada)$

We shall next consider the inferences which refute the validity of the prima facie definition and lead to the final definition, *siddhāntavyāptinirūpanam* or *siddhāntalakṣaṇa*.<sup>1</sup> The first of these inferences is: *jñeyam vācyatvāt* 'it is knowable (it possesses *jñeyatva*) because it is nameable'. Trying to apply definition II,  $s$  being *jñeyatva*,  $\alpha xB(x, s)$  is the entire universe of discourse according to the Naiyāyika assumption: *sarvaṃ jñeyam* 'everything is knowable'. In order to continue the proof a  $y$  has to be found such that  $y \neq \alpha xB(x, s)$ , i.e. an entity which does not belong to the universe of discourse and which is therefore *aprasiddha* 'unexemplified'. Here the definition evidently breaks down, for it is impossible to establish the connexion required for proving the inference *jñeyam vācyatvāt* which is (intuitively) known to be valid.

The above difficulty may be formalized along the following lines. Let  $j$  denote *jñeyatva*, then  $B(x, j)$  denotes 'x possesses *jñeyatva*' or  $x$  *jñeyam*, 'x is knowable'. The assumption *sarvaṃ jñeyam* becomes  $(z)[z = \alpha xB(x, j)]$  (1). In order to derive the contradiction from this we shall have to make use of two logical laws which form part of any formal system and which can be written in the form of bi-conditionals as follows:

<sup>1</sup> *SM*, 220; cf. Ingalls, 62.

$$(x)F(x) \leftrightarrow \sim (Ex) \sim F(x) \quad (2)^1$$

$$(F \leftrightarrow G) \leftrightarrow (\sim F \leftrightarrow \sim G) \quad (3).^2$$

Applying (2) to (1) we arrive at:

$$\sim (Ez)[z \neq \alpha x B(x, j)] \quad (4).$$

Applying (3) to:  $(Ey)[y = \alpha x F(x)] \leftrightarrow (Ey)F(y)$ ,<sup>3</sup> we arrive at:

$$\sim (Ey)[y = \alpha x F(x)] \leftrightarrow \sim (Ey)F(y).$$

This can be applied to (4), which results in:

$$\sim (Ez)\{z = \alpha y[y \neq \alpha x B(x, j)]\}.$$

This last expression means that there is no  $z$  which is equal to  $\alpha y[y \neq \alpha x B(x, j)]$ , in other words, that this restricted-variable denoting *jñeyād anyā* 'what is different from what is knowable' is *aprasiddha*. In this way the required contradiction is reached. It is clearly seen that use is made of the *reductio ad absurdum*. At the same time a formal definition of *aprasiddhatva* is suggested, i.e.:  $x$  is called *aprasiddha* with respect to a property  $F$ , if:  $\sim (Ey)[y = \alpha x F(x)]$  or:  $\sim (Ex)F(x)$ . An entity  $a$ , on the other hand, is called *kevalānvayī* 'unnegatable' if:  $(y)[y = \alpha x B(x, a)]$  or:  $(x)B(x, a)$ —e.g. *jñeyatva* is *kevalānvayī*.

In the other inference which refutes the validity of the original definition, *sattāvān jāteḥ* 'it possesses reality because (it possesses) generic character', the difficulty is of a different kind. In order to appreciate this we have to bear in mind a remark which has been made a few lines earlier in the *SM*. The expression  $A(h, y)$  'occurrence of  $h$  in  $y$ ' can always be further analysed as occurrence with respect to some relation, . . . *sambandhena vṛttitva*. This could be expressed by a subscript within parentheses attached to  $A$  which denotes various types of *sambandha* by names of letters (extending Ingalls<sup>4</sup>):  $c$  for *saṃyoga*,  $h$  for *samavāya* 'inherence', etc. For example, if  $j$  and  $d$  denote *jāti* 'generic character' and *dravya* 'substance' respectively,  $A_{(h)}(j, d)$  symbolizes *samavāya-sambandhena d*,  $j_1$  *vṛttitva*, 'occurrence of generic character in substance through the inherence relation'.

Apart from the possible occurrence of  $h$  in  $y$ , however, the *hetu* always occurs through some relation in its *pakṣa* 'field, residence'. The latter relation is said to be the *hetutāvachedaka-sambandha* 'the limiting relation of *hetutā*'. The additional condition which must be fulfilled in order that the original definition be valid is that this relation be the same as the *vṛttitāvachedakāsambandha*, the relation through which  $h$  resides in  $y$ . If we denote the *pakṣa* by  $p$  this can be expressed as:  $\alpha x A_{(x)}(h, p) = \alpha x A_{(x)}(h, y)$ , where  $y$  is a variable for which

<sup>1</sup> This expression occurs in every predicate calculus. It may be incorporated in the formal system either as an axiom or a definition (e.g. W. V. O. Quine, *Mathematical logic*, Cambridge, Mass., 1951, 102) or as a theorem (e.g. D. Hilbert and W. Ackermann, *Grundzüge der theoretischen Logik*, New York, 1946, 62; S. C. Kleene, *Introduction to metamathematics*, Amsterdam, 1952, 162).

<sup>2</sup> See e.g. Quine, op. cit., 57 (17); Hilbert and Ackermann, op. cit., 8 (26); Kleene, op. cit., 116 (30).

<sup>3</sup> See above, p. 109.

<sup>4</sup> op. cit., 74-7.

the restricted-variable  $\alpha y[y \neq \alpha xB(x, s)]$  has to be substituted. In other words, the definition is further specified as :

$$\sim A_{[\alpha xA_{(x)}(h, p)]}\{h, \alpha y[y \neq \alpha xB(x, s)]\}.$$

This development enables us to return to the inference *sattāvān jāteh*. According to the Naiyāyikas *sattā* 'reality' occurs through the inherence relation only in the first three categories: *dravya* 'substance', *guṇa* 'quality, quale', and *kriyā* 'action',<sup>1</sup> but not in *jāti* 'generic character' or any of the other categories. As this occurrence is termed *vṛtti* this may be written as:  $A_{(h)}(sattā, dravya)$ , etc. The Naiyāyikas seem to assume, at least in this case, that  $A(x, y)$  implies  $B(y, x)$ : locus of an entity is where it occurs.<sup>2</sup> As the *sādhya* in the above inference is *sattā*, the expression  $\alpha xB(x, s)$  covers the other categories, *jātyādayaḥ* 'generic character, etc.'. In the same inference the *hetu* is also *jāti*, and according to another assumption of the Nyāya terminology also *jāti* can inhere only in *dravya*, *guṇa*, and *kriyā*<sup>3</sup>; and these three are said to constitute the *pakṣa* of *jāti*. Hence the occurrence of  $h$  in:  $\alpha y[y \neq \alpha xB(x, s)]$  can never be through the same relation through which  $h$  occurs in the *pakṣa*, i.e. through *samavāya*: the occurrence is *aprasiddha*. Formally speaking,  $\alpha xA_{(x)}(h, p) \neq \alpha xA_{(x)}(h, y)$  and therefore :

$$\sim (\exists y)A_{(z)}(h, y) \text{ if: } z = \alpha xA_{(x)}(h, p)$$

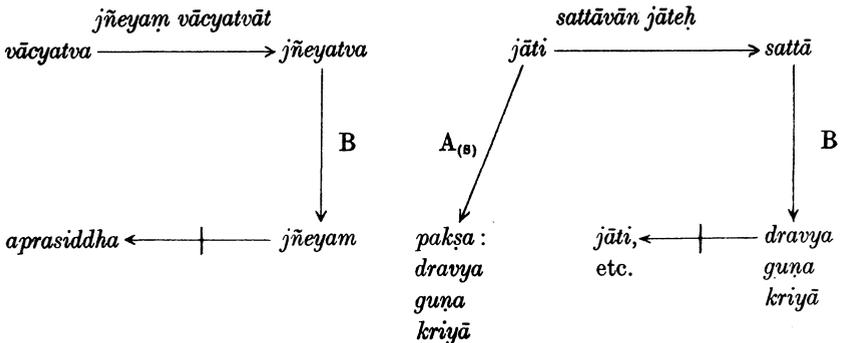
or :

$$\sim (\exists y)A_{[\alpha xA_{(x)}(h, p)]}(h, y).$$

This means that :

$A_{[\alpha xA_{(x)}(h, p)]}\{h, \alpha y[y \neq \alpha xB(x, s)]\}$  is impossible; and hence also its negation is impossible.

The models for the two inferences which we have analysed illustrate dramatically the breaking down of definition II :



#### IV

If we assume, just as the Naiyāyikas seemed to do in the case discussed last,

<sup>1</sup> Ingalls, 53-4, referring to *BP*, 8.

<sup>2</sup> Professor Brough suggested the possibility of the convertibility of  $A$  into  $B$  and vice versa in connexion with the formalizations of definitions I and II.

<sup>3</sup> Ingalls, 75.

that  $A(x, y)$  implies  $B(y, x)$ , the definitions can be written in terms of B only (or A only, if the reverse is also assumed) :

$$\sim B\{ayB[y, ax(x \neq s)], h\} \quad (\text{def. I})$$

$$\sim B\{ay[y \neq axB(x, s)], h\} \quad (\text{def. II}).$$

As it is uncertain to what extent the convertibility between A and B was generally assumed, it seems preferable to use A and B in order to show which terms are used in a given case. If the convertibility were an unquestionable fact, it would be difficult to explain the terminological differentiation which is consistently carried through at least in the *SM*. In the following there is an instance of the use of A and B as two distinct relations, which are interrelated but which cannot be reduced to each other.

As the original definitions had been shown to be invalid, a final definition was proposed which the *BP* expresses as: *hetumannīṣṭhavirahāpratīyoginā sādhyena hetor aikādhikaranyam*. This is further interpreted by the *SM* as follows: *hetumati niṣṭhā — vṛttir yasya sa tathā virahaḥ — abhāvah, tathā ca hetv-adhikarāṇa-vṛttir yo 'bhāvas tadapratīyoginā sādhyena saha hetoḥ sāmānādhikaranyam*. In the first instance *vyāpti* is here defined as: *sādhyena saha hetoḥ sāmānādhikaranyam*, i.e. by the condition:  $axB(x, h) = axB(x, s)$  (1). The rest of the expression enjoins a further condition upon the *sādhyā*: it should not be the *pratīyogī* of an *abhāva* which occurs in the *hetvadhikarāṇa*. As the formalization of the *pratīyogī* with reference to an *abhāva*  $\sim A$  has been given earlier, this condition can be expressed as:  $s \neq ay\{\sim A[y, axB(x, h)]\}$ , which may be interpreted as:  $\sim \sim A[s, axB(x, h)]$  or (for the law of the excluded middle is generally adopted):  $A[s, axB(x, h)]$  (2). Thus the expressions (1) and (2) together constitute the final definition or *siddhāntalakṣaṇa*. If A and B as occurring in these expressions were convertible in accordance with the given rule, one expression would be dependent upon the other, for  $B[axB(x, h), s]$  implies (1) in accordance with the notion of restricted-variable.

In order to apply the final definition the two above inferences may be reconsidered<sup>1</sup>:

*jñeyam vācyatvāt*. Condition (1) is fulfilled, because both loci are the entire universe of discourse. Moreover, (2) is fulfilled also, because it is always assumed that *s*, *jñeyatva*, is present everywhere:  $(x)A(s, x)$ .

*sattāvān jāteḥ*. (1)  $axB(x, s) = dravya, guṇa, \text{ and } kriyā = axB(x, h)$ . (2) *Sattā* is present in  $axB(x, h) = dravya, guṇa, \text{ and } kriyā$ .

The definition of *vyāpti* through *sāmānādhikaranyā* has been submitted to numerous criticisms which have been met by the application of various refinements to the definition. The objections of Naiyāyikas, however, accept generally the main idea of the definition: reference to a locus. This characterization itself was rejected by the Dvaitins, who gave a simple counter-example which seems irrefutable. Caḷari Śeṣācārya, a Mādhva of the fourteenth or fifteenth century, quotes in his *Pramāṇa-candrikā* the inference: *ūrdhvadeśo vṛṣṭimān*

<sup>1</sup> Here we follow the commentary of Svāmī Mādhavānanda, 112, note.

*adhodeśe nadīpūrāt* 'there is rain in the high country because the rivers are full in the low country'.<sup>1</sup> This is an inference, which is known to be valid (if not in the Himālaya, where melting of snow because of sunshine might be the *hetu*, at least in South India, where Śeṣācārya was born), but which constitutes at the same time a good example of *vaiyādhikarāṇya* 'difference of locus': for  $\alpha xB(x, \text{nadīpūram}) = \text{adhodeśa} \neq \alpha xB(x, \text{vr̥ṣṭi}) = \text{ūrdhvaeśa}$ . 'The low country, where the rivers are full, is not the high country, where there is rain.'

In later Advaita, where many Naiyāyika categories are rejected, the definition of *vyāpti* is accepted. In the *Vedāntaparibhāṣā* it is further quantified as follows: *aśeṣa-sādhanāśrayāśrīta-sādhyā-sāmānādhikarāṇya-rūpā*.<sup>2</sup> This can be represented by introducing a class concept  $\hat{x}F(x)$ , 'the class of all  $x$  such that  $F(x)$ '. Then *aśeṣasādhanāśraya* corresponds to:  $\hat{x}B(x, h)$  and *aśeṣa-sādhanāśrayāśrīta-sādhyā* corresponds to:  $\alpha sA[s, \hat{x}B(x, h)]$ , so that the entire expression may be represented by:  $\alpha xB(x, h) = \alpha xB\{x, \alpha sA[s, \hat{y}B(y, h)]\}$ .

In later Advaita the same concepts are used to characterize the relations between the world and Brahman and between the individual soul and Brahman, which are further differentiated by additional qualifications as *bādha-sāmānādhikarāṇya* and *aikyasāmānādhikarāṇya* respectively. While the latter expression may denote nothing but identity,<sup>3</sup> the former is characterized by the technical term *bādha*,<sup>4</sup> which, in conjunction with *sāmānādhikarāṇya* becomes *anirvacanīya* 'unintelligible'. This is, in the Advaita view, in accordance with the fact that it refers to the world. In Advaita, where the Absolute is not even a *dravya* because it is *nirguṇa* 'without qualities' (*dravya* is defined as *guṇāśraya* <sup>5</sup>), a hesitation seemed to have been felt in accepting Brahman as the *āśraya* or locus of the fundamental category of *avidyā* or nescience: while the apparently more consistent (for duality-avoiding) *Vīvarāṇa* school declared Brahman to be the *āśraya* of *avidyā*, the *Bhāmātī* school of Vācaspati Miśra accepted a plurality of nesciences, each possessing an individual soul as its *āśraya*.

In the West correlations between language, logic, and philosophy are often hidden because of the differentiation of the terminology (for instance the pair of categories substance/attribute in philosophy—subject/predicate in logic—noun/adjective in grammar). In India the same terms are often used at different levels.<sup>6</sup> An example is provided by the term *adhikarāṇa* (or equivalents

<sup>1</sup> *Pramāṇa-candrikā*, ed. and trans. S. K. Maitra, Calcutta, 1936, 57, 144.

<sup>2</sup> *Vedānta-paribhāṣā*, 2.10.

<sup>3</sup> cf. also L. Renou, *JA*, CCXXXIII, 1941-2, 165.

<sup>4</sup> See e.g. S. N. Dasgupta, *A history of Indian philosophy*, II, 222.

<sup>5</sup> *Nyāyakośa* s.v. *dravya*. Substance is also *guṇavat* 'possessing qualities' (*Vaiśeṣika-sūtra*, 1.1.15) and *guṇa* is *dravyāśrayī* 'having substance as its locus' (ibid., 16). But in Advaita *brahmaṇo dravyavāsiddhiḥ*, 'it has not been established that Brahman is a substance' (*Vedānta-paribhāṣā*, 2.25).

<sup>6</sup> cf. Renou, *Terminologie*, avant-propos: '... il n'y a pas toujours intérêt à dissocier (la valeur technique d'un mot) de la valeur "mondaine" (*laukika*)'.

such as *ādhāra*, *āśraya*, *adhiṣṭhāna*), which serves to characterize linguistic, logical, and philosophical categories either directly or through relations like  $B(x, y)$ .<sup>1</sup>

The study of some of the expressions of language and of logic with the help of the formal representations of symbolic logic can disclose various structures, which manifest themselves at different levels: in the realm of language, in logic, and in philosophy. The study of the appearance of such structures in metaphysics is also illuminating, even for the so-called 'linguistic philosopher', because metaphysics lingers on the frontiers of language and does therefore show which are the ultimate linguistic and logical categories which man tries to transcend in his incessant attempts to deal with 'that from which all words return—which is not within reach of the mind': *yāto vāco nīvartante — āprāpya mānasā sahā*.<sup>2</sup>

#### APPENDIX

In the above analyses only the first and the last of the definitions of the *Vyāpti-pañcaka* have been formalized. It is relatively easy to formalize the other definitions with the help of the symbols which have been introduced. Adding the three new definitions, the *Vyāpti-pañcaka* may be represented as follows:

- (1)  $\sim A \{h, ayB[y, ax(x \neq s)]\}$
- (2)  $\sim A[h, auB(u, azA \{z, ay[y \neq axB(x, s)]\})]$
- (3)  $axB(x, h) \neq ay[y \neq axB(x, s)]$
- (4)  $(ayB) (\sim A \{h, ayB[y, ax(x \neq s)]\})$
- (5)  $\sim A \{h, ay[y \neq axB(x, s)]\}$ .

Bocheński, to whom we owe the first comprehensive history of formal logic which takes Indian material into account, has given another formalization of these five definitions.<sup>3</sup> If we identify the two terminologies (Bocheński uses more expressions of the logic of relations, but no restricted-variables) the two formalizations can be seen to be partly identical. Bocheński's expressions may therefore first be defined in terms of the expressions used here:

$$\begin{aligned} g &= {}_d h \\ xRy &= {}_d R(x, y) \\ gCs &= {}_d V(h, s) \\ \dot{R} &= {}_d \sim R \end{aligned}$$

<sup>1</sup> *Adhikarāna* (cf. German 'Grund'), the *kāraka*-relation of the locative, may sometimes be used to elucidate what constitutes a reason. Cf. P. Hartmann's comment (*Nominale Ausdrucksformen im wissenschaftlichen Sanskrit*, Heidelberg, 1955, 60, n. 79a), 'dass unter der lokativischen Zuschreibung auch ein Verursachtsein mitgedacht werden kann', based upon a remark by H. Oldenberg. This is already recognized in Patañjali's subdivision of *adhikarāna* into three kinds, the first of which is called *vyāpaka* (Renou, *Terminologie*, 301, and cf. Chakravarti, op. cit., 251).

<sup>2</sup> *Taittirīyopaniṣad*, 2.4. I should like to express here my gratitude to Professor E. W. Beth (Amsterdam) for the interest he has taken in these investigations and for the valuable suggestions he has given.

<sup>3</sup> op. cit. (p. 110, n. 1), 512.

$$\begin{aligned} x\check{R}y &= {}_a\check{R}(x, y) \text{ or : } \alpha X[R(y, x) \rightarrow X(x, y)] \\ x(P/Q)y &= {}_aP[x, \alpha zQ(z, y)] \\ xVy &= xIy = {}_aA(x, y) \\ xOy &= {}_aB(x, y) \\ xAy &= xGy = xDy = {}_a(x \neq y). \end{aligned}$$

With the help of these definitions the first, the third, and the fifth of the definitions of *vyāpti* are seen to be equivalent in both interpretations.

(1)  $g(-V/O/A)$ s becomes :  $\sim A\{h, \alpha yB[y, \alpha x(x \neq s)]\}$ .

(3)  $g(\check{O}/D/G/O)$ s becomes :

$$\begin{aligned} &\check{B}(h, \alpha z\{z \neq \alpha y[y \neq \alpha xB(x, s)]\}) \\ &B(\alpha z\{z \neq \alpha y[y \neq \alpha xB(x, s)]\}, h) \\ &\alpha xB(x, h) = \alpha z\{z \neq \alpha y[y \neq \alpha xB(x, s)]\} \\ &\alpha xB(x, h) \neq \alpha y[y \neq \alpha xB(x, s)]. \end{aligned}$$

This is very similar to the *siddhānta-lakṣaṇa*, but neither do the two negations necessarily cancel each other nor is inequality transitive.

(5)  $g(-V/D/O)$ s becomes :  $\sim A\{h, \alpha y[y \neq \alpha xB(x, s)]\}$ .

The second and the fourth definitions, on the other hand, are not equivalent in both interpretations.

(2)  $g(-V/O/V/O/D)$ s becomes :

$$\sim A[h, \alpha uB(u, \alpha zA\{z, \alpha yB[y, \alpha x(x \neq s)]\})].$$

This does not seem to be correct, as may be seen from the order of the last two relations O/D, where instead the order D/O may be expected, which corresponds to  $\alpha y[y \neq \alpha xB(x, s)]$ , *sādhyavadbhinna* (as in definition (5): *sādhyavadanya*). In Bocheński's terminology the definition should be represented by :  $g(-V/O/V/D/O)$ s, which becomes :

$$\sim A[h, \alpha uB(u, \alpha zA\{z, \alpha y[y \neq \alpha xB(x, s)]\})].$$

(4) This is a slightly different type of definition, about which Bocheński himself says that he is not certain that he has understood it well. His formalization is :  $g(G/A/I/O/A)$ s, which becomes :

$$h \neq \alpha u(u \neq \alpha zA\{z, \alpha yB[y, \alpha x(x \neq s)]\}).$$

The intention of Gaṅgeśa seems to be different and the definition may merely express a generalization of the first definition. This cannot be expressed as easily in Sanskrit as in the modern notation, for Sanskrit has to make use of abstractions (here : *pratijogitva*).<sup>1</sup> Hence the Sanskrit expressions of the first and the fourth definitions differ more than the formal expressions.

<sup>1</sup> cf. the article quoted above, p. 115, n. 1.

## FORMAL STRUCTURES IN INDIAN LOGIC

There is a use of the term 'model' in which it can be said that a linguistic expression, in a natural language, is a model for its sense. A translation of a linguistic expression from one language into another may be said to provide another model for the sense of the original. If the sense of a linguistic expression is of a logical nature, the expression can be translated into an expression of formal logic or into a formula. This is not surprising, for logic and mathematics came into being when expressions of natural languages were translated into formal symbolisms, which were more precise and practical and less cumbersome. Subsequently these artificial languages attained full independence and started a development of their own. Originally, however, these symbolisms could only have been constructed along the lines suggested by the possibilities of expression and the scope of expression of the natural languages themselves. That in mathematics and in modern logic such a linguistic origin of the symbolism has often receded into the background does not imply that the origin of certain symbolisms was independent from the structure of natural languages <sup>1</sup>).

In view of this background it is not surprising that modern logic could provide the tools for the representation of logical expressions used by the Western logicians from Aristotle onwards. This has been shown by Bocheński, Łukasiewicz and many others. It is less evident, on the other hand, that the symbolism of modern logic should be useful in the representation of the only formal logic, regarding which there are good reasons to believe that it developed independently from European logic: namely Indian logic. Nevertheless modern formalisms have been introduced – sometimes hesitatingly – into the study of Indian logic by

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<sup>1</sup>) Some examples of such dependence are discussed in the present author's 'The construction of formal definitions of subject and predicate', to be published in *Transactions of the Philological Society*.

S.Sen <sup>1)</sup>, S. Schayer <sup>2)</sup> and D. H. H. Ingalls <sup>3)</sup>. I. M. Bocheński has now written the first comprehensive history of formal logic which takes Indian material into account <sup>4)</sup>. That a modern symbolism can actually be used for the representation of Indian logic at all need not imply that this symbolism is necessarily universal. For Sanskrit, the language in which the Indian logicians expressed themselves even if their mother tongue was different, is an Indo-European language and its structure is largely similar to the structure of for instance Greek or Latin. This holds for its syntax as well as for its analysis of the parts of speech, both structures which are highly relevant for the development of a formal logic.

The origins of Indian logic are invisible, but the disciplines of reasoning developed in the speculations of later Vedic texts as well as in the researches of the Sanskrit grammarians. The famous grammar of Pāṇini (probably IVth century B.C.) reflects a very high level of logical reasoning and can only be considered as the fruit of a long development, most of the traces of which are lost. The oldest logical text which has come down to us is the *Nyāya-sūtra*, which received its present form in the second or third century A.D. From then onwards an extensive logical literature was produced in India by Hindus, Buddhists and Jains. Logical techniques were adopted by some schools of philosophy and criticized and rejected by others. After a long period of logical discussions, in which the Buddhist logicians (e.g. Vasubandhu, Dīnāga, Dharmakīrti) played a large part, a process of re-orientation took place between the Xth and the XIIth century. This culminated in the gigantic work of Gaṅgeśopādhyāya (Gaṅgeśa) (XIIIth century), founder of the 'New School' (*navya-nyāya*), when logic became largely free from philosophy, epistemology and cosmology, and the attention was mainly confined to the analysis of inference (*anumāna*). Logic thus became an instrument and a method, and as such it was used in various disciplines. Soon knowledge of the logical terminology and familiarity with the techniques of logical analysis became indispensable for anybody writing on matters philosophical, grammatical,

<sup>1)</sup> S. Sen, A study on Mathurānātha's *Tattva-cintāmaṇi-rahasya*, Wageningen 1924.

<sup>2)</sup> S. Schayer, Über die Methode der Nyāya-Forschung, Festschrift M. Winternitz, Leipzig 1933, 247-57; and in other publications.

<sup>3)</sup> D. H. H. Ingalls, Materials for the study of Navya-nyāya logic, Cambridge Mass. 1951. Cf. the present author's review in *Indo-Iranian Journal* 4 (1960), 68-73.

<sup>4)</sup> I. M. Bocheński, Formale Logik, Freiburg/München, 1956, 479-517; 'Die indische Gestalt der Logik'.

ritual and scientific in general. In the following centuries a new flow of logical literature was produced, mainly in Bengal in North East India. Among the general handbooks then written mention may be made of the *Siddhānta-muktāvalī* or *Kārikāvalī-muktāvalī* of Viśvanātha Pañcānana (XVIIth century) upon which the present study is based.

Indian logic has every right to be called formal from Gaṅgeśa, and possibly from the Buddhist logicians onwards. It is formal in as far as it establishes formal rules, the validity of which depends on the structure of the sentence-expressions only. In such expressions variables occur (e.g. 'reason', 'conclusion') for which constants (e.g. 'smoke', 'fire') may be substituted. But while the presence and absence of such constants determine the validity of an empirical expression, they do not affect the validity of a logical expression.

The logical expressions are written in a kind of technical Sanskrit, where use is made of certain features of the Sanskrit language which lend themselves to a formalised treatment. Foremost among these features is nominal composition. As it is relevant in the present context to compare Sanskrit in this respect to other Indo-European languages, a recent formulation may be quoted: 'The capacity to combine independent words into compound words is inherited by Sanskrit from Indo-European, and similar formations are found in other IE languages. Sanskrit differs from the other IE languages in the enormous development which the system has undergone, which is unparalleled elsewhere'<sup>1</sup>).

We have elsewhere studied the relation between these linguistic means of expression and the logical structures<sup>2</sup>). The present paper is based upon a part of the material dealt with in that article, which mainly addresses readers who are familiar with Sanskrit. The present presentation is confined to a representation of Indian expressions by means of symbols and models of modern logic. For the Sanskrit originals the reader may be referred to the other article.

In the following use is made of the terminology of the predicate calculus with equality and in addition of the expression  $axF(x)$  denoting the idea 'x such that  $F(x)$ '. We shall make use of the property:  $(E\gamma)(\gamma = axF(x)) \leftrightarrow (E\gamma)F(\gamma)$ . If there are several values of  $x$  such that  $F(x)$ ,  $axF(x)$  may

<sup>1</sup>) T. Burrow, The Sanskrit language, London 1955, 207-8.

<sup>2</sup>) 'Correlations between language and logic in Indian thought', Bulletin of the School of Oriental and African Studies 23 (1960), 109-122.

denote any of these values: e.g.  $ax(x^2 = 4)$  may denote either  $+ 2$  or  $- 2$ . If  $(E!x)F(x)$ , there is only one  $axF(x)$  which is the same as  $(ix)F(x)$ .

Two special relations will be introduced in order to represent relationships expressed in the original text:  $A(x, y)$  meaning: 'x occurs in y', and  $B(x, y)$  meaning: 'x is the locus of y'. In addition we have:  $(x)B(x, y) \rightarrow \rightarrow (z)A(z, axB(x, y))$ .

We can now proceed to a formulation of the theory of proof. The most direct 'means of knowledge' (*pramāna*) is perception. Unfortunately, perception is not always available. Sometimes an object which is not perceptible itself can be *inferred* from a perception. For instance, we may not be able to perceive fire on a distant mountain, but we may perceive smoke; and hence conclude that there is fire because of the smoke.

If a conclusion  $s$  can be inferred from a reason  $h$  we shall write  $V(h, s)$  (with reference to the initial letters of the Sanskrit terms). A proof or inference consists in showing under which conditions  $V(h, s)$  holds. Such a proof can be applied if the validity of these conditions can be established by direct perception. Then  $V(h, s)$  is valid, and if  $h$  is perceived,  $s$  may be inferred.

It is said in the first instance that  $V(h, s)$  is valid if and only if:

- (1) there is an  $x$  such that  $x \neq s$ ;
- (2) there is a  $y$  such that  $B(y, x)$ , where for  $x$  the condition (1) holds;
- (3)  $\neg A(h, y)$ , where for  $y$  the condition (2) holds.

In other words a first definition of inference can be written as follows:

$$V(h, s) \leftrightarrow \neg A(h, ayB(y, ax(x \neq s))) \quad (\text{Def. I.})$$

Another definition interchanges the order of the conditions (2) and (3) and can accordingly be written as follows:

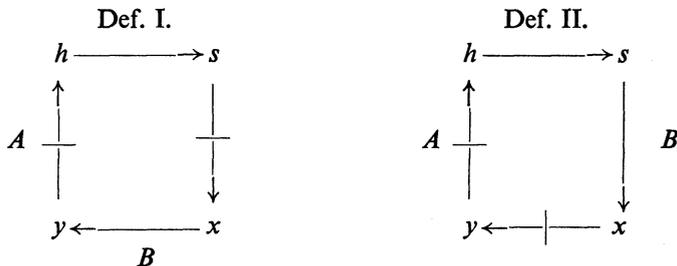
$$V(h, s) \leftrightarrow \neg A(h, ay(y \neq axB(x, s))) \quad (\text{Def. II.})$$

These formulas have been constructed in such a way that there is an isomorphism between the formulas and the Sanskrit expressions in the original. The possibility of this construction is partly due to the use of the  $a$ -terminology. Another isomorphism may be established between the formulas and the following figures.

In terms of these schemes  $V(h, s)$  is proved whenever it is possible to establish the validity of the three steps which lead from  $s$  to  $h$  in the direction indicated by the arrows. The validity in each of these three cases can be established from direct perception.

This approach is not very different from the interpretation of a part of

mathematics as a set of inferences of the form: 'if the axioms  $A_1, A_2, \dots, A_n$  are valid, the theorem  $T_k$  is valid', etc. In both cases the inference is formulated in all generality, whether the premiss is valid or not. In both cases the validity of the inference implies that the conclusion holds whenever the initial conditions or axioms hold.



EXAMPLE

$V$  (smoke, fire) is valid:

Def. I.

$ax(x \neq s)$ : absence of fire.

$ayB(y, \text{absence of fire})$ : lake.

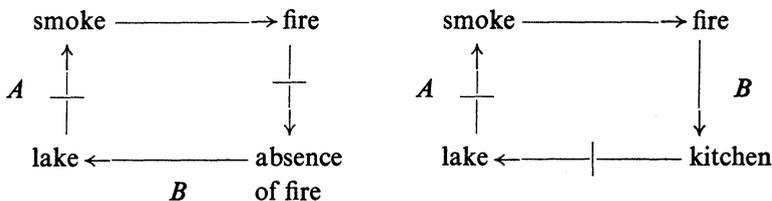
$\neg A$  (smoke, lake).

Def. II.

$axB(x, s)$ : kitchen.

$ay(y \neq \text{kitchen})$ : lake

$\neg A$  (smoke, lake).



COUNTEREXAMPLE

$V$  (fire, smoke) is invalid:

Def. I.

$ax(x \neq s)$ : non-smoke.

$ayB(y, \text{non-smoke})$ :

red-hot iron bar.

$A$  (fire, red-hot iron bar).

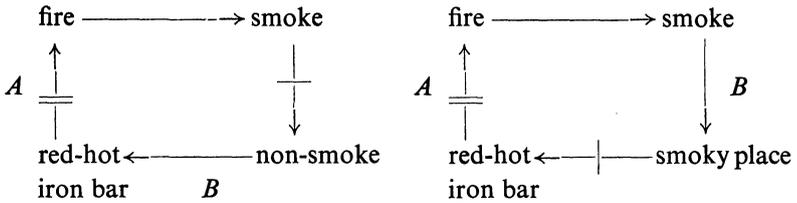
Def. II.

$axB(x, s)$ : smoky place.

$ay(y \neq \text{smoky place})$ :

red-hot iron bar.

$A$  (fire, red-hot iron bar).



After applying these and other similar definitions to many cases and submitting them to various tests, some definitions are accepted whilst others are rejected. The two definitions mentioned here are rejected for two main reasons, which will be shortly referred to.

(A) If  $(y)B(y, x)$  then:  $(z)A(z, ayB(y, x))$  or:  $(z)A(z, ayB(y, ax(x \neq s)))$ , which contradicts definition I. Similarly, if  $(x)B(x, s)$  then:  $\neg (E y)(y \neq axB(x, s))$ , which prevents the application of definition II.

Such so-called unnegatable or omnipresent terms, defined by  $(x)B(x, s)$ , are actually available: for instance 'knowable', which may also occur as a conclusion of a proof, for instance in the inference:  $V$  (nameable, knowable). Hence the definition should account for the validity of such an inference, although it is unable to do so.

(B) There are other cases where the definition does not enable us to prove a conclusion which seems to be intuitively acceptable. As the example produced by the Indian logicians involves abstruse Nyāya categories, a modern example may illustrate the difficulty. Consider again the valid inference  $V$  (smoke, fire):

$axB(x, \text{fire})$ : kitchen.

$ay(y \neq \text{kitchen})$ : my mind.

$A$  (smoke, my mind).

Here we have played a kind of trick: while 'kitchen' is undoubtedly different from 'my mind', 'smoke' must occur in 'my mind' whenever I think of smoke. Hence the inference seems to be shown to be invalid, though it should be valid. The difficulty lies in the kind of occurrence of 'smoke' in 'my mind'. What is evidently needed is a further precision of the occurrence relation  $A$ : the manner in which 'smoke' occurs in 'my mind' when I think about it is different from the manner in which smoke generally occurs, as exemplified by its occurrence (or non-occurrence) in the kitchen. Now the general place where something occurs whenever it occurs 'properly' is called its *residence* and will be denoted by  $p$ . The

additional condition, which should hold in order that the definition be valid, is that  $h$  occurs through  $A$  in the same manner in which it occurs in  $p$ . The different kinds of  $A$  can now be distinguished by means of bracketed subscripts, such as:  $A_{(x)}$ ,  $A_{(y)}$ , ... Then the  $A$  in definition I (and analogously in definition II), should be specified as follows:

$$azA_{(z)}(h, p) = azA_{(z)}(h, ayB(y, ax(x \neq s)))$$

or:

$$V(h, s) \leftrightarrow \neg A_{(azA_{(z)}(h, p))}(h, ayB(y, ax(x \neq s))).$$

If this is applied to the case of occurrence of 'smoke' in 'my mind', it is evident that:  $azA_{(z)}$  (smoke, my mind)  $\neq$   $azA_{(z)}$  (smoke,  $p$ ).

Hence:  $\neg A_{(azA_{(z)}(\text{smoke}, p))}$  (smoke,  $ay(y \neq axB(x, \text{fire}))$ ), which establishes the validity of  $V$  (smoke, fire).

There are several other insertions to the original definitions, enabling them to meet various tests. One source of difficulties is the lack of quantification, which in the above was partly expressed by the absence of quantifiers and partly by the ambiguity inherent in the expression  $axF(x)$ . Several insertions consist therefore of gradual quantifications <sup>1)</sup>. On the whole many definitions were studied and compared on their respective merits. Some were referred to by special names, such as 'the tiger', 'the lion' – the authors being nicknamed the Tiger-cub and the Lion-cub. The objection of the unnegatables applies to several definitions and does not seem to have been challenged itself: the final definition, which is accepted after all the others have been convincingly refuted, does not make use of negative expressions.

The study of Navya-nyāya logic is still in its infancy. Of the huge mass of manuscript material only a fragment has been published. Even of the considerable amount of published material only a small part is read. Yet the study of this logic is indispensable for an understanding of the later phases of Indian philosophy.

To Western logicians Indian logic may be interesting because it developed into a formal logic without being influenced by Western logic and starting from an entirely different background. In studying the problem of the

<sup>1)</sup> For instances cf. the present author's 'Means of formalisation in Indian and Western logic', Proceedings of the XIIth International Congress of Philosophy (Venice 1958). (Chapter 3 in this volume.)

universality of logical principles, or the question of the relation between logic and language, it is a great advantage to be able to look beyond the horizon of Western formal logic to the formal logic of India: ‘denn sie’, says Bocheński <sup>1)</sup>, ‘– und sie allein – bietet dem Historiker eine Möglichkeit von höchster Bedeutung, nämlich die des Vergleichs’.

<sup>1)</sup> Op. cit. 486.

MEANS OF FORMALISATION IN INDIAN  
AND WESTERN LOGIC

I. The representation of logical structures by means of Sanskrit expressions in Indian logic constitutes a formalisation which is similar to the formalisation adopted by Western symbolic logic. The various technical terms, the formation of compounds, the morphological means of expression (e. g. suffixes and case endings) and the syntactical means of expression (e. g. appositional clauses) in the technical Sanskrit of Navya-Nyāya are analogous to the terms, the formulas and the rules of modern Western logic. The following formalisation in terms of symbolic logic of a passage from a Navya-Nyāya text may enable us to study and evaluate the precise function of such Sanskrit expressions, which, when translated into a non-formalised language, can hardly be understood without referring to the original. If a sufficiently large and varied amount of material were analysed in this way, it might enable us to study the relation between logical structure and linguistic expression in general. (This might entail conclusions regarding the dependence of (concepts and results of) Navya-Nyāya upon Sanskrit and of symbolic logic upon some modern languages.) Ultimately it might contribute to the solution of the problem in how far and in which way the structure of Indo-European languages is related to Indian and Western thought.

In the study of Navya-Nyāya symbols of modern formal logic have been adopted by Sen<sup>1</sup> and by Ingalls<sup>2</sup>. Bocheń-

<sup>1</sup> S. SEN, *A study on Mathurānāthā's « Tattva-Cintāmani-Rahasya »*, Wagningen, 1924. The studies by ST. SCHAYER were not accessible to me.

<sup>2</sup> D. H. H. INGALLS, *Materials for the study of Navya-Nyāya logic*, Cambridge, Mass., 1951. Review by I. M. BOCHENSKI, in «The Journal of Symbolic Logic», 17 (1952), 118-19. The reviewer's opinion, that Indian

ski<sup>1</sup> introduced formulas in a first attempt to formalise the five definitions of *vyāpti* of the *VPR*\*, which were analysed by Sen and by Ingalls. As this formalisation does not show in which way the definitions occur and are handled in the argument of the text, it cannot be called entirely successful as yet, whereas it cannot be used to study the functioning of the Sanskrit expressions. From an entirely different standpoint the last question has been dealt with for technical Sanskrit in general by Hartmann<sup>2</sup>. The author frequently quotes the *SM*, but does not analyse expressions characteristic for formal logic, nor include Ingalls' work in his selection of sources<sup>3</sup>.

It will be now attempted to formalise and analyse the four « insertions » resulting in four improvements of the first definition of *vyāpti* as discussed in the *pūrvapakṣavyāptinirūpanam* of the *anumānakhaṇḍa* of the *SM*<sup>4</sup>. The *SM* is an introductory work, less difficult than the *VPR* and nowadays studied in India by beginning students of Navya-Nyāya. It was written in the XVIIth century by Viśvanātha Pañcānana Bhaṭṭācārya as a commentary upon his own work, the *BP*. The four insertions, dealt with here as assumptions 1 and 2 and objections 1 and 2, occur in the text in the order: ass. 1, obj. 1, ass. 2 and obj. 2.

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logic « never rose to the understanding of the difference between connotation and denotation » can be shown to be incorrect: see J. BROUGH, *Theories of general linguistics in the Sanskrit grammarians*, « Transactions of the Philological Society » (1951), 27-46 and cfr. M. B. EMENEAU, *India and linguistics*, « Journal of the American Oriental Society », 75 (1955), 151.

<sup>1</sup> I. M. BOCHENSKI, *Formale Logik*, Freiburg-München, 1956, 512. Cfr. Id., *L'état et les besoins de l'histoire de la logique formelle*, « Proc. Xth Int. Congr. of Philosophy », Amsterdam, 1949, 1062-64.

<sup>2</sup> P. HARTMANN, *Nominale Ausdrucksformen im wissenschaftlichen Sanskrit*, Heidelberg, 1955. Review by J. GONDA, in « Lingua », 5 (1955-56), 441-43.

<sup>3</sup> Hence, for instance in the discussion of abstract nouns ending in *-tva* and *-tā*, no reference has been made to the occurrence in logical Sanskrit — and probably, as a systematical device, in Sanskrit alone — of a generalisation of this procedure through forms ending in *-tvatva* and *-tātva*, denoted by means of subscripts by SEN and INGALLS.

<sup>4</sup> Ed. Hari Rāma Śukla, Banaras, 1951, 216-19. Cfr. transl. E. Röer, Calcutta, 1850, 31; O. Strauss, Leipzig, 1924, 54-56; Svāmī Mādhavānanda, Almora, 1954, 109-11.

\* Abbreviations: *BP* = *Bhāṣā-Pariccheda*; *SM* = *Siddhānta-Muktāvalī*;  
*VPR* = *Vyāpti-Pañcaka-Rahasya*.

II. We shall use familiar symbols of binary predicate logic and, in addition, the expression  $\alpha x R(x)$  (or its abbreviation  $\alpha x R$ ) for a *restricted-variable* denoting the idea «  $x$  such that  $R(x)$  »<sup>1</sup>. We shall make use of properties of this terminology, in particular of:

$$(\alpha x R(x)) Q \} \alpha x R(x) \} \longleftrightarrow (y) \} R(y) \longleftrightarrow Q (y) \} \}^2.$$

Attempts will be made to define the binary predicate  $V(h, s)$ , *vyāpti*, « pervasion », which holds between  $h$ , *hetu*, « reason » and  $s$ , *sādhyā*, « the thing to be inferred ». We know, for instance, that  $V(\text{smoke}, \text{fire})$ , but  $\neg V(\text{fire}, \text{smoke})$ .  $V$  will be defined in terms of two binary predicates  $A$  and  $B$ .

$A(x, y)$  stands for *sambandha*  $x_6 y_7$ , « relation of  $x$  to  $y$  », or *y-vr̥ttitva*  $x_6$ , « presence of  $x$  in  $y$  » (subscripts denote case endings in Pāṇini's order). We know:  $A(\text{fire}, \text{red-hot iron bar})$ ,  $A(\text{smoke}, \text{kitchen})$ ,  $A(\text{smoke}, \text{hill})$ , but  $\neg A(\text{smoke}, \text{lake})$ ,  $\sim A(\text{fire}, \text{lake})$ .

$B(x, y)$  stands for  $y_1 x_7$ , «  $y$  is in  $x$  », or  $x_1 y\text{-vat}$  (-*mat*), «  $x$  has  $y$  »<sup>3</sup>. We know:  $B(\text{kitchen}, \text{fire})$ ,  $B(\text{hill}, \text{fire})$ .

DEFINITION 1:  $V(h, s) \longleftrightarrow \neg A[h, \alpha y \} y \neq \alpha x B(x, s) \} \}.$

Text: *vyāptih sādhyavadanyasminnasambandha udāhṛtah*. Analysis: *sādhyā*:  $s$ ; *sādhyavad* or *sādhyādhikaraṇa*:  $\alpha x B(x, s)$ ; *sādhyavadanya*:  $\alpha y \} y \neq \alpha x B$ ; *sādhyavadanyasminnasambandha* or *sādhyavadanyavṛttitva* (scl. *hetoh*) (the first expression *metri causa*, as the *BP* is written in *anuṣṭubh* verses):  $A[\dots]$ ; *sādhyavadanyasminnasambandha* or *sādhyavadanyāvṛttitva* (scl. *hetoh*):  $\neg A[\dots]$ .

EXAMPLE:

- $\neg A$  (smoke, lake)
- $\neg A$  (smoke, a non-fiery place)
- $\neg A$  [smoke,  $\alpha y \} y \neq$  a fiery place]
- $\neg A$  [smoke,  $\alpha y \} y \neq \alpha x B(x, \text{fire}) \} \}$
- $V$  (smoke, fire).

<sup>1</sup> See: T. HAILPERIN, *A theory of restricted quantification*, « The Journal of Symbolic Logic », 22 (1957), 19-35; 113-29. For another symbolism cfr. J. B. ROSSER, *Logic for Mathematicians*, New York, 1953, 140-50.

<sup>2</sup> This is axiom QR4 of HAILPERIN's System QR (*op. cit.*, 23).

<sup>3</sup> Cfr. HARTMANN, 47-49; INGALLS, 43.

COUNTEREXAMPLE: A (fire, red-hot iron bar)  
 A (fire, a non-smoky place)  
 A [fire,  $\alpha y \} y \neq \alpha x B$  (x, smoke)\{]  
 $\sim V$  (fire, smoke).

ASSUMPTION 1: Assume  $B = S_1$ , where  $S_1$  (*samavāya-saṃbandha*) is defined by:  $S_1$  (parts of fire, fire) (text: *samavāya-saṃbandhena vahnimān vahneravayavaḥ*).

We have:  $V$  (smoke, fire)  
 (def. 1):  $\sim A$  [smoke,  $\alpha y \} y \neq \alpha x B$  (x, fire)\{]  
 (ass. 1):  $\sim A$  [smoke,  $\alpha y \} y \neq \alpha x S_1$  (x, fire)\{]  
 $\sim A$  [smoke,  $\alpha y \} y \neq$  parts of fire\{].  
 $\sim A$  (smoke, kitchen).

But we know: A (smoke, kitchen). Hence  $B \neq S_1$ .

Thereupon B is defined as follows:

DEFINITION 2:  $B \longleftrightarrow (\cdot R) R$  (p, s), where p stands for *pakṣa*, «subject». Text: *yena saṃbandhena sādhyam, tena saṃbandhena sādhyavān*<sup>1</sup>.

ASSUMPTION 2: Assume  $A = S_2$ , where  $S_2$  (*samavāya-saṃbandha*) is defined by:  $S_2$  (smoke, parts of smoke) (text: *dhūmavayave dhūmasya samavāya-saṃbandha*; both  $S_1$  and  $S_2$  are *samavāya-saṃbandha* S, for S is a symmetrical relation, i. e.  $S(x, y) = S(y, x)$ <sup>2</sup>).

We have:  $V$  (smoke, fire)  
 (def. 1):  $\sim A$  [smoke,  $\alpha y \} y \neq \alpha x B$  (x, fire)\{]  
 (ass. 2):  $\sim S_2$  [smoke,  $\alpha y \} y \neq \alpha x B$  (x, fire)\{]  
 $\sim S_2$  [smoke,  $\alpha y \} y \neq$  kitchen\{]  
 $\sim S_2$  (smoke, parts of smoke).

This contradicts the definition of  $S_2$ , hence  $A \neq S_2$ .

Thereupon A is defined as follows:

DEFINITION 3:  $A \longleftrightarrow (\cdot R) R$  (h, p).

Text: *yena saṃbandhena hetutā tenaiva saṃbandhena sādhyavadanyāvṛttitvam*.

<sup>1</sup> On appositional clauses expressing identity cfr. HARTMANN, 153. See also L. RENO, in « Journal Asiatique », 245 (1957) 125, 126, discussing *sāmarthya* as used by Śaṅkara, *Brahmasūtrabhāṣya*, l. l. 24, commenting upon CU 3. 13. 7. For the background (the idea of «inclusion») see J. GONDA, *The original character of the Indo-European relative pronoun io-*, «Lingua», 4 (1954-55), 1-41.

<sup>2</sup> Cfr. INGALLS, 75; A. FOUCHER, *Le compendium des topiques*, Paris, 1949, 168-71; *Tarkasamgraha*, ed. Y. V. Athalye, Poona, 1930, 96-99.

OBJECTION 1: We have: V (smoke, fire), and hence (2).

But also: B (kitchen, fire), hence:

$\neg$  A [smoke,  $\alpha y$ ]  $y$  = kitchen { }.

We have: hill  $\neq$  kitchen, hence:  $\neg$  A (smoke, hill) (3).

But we know: A (smoke, hill). Hence def. 1 is invalid. We adopt:

DEFINITION 4:  $V(h, s) \leftrightarrow (\alpha xB) \neg A [h, \alpha y] y \neq \alpha xB(x, s)$  { } { }

Text: *sādhyavadanyaśca sādhyavattvāvachchinnapratiyogitākābhāvān.*

Analysis: *sādhyavadanya*:  $\alpha y$  { }....{ as before; the *pratyogitā* refers to the negation in *bheda* (or *anya*):  $\neq$ ; *sādhyavattvāvachchinnā*: generalisation of the restricted-variable  $\alpha xB$  (negated under this *pratyogitā*) in *sādhyavattva*; *sādhyavattva*: occurrence of *sādhyavad*; *sādhyavad*:  $\alpha xB(x, s)$  as before.

SOLUTION OF OBJECTION 1: We have not only: B (kitchen, fire), but (def. 4) also: B (hill, fire), etc., and hence we cannot infer (3).

OBJECTION 2: Assume: V (fire, smoke) (4)

(def. 4):  $(\alpha xB) \neg A$  [fire,  $\alpha y$ ]  $y \neq \alpha xB(x, \text{smoke})$  { } { }

or:  $\neg A$  [fire,  $\alpha y$ ]  $y \neq$  a smoky place { }, hence  $\neg A$  (fire, lake) (5).

This holds true, but (4) should be expected to be incorrect. Hence def. 4 is invalid. We adopt:

DEFINITION 5:  $V(h, s) \leftrightarrow (\alpha yD)(\alpha xB) \neg A [h, \alpha y] y \neq \alpha xB(x, s)$  { } { }, where D abbreviates ( $y \neq \alpha xB$ ).

Text: *sādhyavadanyāvṛttitvam ca sādhyavadanyāvṛttitvatvāvachchinnapratiyogitākābhāvāḥ.* Analysis: *sādhyavadanyāvṛttitva*:

$\neg A$  [....] as before; the *pratyogitā* refers to the negation in *abhāva* (or *avṛttitva*):  $\neg$ ; *sādhyavadanyāvṛttitvatvāvachchinnā*: generalisation of the restricted-variable  $\alpha yD$  (negated under this *pratyogitā*) in *sādhyavadanyāvṛttitvatva*; *sādhyavadanyāvṛttitvatva*: occurrence of *sādhyavadanyāvṛttitva*; *sādhyavadanyāvṛttitva*: A [....] as before.

SOLUTION OF OBJECTION 2: (4) implies (5), but (def. 5) also:  $\sim A$  (fire, red-hot iron bar). But we know: A (fire, red-hot iron bar), hence there is a contradiction. Hence (4) is incorrect.

Definitions 2 and 3 may be substituted in 5. With the help of (1), definitions 4 and 5 can be written as:

DEFINITION 4\*:  $V(h, s) \leftrightarrow (u) [B(u, s) \rightarrow \neg A] h, \alpha y (y \neq u)$  { }.

DEFINITION 5\*:  $V(h, s) \leftrightarrow (u) [B(u, s) \rightarrow (v)] (u \neq v) \rightarrow \neg A(h, v)$  { }.

If  $V(h, s)$  were to denote  $h \rightarrow s$ , these definitions would be incorrect, as the *definiendum* occurs in the *definiens*.

III. It may be premature to draw many conclusions before other passages will have been formalised and analysed. Along the above lines a formalisation could be effectuated of several insertions to the first definition of the *VPR*. Here the second negation applies to the *sādhya* itself, i. e. the definition is of the form:  $\neg A \} h, \alpha xB (x, \neg s)\{$ : *sādhyaḥbhāvavadavṛttitva*. Thereupon the exact significance of *kevalānvayitva* could be assessed and the conclusion, that the five definitions have defined *avyabhicāritatva* in stead of *vyāpti*, could be drawn. The present formalisation may be concluded with four remarks.

From the analysis of the Sanskrit expressions of the definitions it can be seen how far more powerful is symbolic representation of logical structures than natural language. At the same time we reach a level ourselves from which it becomes possible to admire the acuteness and penetration of Indian logicians. We now see how exactly the term *avacchinna* expresses generalisation, a fact noted already by Ingalls<sup>1</sup>. We also observe how in Indian logic quantification is arrived at and expressed without the help of variables.

The gradual improvement of the original definition corresponds in modern logic to the introduction of universal quantifiers. It is reached through the creation of counterexamples by means of substitutions of individual constants (smoke, fire). The old Nyāya has often been blamed for violating a principle the schoolmen referred to as: *exempla non probant*. It is well known that the modern school rejected this old view. The actual discovery by Navya-Nyāya in this respect may lie in the recognition, that examples cannot prove, but that counterexamples can refute. Hence their skill in handling them goes partly back to the old school. Also in modern Western logic the importance of the counterexample is great. It has been even shown how a general method of deduction can be based upon it<sup>2</sup>.

It is for the same reason that one cannot entirely agree with Hartmann's conclusion regarding the abstractive power of no-

<sup>1</sup> *Op. cit.*, 49-50.

<sup>2</sup> See E. W. BETH, *Semantic entailment and formal derivability*, « Med. Kon. Ned. Akad. v. Wet. », New Series, XVIII, 13, Amsterdam, 1955.

minimal structures in technical Sanskrit, « dass bei dem betonten Blick auf das Begriffliche, Allgemeine, das Bewusstsein der konkreten Sachlage ungebührlich weit zurücktreten muss, wenn diese nicht bei jeder einzelnen abstrakten Nennung oder Aussage deutlich und mit allen Vorgängen vergegenwärtigt wird »<sup>1</sup>. In Nyāya (which, in philosophy, holds a realistic position), on the other hand, the facility with which a concrete individual (« fire ») can be substituted for a technical term (« *sādhyā* ») safeguards the possibility for abstractions to be applied in concrete cases.

Of course, it should be possible to give an enumeration of all the logical laws of which the above derivations make use. It seems likely that the term *parāmarsā*, enunciated prior to the first definition and translated in various ways, corresponds to the modern idea of substitution. Attention may be drawn to the application of the laws of contradiction and of the excluded third (in assumptions 1 and 2), as it is sometimes suggested that Indian thought might discard such laws (e. g. when dealing with concepts such as *anirvacanīya* or *catuṣkoṭi*). But these laws are not only regularly applied by the logicians (in accordance with Aristotle's argument, that the denial of the law of contradiction renders all intelligible discourse impossible<sup>2</sup>, but they are also utilised by the metaphysicians. Śaṅkara himself says<sup>3</sup>: different Vedānta texts cannot teach different cognitions of Brahman, for it is certain that, if at all they differ, « only one of them is the right one, the others are erroneous » (*teṣāmekamabhrāntaṃ bhrāntānitarāni*)<sup>4</sup>.

Reynier Vinkeleskade 74  
AMSTERDAM (The Netherlands)

<sup>1</sup> *Op. cit.*, 202.

<sup>2</sup> *Met.*, Γ, 4, 1006 a, 18-22.

<sup>3</sup> *Brahmasūtrabhāṣya*, 3. 3. 1.

<sup>4</sup> My teacher Prof. Dr. E. W. BETH (University of Amsterdam) suggested some improvements to an earlier draft of this paper, which I gratefully took into account. I should also like to express my gratitude to Prof. DINESH CHANDRA GUHA, M. A., Kāvya-Nyāya-Tarka-Vedānta-tīrtha (College of Indology, Banaras Hindu University), for the inspiring enthusiasm with which he initiated me into Navya-Nyāya at Banaras during 1956-57.

THE THEORY OF DEFINITION IN INDIAN LOGIC

1. THE THEORY OF DEFINITION according to the Indian logicians has been repeatedly studied in a wider context. Foucher has given a description of its treatment in the *Tarka-saṃgraha*, a well-known manual of Indian logic written in the seventeenth century, and in some of its commentaries;<sup>1</sup> Ingalls has studied its use in the new school (*navya-nyāya*);<sup>2</sup> and Mile. Biardeau has considered its place in the development of Indian thought in general.<sup>3</sup> In the present paper a formalization will be given of the theory of definition according to its exposition in the *Tarka-saṃgraha* and in some definitions quoted by the *Nyāya-kośa* (NK) from various logical works.<sup>4</sup> In addition we will consider a related concept of Indian logic, expressed by the term *saṃkara*.

The terms used for definiens (*lakṣaṇa*) and definiendum (*lakṣya*) in logic as well as the idea of definition have an interesting prehistory in the work of the Sanskrit grammarians, to which we direct our attention here before considering the logical doctrines. Such a grammatical prehistory of a logical doctrine is not surprising as grammar

was the first discipline in India which developed a scientific technique which in turn influenced or foreshadowed other domains of Indian thought.<sup>5</sup>

In the *Ākṣapṛāśikhya* (? fourth century B. C.) *lakṣaṇa* refers to a grammatical rule.<sup>6</sup> In Kātyāyana's *vārttikas* (? third century B. C.) as quoted by Patañjali in the *Mahābhāṣya*, *lakṣaṇa* and *lakṣya* denote respectively a grammatical rule and its object: *śabdo lakṣyaḥ sūtraṃ lakṣanam* "the word is the object of the rule, the *sūtra* (of Pāṇini) is the rule." It is said, then, that it is the task of grammar to deal with both *lakṣaṇa* and *lakṣya*.<sup>7</sup> Actually, Pāṇini's grammar (fourth century B. C.) owes part of its fame to the fact that the rules in it do not prescribe how to speak correct Sanskrit, but describe the facts of the language; the examples (*dṛṣṭānta*) for the rules are taken from everyday usage (*loka*). The underlying principle of empiricism was laid down explicitly only much later. Kṣīrasvāmin (twelfth century A. D.) formulates it as: *lakṣyamūlaṃ lakṣaṇam* "lakṣaṇa has lakṣya for its base." Nāgojibhaṭṭa says: *lakṣyānusāri vyākhyānam* "interpretation

<sup>1</sup> A. Foucher, *Le compendium des topiques* (Paris, 1949) 7-14.

<sup>2</sup> D. H. H. Ingalls, *Materials for the study of Navya-nyāya logic*, HOS 40 (Cambridge, Mass., 1951) especially 80-1.

<sup>3</sup> M. Biardeau, "La définition dans la pensée indienne," *Journal Asiatique* 245 (1957) 371-84.

<sup>4</sup> MM. Bhīmācārya Jhalakikar, *Nyāya-kośa* (Poona, 1928) s. vv. *lakṣaṇa*, *ativyāpti*, *avyāpti*, *asaṃbhava*, *saṃkara*.

<sup>5</sup> See especially L. Renou, "Connexions entre le rituel et la grammaire en sanskrit," *Journal Asiatique* 233 (1941-42) 105-65; *id.*, "Grammaire et Vedānta," *Journal Asiatique* 245 (1957) 121-33.

<sup>6</sup> *Ākṣapṛāśikhya* 13. 31 quoted by L. Renou, *Terminologie grammaticale du sanskrit* (Paris, 1957) 483.

<sup>7</sup> Quoted by Renou, *Terminologie* 261. Cf. D. S. Ruegg, *Contributions à l'histoire de la philosophie linguistique indienne* (Paris, 1959) 33; cf. the present author in: *Philosophy East and West* 10 (1960) 54.

follows *lakṣya*.”<sup>8</sup> For a grammatical rule to be correct, it has to conform to usage. This idea has been further developed in Indian musical theory, where *lakṣaṇa* and *lakṣya* come to denote theory and practice respectively and where it is stressed that the actual practice of musicians is the only sound base for a theory of music. For instance, Rāmāmāyā (fifteenth century A. D.) says in the beginning of his musical treatise *Svaramelakalānidhi*:

*tasmāl lakṣya pradhānatvaṃ kṛtvā vakṣyē'sya lakṣanam*

“therefore I shall speak about the *lakṣaṇa* (of music) basing myself upon (its) *lakṣya*.”<sup>9</sup>

The Sanskrit grammarians made use of the idea of definition in dealing with the technical terms of grammar. A technical term is called *saṃjñā*, and a *sūtra* of Pāṇini which introduces such a term by definition is called a *saṃjñāsūtra*. The denotation of the *saṃjñā* is called the *saṃjñin*. For instance, in the first rule of Pāṇini's grammar: *vr̥ddhir ādaic “ā, ai and au* are called *vr̥ddhi*,” the three vowels constitute the *saṃjñin* and the technical term *vr̥ddhi* introduced to denote these is the *saṃjñā*.<sup>10</sup> Pāṇini is aware of the fact that the technical terms occupy a special position in his grammar in that they denote something else (*i. e.*, the *saṃjñin*), whereas other words which are quoted in rules do not denote the object to which they refer in ordinary usage, but denote their “own form.” Hence a word occurring in a rule cannot generally be replaced by any of its synonyms, but a *saṃjñā* occurring in a rule has to be

replaced by its *saṃjñin*. This is expressed in the rule: *svaṃ rūpaṃ śabdasyāśabdasamjñā* “a word (in a grammatical rule) which is not a *saṃjñā* denotes its own form.”<sup>11</sup>

The technical terms belong to the metalanguage, otherwise consisting of the *paribhāṣā* rules which are rules of interpretation or rules which indicate how the rules of grammar have to be manipulated. A typical *paribhāṣā* rule is a rule which discusses the relative strength of various *sūtras* or enunciates general principles regarding the order in which *sūtras* have to be applied. The status of a *paribhāṣā* corresponds to the status of a metatheorem in modern logic.

These two types of element of the metalanguage, *saṃjñā* and *paribhāṣā*, are combined in two statements regarding the scope of their applications. These two statements are themselves *paribhāṣā*-rules.<sup>12</sup> The first is: *yathoddeśaṃ saṃjñāparibhāṣam* “*saṃjñā* and *paribhāṣā* (apply) where they are taught.” The second: *kāryakālaṃ saṃjñāparibhāṣam* “*saṃjñā* and *paribhāṣā* (apply) at the time the operations (to which they apply take place).” The first enunciation limits the applicability of a *saṃjñā* or *paribhāṣā* to its immediate vicinity, *i. e.*, to the immediately preceding and following rules. The second, on the other hand, declares the universal validity of a *saṃjñā* or *paribhāṣā*, whereby the latter fully acquires the character of a metatheorem. There is no general pronouncement as to the definite choice between these two alternatives, for this decision depends on each case individually. After Pāṇini the problem has become to find in each particular case of word formation a *paribhāṣā* which enables us to manipulate the *lakṣaṇa* of Pāṇini's *sūtras* in such a way that the result of their application conforms to the *lakṣya*. Kielhorn discusses examples of this in connection with the above two *paribhāṣā* rules in his translation of the *Paribhāṣendūśekhara*.<sup>13</sup> This *rapprochement* of *lakṣaṇa* to *lakṣya* is not obtained by changing the form of the *lakṣaṇa* of Pāṇini's permanent rules, but by changing the interpretation. Similarly, apparently superfluous

<sup>8</sup> Quoted by Renou *ibid.* Cf. also L. Renou, *La Durghaṭavṛtti de Saraṇadeva* (Paris, 1940) I 1, 136. The philosopher Śaṅkara (eighth century A. D.) utilizes an analogous terminology when he speaks about the subordination of reason to revelation: *āgamānusāri-tarka* “reasoning following scripture” (*Brahma-sūtra-bhāṣya* 2.1.11). In Advaita, revelation is to logic in the supra-sensible realm what sense-perception is to logic in the realm of ordinary experience. Inference is not based upon experience or revelation, for it is a separate means of knowledge, but it depends for its application on the data of experience or revelation.

<sup>9</sup> *Svaramelakalānidhi* 2.17 ed. M. S. Ramaswami Aiyar (Annamalai, 1932) 9.

<sup>10</sup> In the formulation the general practice is that the *saṃjñin* (the predicate of the sentence) precedes the *saṃjñā* (the subject). The above rule forms an exception *maṅgalārtham* “for the sake of auspiciousness,” for it enables Pāṇini to commence his grammar with an auspicious word meaning “growth, increase.”

<sup>11</sup> Pāṇini 1.1.68: translated and lucidly discussed by J. Brough, “Theories of general linguistics in the Sanskrit grammarians,” *Transactions of the Philological Society* (1951) 28 ff. This shows that the Indians distinguished clearly between connotation and denotation (cf. the present author in: *Indo-Iranian Journal* 4 [1960] 72).

<sup>12</sup> Nrs. 2 and 3 in Nāgōjibhaṭṭa's *Paribhāṣendūśekhara*.

elements in Pāṇini's *sūtras* are not deleted but are pronounced *jñāpaka* rules which disclose *lakṣya* material to which the *lakṣaṇa* is adjusted. Here is a science which is at the same time empiric in that it respects and depends on *lakṣya*, as also dogmatic in that it adheres to the *lakṣaṇa* which was once established.

A *paribhāṣā* is often called *nyāya*,<sup>14</sup> the term which later denotes (the system of) logic. Likewise the *paribhāṣā* rules of the Mīmāṃsā, which are often rules for the interpretation of the ritual *sūtras*,<sup>15</sup> are called *nyāya*.<sup>16</sup> Renou has rightly stressed that both Mīmāṃsā and Vedānta "visent à donner un enseignement *pāribhāṣika*, c'est-à-dire à constituer une herméneutique, portant sur les portions du Veda auxquelles ils confèrent un rôle privilégié."<sup>17</sup> The early use of the term *nyāya* as synonymous with *paribhāṣā* may be an indication that elements and laws of Indian logic can be traced back to the discussion on problems of language and metalanguage in earlier systems and especially in grammar.

2. After this prelude let us consider the theory of definition in the later logical doctrines. The theory deals with the various relations which may hold between the definiens (*lakṣaṇa*, to be represented by L) and the definiendum (*lakṣya*, to be represented by l.) These relationships may be formalized with the help of a Boolean algebra, where letters denote classes and where the notions of *null-class*, *complement*, *product*, *equality* and *inequality* are defined as follows:

- DEF. 1. 0, the null-class, denotes the class to which no elements belong.
- DEF. 2.  $\bar{x}$ , the complement of x, denotes the class consisting of the elements which are not elements of x.
- DEF. 3.  $x.y$ , the product of x and y, denotes the class consisting of the elements which are elements of x and of y.

- DEF. 4.  $x = y$  means: every element of x is an element of y and vice versa.
- DEF. 5.  $x \neq y$  means: not every element of x is an element of y and vice versa.

With the help of these notations the logical content of the Sanskrit expressions can be represented. In order to obtain a formalization which is close to the structure of the Sanskrit expressions, we shall in addition make use of a relation A which holds between two classes x and y in the following manner:<sup>18</sup> A(x, y) means: "there is occurrence (*vṛttitva*, *sattva*) of x in y." Hence A(x, y) implies:  $x.y \neq 0$ , and  $\sim A(x, y)$  is equivalent to  $A(\bar{x}, y)$ .

3. The *Tarka-dīpikā* defines a valid definiens by: *lakṣyatāvachedaka-samanīyatatvam* "co-extensiveness (of L) with the limitor of the class of l,"<sup>19</sup> which may be interpreted as:  $L = l$ . Another definition for L is given by the *Tarka-bhāṣā* in the following terms: *dūṣaṇa-traya-rahito dharmah* "a property which is free from the three defects."<sup>20</sup> In each of these three defects we may presuppose that  $L \neq l$ , for this constitutes the definition of defect. The three defects referred to here are *ativyāpti* "overpervasion," *avyāpti* "nonpervasion" and *asambhava* "impossibility." They are defined as follows:<sup>21</sup>

(1) *alākṣya-vṛttitvam* (scl. *lakṣaṇasya*) *ativyāptih* "overpervasion is occurrence (scl. of L) in the complement of l," which may be formalized as:  $A(L, \bar{l})$  or:  $L.\bar{l} \neq 0$ . Elsewhere<sup>22</sup> this definition is further extended with the expression: *lakṣya-vṛttitve sati* "while (L) occurs in l," which may be formalized as:  $A(L, l)$  or:  $L.l \neq 0$ . Hence the

<sup>18</sup> Cf. the present author's "Correlations between language and logic in Indian thought," *Bulletin of the School of Oriental and African Studies* 23 (1960) 109-22, and: "Formal structures in Indian logic," *Synthese. An international quarterly for the logical and psychological study of the foundations of science* 12 (1960) 279-86.

<sup>19</sup> Quoted NK 696 and Foucher *op. cit.* 10. The term *avachedaka* is utilized on account of the occurrence of the abstract (*lakṣyatā*). Cf. the present author's "Means of formalisation in Indian and Western logic," *Proceedings of the XIIth International Congress of Philosophy* (Venice, 1958). Cf. also A. Uno, "The determination of terms in *Navya-nyāya*," *Journal of Indian and Buddhist Studies* 7 (1958) 61-5.

<sup>20</sup> NK *loc. cit.*

<sup>21</sup> *Tarka-dīpikā* ad *Tarka-samgraha* 4. Cf. also *Tarka-kamudī* quoted NK 99.

<sup>22</sup> Cf. Foucher *op. cit.* 11.

<sup>13</sup> Bombay 1868, 11 ff.

<sup>14</sup> Kielhorn, *op. cit.* iv, n. 1.

<sup>15</sup> There were ritual *sūtras* possessing their own *paribhāṣā* sections (e. g., *Āpastamba-śrauta-sūtra* 24). Cf. L. Renou, *Les écoles védiques et la formation du Veda* (Paris, 1947) 172.

<sup>16</sup> See e. g., D. V. Garge, *Citations in Sabara-bhāṣya* (Poona, 1952) 252-65. Cf. Renou in *Journal Asiatique* 233 (1941-42) 119 n. 2.

<sup>17</sup> L. Renou, *Études védiques et pāṇinienes* VI (Paris, 1960) 54.

definition of *ativyāpti* is:  $L.\bar{l} \neq 0 \wedge (L.l \neq 0)$ .

EXAMPLES: *goḥ śṛṅgīlvaṃ* "cows are horned beasts," or: *manuṣyo brāhmaṇaḥ* "Brahmans are human beings."

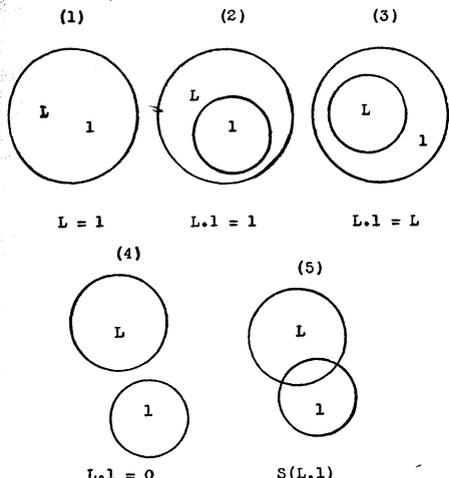
(2) *lakṣyaikadeśāvṛttitvaṃ* (scl. *lakṣaṇasya*) *avyāpīṭh* "non-pervasion is non-occurrence (scl. of L) in a part of l," which may be formalized as:  $\sim A(L,l)$  or:  $A(L,l)$  or:  $L.l \neq 0$ . Though this definition is not further explained by an additional expression, it contains itself the term *-ekadeśa* "part of" which seems to imply that L does occur in another part of l, i. e., that  $A(L,l)$  or:  $L.l \neq 0$ . Hence the definition of *avāpti* is:  $(L.l \neq 0) \wedge (L.l \neq 0)$ .

EXAMPLES: *gor nīlarūpavattvaṃ* "cows are black beasts," or: *śikhāsūtravān brāhmaṇaḥ* "Brahmans are wearing a tuft and a thread."

(3) *lakṣyamātrāsattvaṃ* (scl. *lakṣaṇasya*) *asambhavaḥ* "impossibility is non-occurrence (scl. of L) in the whole of l," which may be formalized as:  $L.l = 0$ .

EXAMPLES: *gor ekaśapatvaṃ* "cows are solid-hoofed beasts," or: *śuṅḍadaṇḍavān brāhmaṇaḥ* "Brahmans are wearing an elephant's trunk."

The question arises whether this classification is exhaustive. In general there are five possibilities for the relations which may hold between the classes L and l:



where  $S(L,l)$  means: "L overlaps l," i. e.:

$$(L.l \neq 0) \wedge (L.l \neq L) \wedge (L.l \neq 0)$$

If *ativyāpti* denotes merely  $L.l = 1$  and *avyāpti* denotes merely  $L.l = L$ , case (5) would have been neglected by the Indian logicians. Actually the examples given to illustrate *avyāpti* are of the type (5). The first example is explained by the *Nayāya-bodhinī*<sup>23</sup> by observing that there are cows which are not black beasts. But it could equally well be noted that there are black beasts which are not cows. In other words, the example is a case of  $S(L,l)$ . The second example seems at first sight to constitute a good example of a valid definition. But the commentator explains: there are Brahmins who do not wear a tuft and a thread, e. g., *sannyāsins*. In this case also it could be noted that there are *kṣatriyas* who do wear a tuft and a thread, so that this example too illustrates overlapping.

Cases of overlapping are actually referred to as being defects because of both *avyāpti* and *ativyāpti*.<sup>24</sup> Hence  $S(L,l)$  is defined as:

$$[(L.\bar{l} \neq 0) \wedge (L.l \neq 0)] \wedge \wedge [(L.l \neq 0) \wedge (L.l \neq 0)]$$

or:

$$(L.\bar{l} \neq 0) \wedge (L.l \neq 0) \wedge (L.l \neq 0)$$

4. Elsewhere in Indian logic this concept is actually defined and exemplified. One of the definitions of *saṃkara* "overlapping" quoted by the NK<sup>25</sup> is the following: *parasparātyantābhāva-sāmānādhikarāṇye sati jātyantareṇa sāmānādhikarāṇyam* "that which possesses common-locusness with another generic character, while there is common-locusness of mutual relational absence."<sup>26</sup> Another definition speaks analogously of properties: *parasparātyantābhāva-sāmānādhikarāṇayor dharmayor ekatra samāveśaḥ* "the coming together in one place of two properties, which possess common-locusness of mutual relational absence." Both definitions mean that of two classes, which mutually occur in each other's absence, the one also occurs in the other. Or, further explained: of two

<sup>23</sup> *Ibid.*

<sup>24</sup> I am grateful to Professor Brough for calling my attention to this.

<sup>25</sup> NK 901. An ancient practice in the Sāmaveda was the "overlapping" of *ṛks* in order to form a *sāman*: see the present author's *Nambudiri Veda Recitation* ('s-Gravenhage, 1961) 72-3.

<sup>26</sup> For "relational absence" cf. the present author in: *Indo-Iranian Journal* 4 (1960) 71.—The treatment of the example in the NK seems to show that the term *paraspara* "mutual" in the above expression does not refer to *anyonyābhāva* "mutual absence."

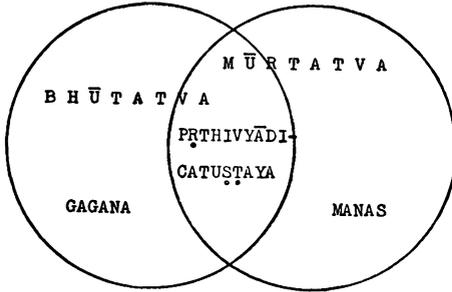
classes  $x$  and  $y$ ,  $x$  occurs in the complement of  $y$ ,  $y$  occurs in the complement of  $x$ , and  $x$  occurs in  $y$ . This may be formalized as follows:

$$A(x, \bar{y}) \wedge A(y, \bar{x}) \wedge A(x, y)$$

or:

$$(x.\bar{y} \neq 0) \wedge (\bar{x}.y \neq 0) \wedge (x.y \neq 0).$$

The example given corroborates this interpretation.<sup>27</sup> The relation *saṃkara* is said to hold between *bhūtatva* "materiality," and *mūrtatva* "limitation, form." Now *bhūtatva* occurs in *prthivyādi-catuṣṭaya* "the quadruple earth etc." (i. e., earth, water, fire and air) and also in *gagana* or *ākāśa* "ether." On the other hand, *mūrtatva* occurs in the same quadruple and also in *manas* "mind." In other words: materiality and limitation overlap. For the quadruple is both limited and material; ether is material but not limited; and mind is limited but not material.



<sup>27</sup> *Dinakarīya ad Kārikāvalī-muktāvalī ad Bhāṣā-pariccheda* 8 (ed. Hari Rāma Śukla [Banaras, 1951] 58); cf. NK *loc. cit.* and Svāmi Mādhavānanda, *Bhāṣā-pariccheda* with *Siddhānta-muktāvalī* (Almora, 1954) 11 n. 4.

Another example is the overlapping of *udbhūtatva* "perceptibility" and *śuklatva* "whiteness"; for while some white things are manifest, there is whiteness without perceptibility (e. g., in the eye) and also perceptibility without whiteness (e. g., in blueness).<sup>28</sup>

The concept of *saṃkara* is important because of a doctrine according to which every entity is determined by at least one general property, which need not necessarily be a generic character (*jāti*): for generic characters are either distinct, or one includes the other. Overlapping (*saṃkara*) is an impediment to being a *jāti*.<sup>29</sup> Hence *udbhūtatva* "perceptibility" is not a *jāti* because it overlaps *śuklatva* "whiteness." "Imperceptibility" (*anudbhūtatva*), on the other hand, is a *jāti*: for it is considered as various (*nānā*), so that each colourhood has its own case of imperceptibility.

Another impediment to being a *jāti* is (giving rise to) infinite regress (*anavasthiti*, *anavasthā*) (cf. *τρίστρος ἀσθρονος*).—These doctrines seem to reflect difficulties which are not dissimilar to those dealt with in the theory of types in modern logic.

According to Bocheński a point of difference between Indian and Western logic is the intensional character of Indian logic.<sup>30</sup> While this view, at the present state of our knowledge, remains valid for a large number of cases, the doctrines considered in the above show a marked extensional character.

<sup>28</sup> *Kārikāvalī-muktāvalī* 188 and cf. Svāmi Mādhavānanda *op. cit.* 91-2.

<sup>29</sup> *saṃkara jātitve bādhaḥ*: *Nilakaṇṭhi ap. NK loc. cit.* Cf. Ingalls *op. cit.* 42, n. 49.

<sup>30</sup> I. M. Bocheński, *Formale Logik* (Freiburg/München, 1956) 517.

CONTRAPOSITION IN INDIAN LOGIC

I

An inference in Indian logic can generally be written in the following form:

$$(1) \quad (x)(A(h, x) \rightarrow A(s, x)).$$

Here  $A(y, x)$  denotes the relation of occurrence of a term  $y$  in its locus  $x$ ;  $h$  denotes the reason (*hetu*) and  $s$  denotes the thing-to-be-inferred (*sādhya*). In the stock example  $h$  denotes smoke and  $s$  denotes fire. The relationship of inference is often called *vyāpti*, "pervasion", which may be written  $V(h, s)$ . It is said that the *hetu* is "pervaded" (*vyāpya*) by the *sādhya*, its "pervader" (*vyāpaka*).

In any given inference the place of the variable  $x$  is taken by a preferential locus, which is called *pakṣa* and which will be denoted by  $p$ . The question is then asked whether  $s$  occurs in  $p$  if  $h$  occurs in  $p$ . In the stock example  $p$  is the mountain where fire is to be inferred from smoke. The conclusion that  $s$  actually occurs in  $p$  when  $A(h, p)$  is given is proved by means of a quantified inference of the form (1).

In the VIIth century A.D. the Buddhist logician Dharmakīrti mentions three conditions which are required for establishing the validity of such an inference. This doctrine originated, in an unquantified form, before Dignāga, who lived in the Vth or VIth century.<sup>1</sup> In the formulation of Dharmakīrti, quantification is expressed by the position of the particle *eva*, "only, just", in the following manner:

(a) "occurrence in the inferred (*anumeya*) of  $h$  only"<sup>2</sup> which can be expressed by: (*anumeya*)  $A(h, \text{anumeya})$ . The words "of  $h$ ", mentioned only here, belong to all three conditions;

(b) "occurrence of  $h$  in similar instances (*sapakṣa*) only",<sup>3</sup> which can be expressed by:  $(x)(A(h, x) \rightarrow (x = \text{sapakṣa}))$ ; and:

(c) "non-occurrence in dissimilar instances (*asapakṣa*) of  $h$  only",<sup>4</sup> which can be expressed by: (*asapakṣa*)  $\sim A(h, \text{asapakṣa})$ .

The meaning of these conditions depends on the meanings of the terms *anumeya*, *sapakṣa* and *asapakṣa* which are subsequently defined as follows:

<sup>1</sup> A *Tarka-sāstra*, ed. by G. Tucci (*Pre-Dignāga Buddhist text on logic from Chinese sources*, Baroda, 1930) mentions the doctrine of the three conditions (G. Tucci, *Buddhist logic before Dignāga*, *Journal of the Royal Asiatic Society*, 1929, p. 483).

<sup>2</sup> *liṅgasyānumeये sattvam eva*: Dharmakīrti's *Nyāyabindu* with Dharmottara's *ṭīkā* (NB), ed. by Candrasekhara Śāstrī, Banaras, 1954, p. 22. Cf. Th. Stcherbatsky, *Buddhist Logic II*, 's-Gravenhage 1958 (reprint of: *Bibliotheca Buddhica XXVI 2*, Leningrad, 1930), pp. 51-9.

<sup>3</sup> *sapakṣa eva sattvam*: NB, p. 23.

<sup>4</sup> *asapakṣe cāsattvam eva*: loc. cit.

“*anumeya* is here the substratum (*dharmīn*), the characteristic (*viśeṣa*) of which it is desired to know”.<sup>5</sup> The commentator Dharmottara (VIIIth or IXth century) explains *viśeṣa* by *dharmā*, “property”.

“*sapakṣa* is what is similar through similarity with the property *s*”;<sup>6</sup> and:

“*asapakṣa* is what is not *sapakṣa*”.<sup>7</sup>

In later texts the place of *anumeya* is taken by *pakṣa* and the place of *asapakṣa* by *vipakṣa*; in both cases these latter terms will be adopted here. As in a given inference there is only one *p*, the first condition amounts to:  $A(h, p)$ . Dharmottara explains similarity (*sāmānya*) as similarity of the *sapakṣa* with *p*. This similarity consists in the fact that *s* actually occurs in the *sapakṣa* cases and does not occur in the *vipakṣa* cases.

In order to formalize these conditions in a manner which preserves as far as possible the structure of the Sanskrit expressions, use will be made of the expression  $\alpha xF(x)$  for a *restricted-variable* denoting any of the values of *x* such that  $F(x)$ . The use of this expression may be illustrated by the formalizations of: “*h* is what is pervaded by *s*” as:  $h = \alpha xV(x, s)$  and of: “*s* is what pervades *h*” as:  $s = \alpha xV(h, x)$ . The use of the expression  $\alpha xF(x)$ , which may occur either free or bound, is governed by various laws, the following three of which will be utilized:

$$(2) \quad (\alpha xF(x))G(\alpha xF(x)) \leftrightarrow (x)(F(x) \rightarrow G(x)),$$

$$(3) \quad (Ey)(y = \alpha xF(x)) \leftrightarrow (Ex)F(x),$$

$$(4) \quad (y)(y = \alpha xF(x)) \leftrightarrow (x)F(x).$$

The terms *sapakṣa* and *vipakṣa* can now be defined as follows:  
*sapakṣa*:

$$(5) \quad \alpha x((x \neq p) \wedge A(s, x));$$

*vipakṣa*:

$$(6) \quad \alpha x \sim A(s, x).$$

If these two expressions are substituted, respectively, in the second and third conditions of Dharmakīrti mentioned above, the result is:

$$(7) \quad (y)(A(h, y) \rightarrow (y = \alpha x((x \neq p) \wedge A(s, x))))),$$

$$(8) \quad (\alpha x \sim A(s, x)) \sim A(h, \alpha x \sim A(s, x)).$$

The question now arises whether (7) and (8) yield (1), as is the contention of Dharmakīrti. Starting from (7) we derive:

$$(y)A(h, y) \rightarrow (y)(y = \alpha x((x \neq p) \wedge A(s, x))),$$

<sup>5</sup>*anumeyo'tra jñāsitaviśeṣo dharmī: NB, p. 24.*

<sup>6</sup>*sādhyā-dharma-sāmānyena samāno'rthah sapakṣah: loc. cit.*

<sup>7</sup>*na sapakṣo'sapakṣah: loc. cit.*

and hence with the help of (4):

$$\begin{aligned} (y)A(h, y) &\rightarrow (y)((y \neq p) \wedge A(s, y)), \\ (y)(A(h, y) &\rightarrow ((y \neq p) \wedge A(s, y))), \\ (y)(A(h, y) &\rightarrow A(s, y)), \end{aligned}$$

or (1), which has been thus derived from (7) only.

The expression (8), on the other hand, yields with the help of (2):

$$(9) \quad (x)(\sim A(s, x) \rightarrow \sim A(h, x)).$$

This is the contrapositive of (1), which has been thus derived from (8) only. Summarizing these results it may be said that the second condition implies (1) directly, whereas the third condition implies either (1) or the second condition through contraposition.

The original Buddhist theory of the three conditions in their unquantified form may mainly have been a kind of argument by reference to analogous cases. Its indeterminate status prevents a judgment on the interdependence of the three conditions. Dharmakīrti quantifies the conditions by means of *eva*. Thus it was recognized that the second and third conditions are equivalent and that this equivalence constitutes a logical relationship, namely, contraposition.

Although Stcherbatsky and, following him, Bocheński, have studied the three conditions of Dharmakīrti and have explicitly mentioned the special use of *eva* in his formulation, they have neither stated what was in their opinion the exact significance of these conditions, nor have they investigated whether an inference can be proved from any of these three conditions separately, or only from all the three together. Bocheński, after having mentioned Dharmakīrti's use of *eva*, again expresses the opinion that Buddhist logic was "noch ganz durch die Beispiele bestimmt".<sup>8</sup> Frauwallner does not seem to have paid much attention to the problems of quantification and independence of the three conditions, but apparently attributes the discovery of contraposition to Dignāga on account of the latter's *hetucakra*.<sup>9</sup> We do not know, however, how the various clauses of the *hetucakra* were thought to be related to each other and this seems to prevent a very definite conclusion. The mere occurrence of contrapositive clauses does not prove much, since the classification is exhaustive. Randle states rightly, referring to the second and third conditions, that "both clauses say exactly the same

<sup>8</sup>I. M. Bocheński, *Formale Logik*, Feiburg/München, 1956, p. 506.

<sup>9</sup>Frauwallner's formulations seem to attribute quantified expressions to logicians prior to Dignāga, while suggesting at the same time that the three conditions are jointly necessary: E. Frauwallner, Dignāga, sein Werk und seine Entwicklung. *Wiener Zeitschrift für die Kunde Süd- und Ostasiens und Archiv für indische Philosophie*, Vol. 3 (1959) p. 85. Later he refers to contraposition when speaking about the same period (*op. cit.* pp. 93-4). For Plato's "struggling" with this (or a very similar) false principle, cf. I. M. Bocheński, *Ancient formal logic*, Amsterdam, 1951, p. 17.

thing",<sup>10</sup> but he does not pay much attention to the law of contraposition which underlies this statement.

That *either* the second *or* the third condition yields the required inference is due to the fact that the third condition is the contrapositive of the second condition. If in either condition the constant  $p$  is substituted for the variable  $x$ , the first condition  $A(h, p)$  immediately yields the required result  $A(s, p)$ . This result has been derived by first formalizing the second and third conditions in (7) and (8), respectively, and by subsequently deriving conclusions from these formulas. Such a purely logical interpretation can be assumed to be justified, since it seems reasonable to treat the Sanskrit text itself as a product of logical activity. That Indian and Western arguments are not different in this respect follows from the commentary on Dharmakīrti's text, which confirms the above conclusion. After having commented upon all the three conditions, Dharmottara asks: "But does the expression 'occurrence in the *sapakṣa* only' not entail 'only non-occurrence in the *vipakṣa*'?"<sup>11</sup> Asking this question can only be explained by assuming that the author knew the law of contraposition and its relevance to the present context. He then continues: "Why then are two clauses (*upādāna*) mentioned? The answer is as follows. *Either* the *anvaya*, *or* the *vyatireka* should be used, each with necessity, and not otherwise".<sup>12</sup> Here *anvaya* refers to the second condition, and *vyatireka* to the third; in other words, *vyatireka* denotes the contraposition or *anvaya*. The required necessity (*niyama*) refers to the use of *eva* or to the quantification of the expressions, as is evident from the subsequent lines, where it is said that without this necessity, even when both conditions are assumed to be valid, an invalid inference can be derived. The example which is given to show this is the following: "it is black because it is that person's child".<sup>13</sup> Here blackness occurs in *some* of the children (*sapakṣa*) but it is also absent from *some* other children (*vipakṣa*). This inference is invalid. Hence the conditions should be properly quantified and "of both only one should be used and not two"<sup>14</sup> Lastly Dharmottara summarizes his interpretation as follows: "With necessity, either the *anvaya* or the *vyatireka* should be used".<sup>15</sup>

Many of these sentences were translated in similar terms by Stcherbatsky,<sup>16</sup> but he failed to appreciate their significance adequately, mainly on account of his special interest in "universal and necessary propositions",

<sup>10</sup>H. N. Randle, *Indian logic in the early schools*, London, etc., 1930, p. 183.

<sup>11</sup>*nanu ca sapakṣa eva sattvam ity ukte vipakṣe'sattvam eveti gamyata eva: NB*, p. 24.

<sup>12</sup>*anvayo vyatireko vā niyamavān eva prayoktavyo nānyatheti: loc. cit.*

<sup>13</sup>*śyāmas tatputratvāt* (Stcherbatsky translates this differently, with reference to the *upādhi*). This type of (invalid) inference may be a forerunner of the type of (valid) inference in the latter schools: *kapisamyogy etadvṛkṣatvāt*, "it has contact with a monkey because it is this tree".

<sup>14</sup>*dvayor eka eva prayoktavyo na dvāv iti: loc. cit.*

<sup>15</sup>*niyamavān evānvayo vyatireko vā prayoktavya iti: loc. cit.*

<sup>16</sup>*Op. cit.* pp. 57–8.

i.e., mainly because of his Kantian bias. In the chapter on contraposition in the first volume of his monumental "Buddhist Logic"<sup>17</sup> Stcherbatsky states that the second and third conditions "are two manners of expressing the same idea", but he then adds the apparently meaningless sentence: "But in order to express the *necessary* dependence of the reason upon the predicate both must be stated, either expressedly or by implication".<sup>17</sup> The same insistence on necessary propositions causes a term like *samāna* in the same passage to be variously translated by "similar", "common", or "universal".

In the above translation no attention has been paid to the occurrence of the term *niścitam*, "ascertained, necessary" at the end of the third condition, which Stcherbatsky accordingly translates as: "its absolute absence in dissimilar cases is necessary". With this translation the problem arises why only the third condition would have to be ascertained as necessary. This problem is solved by Dharmottara, according to whom the term should be applied to all the three conditions. In the commentary upon the first condition he says: "Even though the word *niścita* is not mentioned here, it is mentioned at the end and should be considered with reference to the two preceding expressions".<sup>18</sup>

## II

After a millenium, the requirement of necessity forms part of the definitions of *sapakṣa* and *vipakṣa* in the texts of the modern school of Indian logic (*navya-nyāya*). The *pakṣa* is again defined with reference to the occurrence of *s*, which is not ascertained but merely desired. In a well-known manual of Indian logic, the *Tarka-samgraha* (*TS*) of Annaṃbhaṭṭa, written in the XVIIth century A.D., the same concepts are utilized but form part of a newly elaborated system of formal logic.<sup>19</sup> The definitions are given as follows:

"*p* is the locus of *s* which is doubted",<sup>20</sup> e.g., the mountain;

"*sapakṣa* is a locus of *s* which is ascertained",<sup>21</sup> e.g., the kitchen; and:

"*vipakṣa* is a locus of absence of *s* which is ascertained",<sup>22</sup> e.g., the lake.

With the help of these terms three types of inference can be distinguished, which are called: (I) *kevalānvayī*, "universally positive, unnegatable"; (II) *kevalavyatirekī*, "universally negative, unexampled"; and: (III)

<sup>17</sup>*Buddhist logic I*, 's-Gravenhage, 1958 (reprint of: *Bibliotheca Buddhica XXVI 1*, Leningrad, 1932), pp. 301-3. The Kantian predisposition is so strong as to obliterate the problem in: Th. Stcherbatsky, *Erkenntnistheorie und Logik nach der Lehre der späteren Buddhisten*, München/Neubiberg, 1924, p. 206.

<sup>18</sup>*yady api cātra niścita-grahaṇam na kṛtam tathāpyante kṛtam prakṛāntayor dvayor api rūpayor apekṣaṇīyam*: *NB*, p. 22.

<sup>19</sup>Cf. the present author's *Formal structures in Indian logic*, *Synthese*, Vol. 22 (1960), pp. 279-86.

<sup>20</sup>*samdigdha-sādhyavān pakṣaḥ*: *TS*, ed. by Y. V. Athalye, Bombay, 1930, pp. 43, 290-3.

<sup>21</sup>*niścita-sādhyavān sapakṣaḥ*: *loc. cit.*

<sup>22</sup>*niścita-sādhyābhāvavān vipakṣaḥ*: *TS*, p. 44.

*anvaya-vyatirekī*, “positive-negative”. This classification was already given by logicians of the old Hindu school, such as Uddyotakara (early VIIth century),<sup>23</sup> and is dealt with in the *TS*.<sup>24</sup> The three types are defined by means of the above terminology in the *Siddhānta-muktāvalī* (*SM*) of Viśvanātha Pañcānana (XVIIth century) as follows:

(I) “Universally positive is what has no dissimilar instances”,<sup>25</sup> which can be written with the help of (6) as:

$$\sim(Ey)(y = \alpha x \sim A(s, x)),$$

and hence with the help of (3) as:

*kevalānvayī*:

$$(10) \quad \sim(Ex) \sim A(s, x) \quad \text{or:} \quad (x)A(s, x).$$

EXAMPLE: the inference  $A(\text{namability}, x) \rightarrow A(\text{knowability}, x)$ .<sup>26</sup>

(II) “Universally negative is what has no similar instances”,<sup>27</sup> which can be written with the help of (5) as:

$$\sim(Ey)(y = \alpha x((x \neq \phi) \wedge A(s, x))),$$

and hence with the help of (3) as:

*kevalavyatirekī*:

$$\sim(Ex)((x \neq \phi) \wedge A(s, x)).$$

This may be written as:

$$(11) \quad (x)(A(s, x) \rightarrow (x = \phi)).$$

EXAMPLE: the inference

$$A(\text{smell}, \text{earth}) \rightarrow A(\text{being different from other things}, \text{earth}).$$

This may be explained along the following lines. In the traditional physics adopted by the logicians, earth (the *pakṣa*) is the only substance which smells or has smell (*gandha*) or *g*:

<sup>23</sup>See: S. C. Vidyabhusana, *A history of Indian logic*, Calcutta, 1921, p. 130.

<sup>24</sup>*TS*, pp. 40, 281–90. Cf. A. Foucher, *Le compendium des topiques*, Paris, 1949, p. 132–5.

<sup>25</sup>*asadvipakṣaḥ kevalānvayī*: *SM*, ed. by Hari Rāma Śukla, Banaras, 1951, p. 501.

<sup>26</sup>The relation of *kevalānvayī* cases to the definitions of *vyāpti* is further discussed in the present author's *Correlations between language and logic in Indian thought*, *Bulletin of the School of Oriental and African Studies*, Vol. 23 (1960), pp. 116–7.

<sup>27</sup>*asatsapakṣaḥ kevalavyatirekī*: *loc. cit.* It is generally the inference which is called *kevalavyatirekī*, but it is the *hetu* according to the *Nyāyakośa* (Poona, 1928, p. 241) in a quotation from the *TS* or the *Nyāya-siddhānta-mañjarī*. As in some other cases, this reading differs considerably from the reading of the printed edition by Jivanātha Miśra (Banaras, 1916, p. 97; a critical edition with translation is being prepared by T. Gelblum).

$$p = (E!x)A(g, x),$$

or:

$$(12) \quad A(g, p) \wedge (x)(A(g, x) \rightarrow (x = p)).$$

The right member of this conjunction and (11) show that the inference is universally negative.

While (I) and (II) are relatively exceptional cases, the general situation is dealt with in the remaining possibility:

(III) "Positive-negative is what has both similar and dissimilar instances",<sup>28</sup> which can be written as:

*anvayavyatirekī*:

$$(13) \quad (Ex) \sim A(s, x) \wedge (Ex)((x \neq p) \wedge A(s, x)).$$

EXAMPLE: the inference  $A(\text{smoke}, \text{hill}) \rightarrow A(\text{fire}, \text{hill})$ .

PROOF:  $ax \sim A(s, x)$  is *lake*;  $ax((x \neq p) \wedge A(s, x))$  is *kitchen*.

The term *anvayavyatirekī* can at the same time denote the law of contraposition, which transforms a "positive pervasion" (*anvaya-vyāpti*) into a "negative pervasion" (*vyatireka-vyāpti*). We have seen that Dharmottara used *anvaya* and *vyatireka* already in this sense. The Mīmāṃsaka Kumārila Bhaṭṭa (VIIIth century), a famous and well-informed opponent of Buddhist doctrines, describes contraposition in the following verse of the *Śloka-vārttika*:

If the relation of pervader and pervaded holds between two entities, it follows that the converse (*viparīta*) relation holds between their negations.<sup>29</sup>

The commentator Bhaṭṭomveka (late VIIIth century?) explains these two lines as referring to *anvaya* and *vyatireka*, respectively.<sup>30</sup> Similar formulations are given by the later logicians. Keśava Miśra (between 1200 and 1400) quotes the same lines in his *Tarka-bhāṣā* and adds the following verse:

In positive inference the reason is pervaded and the thing-to-be-inferred is the pervader; in the other case the negation of the thing-to-be-inferred is pervaded and the negation of the reason is the pervader. First the pervaded is stated and next the pervader. Thus examined the true nature of pervasion becomes manifest.<sup>31</sup>

The *TS* uses the terms as follows: "Where there is smoke, there is fire, as in the kitchen' is a positive pervasion. "Where there is no fire, there is no smoke either, as in the lake' is a negative pervasion".<sup>32</sup>

<sup>28</sup>*satsapakṣavipakṣo'nvayavyatirekī*: *SM*; *loc. cit.*

<sup>29</sup>*vyāpya-vyāpaka-bhāvo hi bhāvayor yādyg iṣyate/tayor abhāvayos tasmād viparītaḥ pratīyate*||*Śloka-vārttika anumāna*: 121b-122a.

<sup>30</sup>*Śloka-vārttika-vyākhyā* (ed. by S. K. Rāmanātha Śāstrī, Madras, 1940, p. 335).

<sup>31</sup>*anvaye sādhanam vyāpyam sādhyam vyāpakam iṣyate/sādhyābhāvo'nyathā vyāpyo vyāpakah sādhanāyayah||vyāpyasya vacanam pūrvam vyāpakasya tataḥ param/evam parikṣitā vyāptiḥ sphuṭī bhavati taitvataḥ*||*Tarka-bhāṣā*, ed. by S. N. Dvivedi, Banaras, 1952, p. 22. Cf. transl. by P. Tuxen, Copenhagen, 1914, p. 22.

<sup>32</sup>*TS*, p. 40.

In these various texts contraposition is postulated with reference to pervasion and inference in the following terms:

$$V(h, s) \leftrightarrow V(\text{negation of } s, \text{ negation of } h),$$

or:

$$(14) \quad (x)(A(h, x) \rightarrow A(s, x)) \leftrightarrow (x)(\sim A(s, x) \rightarrow \sim A(h, x)).$$

The expressions (13) and (14) are related to each other in the following manner: (14) is only meaningfully applicable if (13) holds. This can be shown with reference to the left member *L* of (13) and the right member *R* of (14) as follows:

A. If:  $\sim(Ex) \sim A(s, x)$ , i.e., in a case of *kevalānvaya*, *R* is not defined.

B. If:  $(x)(\sim A(s, x) \rightarrow \sim A(x = p))$ , i.e., in a case of *kevalavyatireka*, *L* implies  $x = p$ ; but this would be useless, for it is impossible to establish an inference if only the *pakṣa*, where *s* is doubted, is available, while the similar instances, where *s* would have been ascertained previously, are absent.

### III

Next some refinements of (14), which may illustrate the subtleties and complexities of the *navya-nyāya* style and formalisms, will be considered.

The occurrence relation  $A(x, y)$  is always an occurrence through a relation (*sambandhena*<sup>33</sup> *vr̥ttitva*), which relation should be specified if certain ambiguities are to be avoided. The various relations, such as *samyoga*, “contact” (*c*), *samavāya*, “inherence” (*i*), etc., will be written as subscripts attached to  $A$ :  $A_c, A_i$ , etc.<sup>34</sup> For instance we have  $A_c(\text{smoke, hill})$ , but on the other hand:  $A_i(\text{smoke, particles of smoke})$ . In general the occurrences in an inference may be specified, for instance as follows:

$$(x)(A_p(h, x) \rightarrow A_q(s, x)).$$

In a contraposition of the form (14) the negated occurrence is required to occur through the same relation as the originally affirmative occurrence. In other words, two conditions should be fulfilled for a contrapositive to be valid:

$$(15) \quad \alpha y \sim A_y(h, x) = \alpha y A_y(h, x),$$

$$(16) \quad \alpha y \sim A_y(s, x) = \alpha y A_y(s, x).$$

In Sanskrit (15) could be expressed by the formula: *yatsambandhena hetor vr̥ttitvaṃ tatsambandhāvachhinna-pratīyogitāka-hetvabhāva*, “the absential

<sup>33</sup>For this use of the instrumental case ending cf. H. Jacobi, Über den nominalen Stil des wissenschaftlichen Sanskrits, *Indogermanische Forschungen*, Vol. 14 (1903), pp. 239-40; L. Renou, *Histoire de la langue sanskrite*, Paris, 1956, p. 141.

<sup>34</sup>In the article quoted above, note 26, these subscripts are within parentheses, as unbracketed subscripts are used to denote case-endings.

correlate<sup>35</sup> of the negation of  $h$  is limited by the relation through which  $h$  occurs". Analogously (16) could be expressed by: *yatsambandhena sādhyavṛttitvaṃ tatsambandhāvachchinna-pratīyogitāka-sādhyābhāva*, "the absential correlate of the negation of  $s$  is limited by the relation through which  $s$  occurs".

By introducing the conditions (15) and (16) into the contraposition formula (14), we arrive at:

$$(17) \quad (x)(A_p(h, x) \rightarrow A_q(s, x)) \leftrightarrow (x)(\sim A_{\alpha\nu A_p(s, x)}(s, x) \rightarrow \sim A_{\alpha\nu A_p(s, x)}(h, x)).$$

In this formula:  $\sim A_{\alpha\nu A_p(s, x)}(s, x)$  could be expressed in Sanskrit by: *sādhyavṛttivāvacchedaka-sambandhāvachchinna-pratīyogitāka-sādhyābhāva*, "negation of  $s$ , the absential correlate of which is limited by the limiting relation through which  $s$  occurs". Analogously:  $\sim A_{\alpha\nu A_p(h, x)}(h, x)$  could be expressed in Sanskrit by: *hetu-vṛttivāvacchedaka-sambandhāvachchinna-pratīyogitāka-hetvābhāva*, "negation of  $h$ , the absential correlate of which is limited by the limiting relation through which  $h$  occurs".

If the suggested substitutions are made, (17) may be further simplified to:

$$(18) \quad (x)(A_p(h, x) \rightarrow A_q(s, x)) \leftrightarrow (x)(\sim A_q(s, x) \rightarrow \sim A_p(h, x)).$$

The four Sanskrit expressions in the foregoing lines may illustrate how certain logical relationships can be expressed in the formalized and technical Sanskrit of *navya-nyāya*. They do not actually occur in the *SM*, but are constructed along similar lines as the more complicated expressions given there. This additional complication is due to the fact that certain other conditions are required at the same time to hold as well. This requirement corresponds to universal quantification in Western logic, which is expressed by means of abstractions in Indian logic.<sup>36</sup> The abstraction of a term  $x$ , which will be written as  $\hat{x}$ , can be expressed in Sanskrit by adding the suffixes *-tva* or *-tā* to the stem, or by the use of such terms as *jāti*, "class" or *dharma*, "property".

If an expression  $F(x)$ , in which a term  $x$  occurs, is universally quantified with respect to  $x$ , this is expressed in Sanskrit by saying that the expression  $F(x)$  is limited by  $\hat{x}$ , the abstraction of  $x$ . In the formalism which represents the Sanskrit expressions this could be expressed by adding an additional subscript to the functions and relations which are generalised with respect

<sup>35</sup>The "thing-negated" or *abhāvīya-pratīyogī*. See: D. H. H. Ingalls, *Materials for the study of Navya-nyāya logic*, Cambridge, Mass., 1951, p. 55. A formalisation is suggested by the present writer in: *Indo-Iranian Journal*, Vol. 4 (1960), pp. 68-73. While Ingalls translated this *pratīyogī* by "counterpositive", J. Brough suggested in his review (*Journal of the Royal Asiatic Society*, 1954, p. 88) "absential correlate" in order to avoid confusion with "contrapositive". In the present context this translation has naturally been adopted, although *abhāva* is here translated by "negation" and not by "absence".

<sup>36</sup>See the present writer's Means of formalisation in Indian and Western logic, *Proceedings of the XIIth International Congress of Philosophy* (Venice, 1958). Firenze, 1960, pp. 221-227.

to their variables. This cannot give rise to confusion with a relational subscript, for the subscript which denotes universal quantification is always marked with the circumflex accent, while there are few relations which are abstractions.<sup>37</sup> Hence universal quantification can be expressed as follows:

$$(19) \quad F_{\hat{z}}(x) = {}_a(x)F(x)$$

where the left member corresponds to the Sanskrit expression.

The *SM* expresses the various conditions referred to earlier in the following sentence,<sup>38</sup> parts of which have been numbered 1–8: *yatsaṃbandhena<sup>1</sup> yadavacchinam prati<sup>2</sup> yena saṃbandhena<sup>3</sup> yena rūpeṇa<sup>4</sup> vyāpakatā grhyate tatsaṃbandha<sup>5</sup>-avacchinna-pratīyogitāka taddharma<sup>6</sup>-avacchinnaḥāvavattā-jñānāt tatsaṃbandha<sup>7</sup>-avacchinna-pratīyogitāka taddharma<sup>8</sup>-avacchinnaḥāvasya siddhiḥ*. A literal translation would run as follows:<sup>39</sup> “by which form<sup>4</sup> through which relation<sup>3</sup> with respect to what (i.e., *s*) is limited by which<sup>2</sup> through which relation<sup>1</sup> the pervasion (i.e., of *h*) is understood, the negation (of *h*) is inferred which is limited by that property<sup>8</sup> the absential correlate of which is limited by that relation<sup>7</sup> on account of the knowledge of the negation (of *s*) which is limited by that property<sup>6</sup> the absential correlate of which is limited by that relation<sup>5</sup>”. As *h* and *s* are not mentioned explicitly, this is a formulation of generalized contraposition without special reference to the inferential process. An English translation is bound to be hardly intelligible, but a translation into a formula expresses the meaning more directly and can also give a picture of the structure of the Sanskrit original. Restricted-variables represent the four pairs of correlative pronouns (relative *yad* — demonstrative *tad*) which express identity.<sup>40</sup> As in the text the abstractions (expressed twice by *dharma*, once by *rūpa* and once merely by *-yad*) are given with reference to the negation, negation will now be expressed by means of a negating function, as in Łukasiewicz’ terminology, by defining:

$$(20) \quad Nx = {}_a \sim x.$$

Then:  $\sim A_{\hat{m}p}(x, y)$  (where:  $x = x(m)$  and/or  $y = y(m)$ ) may be written as:  $NA_{\hat{m}p}(x, y)$  and as the occurrence *A* is negated with respect to the relation *p*, this may be written as:  $N_p A_{\hat{m}}(x, y)$ .

The Sanskrit expression may now be represented by the following formula,

<sup>37</sup>One such exception is the relation *viśeṣanātā*: *Nyāyakośa* ss. vv. *viśeṣanam, svarūpasambandhaḥ*. This relation does not occur in the present context.

<sup>38</sup>*SM*, pp. 501–2.

<sup>39</sup>Without the excellent translation and notes of Svāmī Mādhavānanda (Calcutta, 1954, p. 235) it would have been difficult to understand the original. There is one confusing error. The last three lines of page 235 should be read as: “. . . that non-existence which is determined by that attribute, and the counterpositiveness of which is determined by that relation”. In his notes the Svāmī gives the following instances for the parts of the expression which have been numbered 1–8 in the above: 1, 3, 5, 7: *conjunction*; 2, 8: *smokehood*; 4: *as fire* (i.e., *firehood*); 6: *firehood*.

<sup>40</sup>Cf. P. Hartmann, *Nominale Ausdrucksformen im wissenschaftlichen Sanskrit*, Heidelberg, 1955, pp. 66, 140–4.

corresponding parts of which are numbered with the same numbers 1-8, respectively, as the above:

$$(21) \quad (x)(A_{\hat{m}_p}(h, x) \rightarrow A_{\hat{n}_q}(s, x))$$

$$\leftrightarrow (x)(N_{\alpha\gamma A_{\hat{p}_q}(s, x)} A_{\alpha\gamma A_{\hat{n}_p}(s, x)}(s, x) \rightarrow N_{\alpha\gamma A_{\hat{p}_p}(h, x)} A_{\alpha\gamma A_{\hat{n}_q}(h, x)}(h, x)).$$

This expression can be further simplified as follows:

$$(x)(A_{\hat{m}_p}(h, x) \rightarrow A_{\hat{n}_q}(s, x)) \leftrightarrow (x)(N_{\hat{n}} A_q(s, x) \rightarrow N_{\hat{m}} A_p(h, x)).$$

Hence, because in the case of inference:  $m = h$  and  $n = s$ :

$$(22) \quad (h)(s)((x)(A_p(h, x) \rightarrow A_q(s, x)) \leftrightarrow (x)(\sim A_q(s, x) \rightarrow \sim A_p(h, x))),$$

which is a generalization of (18).

Whoever has mastered this terminology may well ask what purpose it serves. The answer is that it excludes certain invalid inferences. For instance, if again  $c$  denotes *saṃyoga*, “contact”, and  $i$  denotes *samavāya*, “inherence”, we have:

$$A_c(\text{smoke, hill}) \rightarrow A_c(\text{fire, hill}),$$

and hence with (22):

$$\sim A_c(\text{fire, lake}) \rightarrow \sim A_c(\text{smoke, lake}),$$

but we are not entitled to infer:

$$\sim A_i(\text{fire, particles of smoke}) \rightarrow \sim A_i(\text{smoke, particles of smoke}),$$

for in the traditional physics which the Naiyāyikas adopt, fire does not occur in particles of smoke through inherence, while smoke does occur in particles of smoke through inherence.

#### IV

The Indian logicians had a special reason for being interested in contraposition and in inferences with negative premisses. It enabled them to do without an additional category, which some other schools of Indian thought held to be indispensable. The high evaluation of such economy of thought may have been taken from the Sanskrit grammarians.<sup>41</sup> But while the grammarians often preferred the shortest possible expression (*sūtra*), the logicians, not shrinking, as we have seen, from long and complicated expressions, attempted to manipulate a relatively small number of elementary concepts.

<sup>41</sup>Both grammarians and logicians prefer a “light” (*laghu*) expression to a “heavy” (*guru*) expression “for the sake of lightness” (*lāghavārtham*). See: L. Renou, *La Durgḥāvṛtti de Śaraṇadeva*, Paris, 1940, Vol. I (*Introduction*), pp. 1, 81-2 note. For *navya-nyāya* see Ingalls, *op. cit.*, p. 47. Cf. also the discussions concerning *anyathā-siddhi*, litt. “establishment by other means”, i.e., “superfluity”: *SM* pp. 86-91.

Accordingly the logical style can be characterized not only by its extreme nominalisation and paucity of verbal forms, but also by its limited vocabulary.

The additional category which can be avoided by considering it a special kind of inference is the category of *arthāpatti*, "presumption", adopted by the Mīmāṃsakas.<sup>42</sup> The latter argued that there are cases where neither the category of direct perception, nor that of inference, is a valid means of knowledge. The following is an example. Suppose it were known from astrology that Devadatta shall live a full hundred years, and suppose that it were known from direct perception that he is not at home.<sup>43</sup> It may then be presumed that Devadatta is outside his house. On account of this presumption the Mīmāṃsakas postulate a special means of knowledge, namely, *arthāpatti*. The Naiyāyikas on the other hand class it under inference, because the reasoning involved can be represented by the following metatheorem:

$$(23) \quad A \rightarrow (B \vee C), A, \sim B \vdash C.$$

This metatheorem involves the use of an inference with a negative premiss ( $\sim B \rightarrow C$ ). While this metatheorem is not explicitly stated, it is utilized and it is recognized that it necessitates an inference with a negative premiss. Such inferences were rejected by the Mīmāṃsakas, but the Naiyāyikas proved their validity by referring to the principle of contraposition. Hence the latter principle helped them to explain *arthāpatti* as a special kind of inference.<sup>44</sup>

The same metatheorem is applied in another example, the treatment of which is formulated in the *SM* in the following terms: "In cases such as 'Fat Devadatta does not eat by day', it is known that fatness is pervaded by eating, and hence eating is proved. And as eating by day is contradicted, eating at night is proved".<sup>45</sup>

Not only the Mīmāṃsakas but also the Advaitins adopted *arthāpatti*. Both

<sup>42</sup>See G. N. Jhā, *Pūrva-Mīmāṃsā in its sources*, Banaras, 1942, pp. 157-60, and the manuals of Indian philosophy. The two schools of the Mīmāṃsā disagreed on the nature and function of *arthāpatti*. The followers of Kumārila Bhaṭṭa expressed their dislike of the views of the rival school thus: "Well, alas, who are they who, kissing the backs of the followers of Prabhākara, babble thus?" (*Mānameyodaya*, ed. and transl. by C. Kunhan Raja and S. S. Suryanarayana Sastri, Madras, 1933, p. 124). For views of the Advaitins, cf. D. M. Datta, *The six ways of knowing*, London, 1932, p. 216 sq.; A. B. Shastri, *Studies in Post-Śaṅkara dialectics*, Calcutta, 1936, pp. 67-7.

<sup>43</sup>In order to account for such negative knowledge the Mīmāṃsakas postulate another special means of knowledge, *anupalabdhi*, "non-apprehension", which the Naiyāyikas again reject as a distinct category and class under *pratyakṣa*, "perception".

<sup>44</sup>*Presumption* was rejected already in the *Nyāyasūtra* (2.2.3) and in the *Tarkaśāstra* (see above note 1), p. 25. Cf. JRAS (1929), pp. 485-6.

<sup>45</sup>*pīno devadatto divā na bhūṅkta ity ādau pīnatvasya bhojana-vyāpyatvāvagamād bhojanam siddhyati|divā-bhojana-bādhe ca rātri-bhojanam siddhyati: SM*, pp. 503-4; transl. pp. 238-9.

agreed, accordingly, in rejecting the *kevalānvaṃyī* and *kevalavyatirekī* cases of inference recognized by the Naiyāyikas, but for different reasons. The Advaita position directly implies that *kevalānvaṃyī* cases are impossible. For according to the Advaitins there is an absolute entity, *brahman*, and no other entity can reside where *brahman* resides. In the *Vedāntaparibhāṣā* (XVIIth century) this is expressed as follows: "In our view every property is an absential correlate of relational negation located in the Absolute".<sup>46</sup> If *brahman* be denoted by *b*, this may be expressed by:

$$(24) \quad (x) \sim A(x, \alpha y A(b, y)).$$

Hence for any *sādhyā* *s* we have:

$$\sim A(s, \alpha y A(b, y),$$

hence:  $(Ex) \sim A(s, x)$  or:  $\sim(x) A(s, x)$ , which contradicts (10); i.e., *kevalānvaṃyī* inferences are impossible. The *kevalavyatirekī* inferences are rejected as well, this time not by means of a definite proof, but in an apparently circular argument. The *Vedāntaparibhāṣā* says that there is no use for *kevalavyatireka*, since in cases where no positive inference is available, another category, namely, *arthāpatti*, will be shown to provide the required knowledge.<sup>47</sup> In a later chapter, when dealing with *arthāpatti*, the text establishes its use as an independent means of knowledge by saying that it was previously shown that no inference can be *kevalavyatirekī*.<sup>48</sup>

The Mimāṃsakas of the Bhaṭṭa school rejected *kevalavyatireka* cases as they would constitute an exception to a general law. The argument of the *Mānameyodaya* (early XVIIth century) runs as follows: "(for) by inference, what has been established somewhere is established elsewhere",<sup>49</sup> i.e., it is always presupposed that:

$$(25) \quad (Ex)((x \neq p) \wedge A(s, x)).$$

In *kevalavyatirekī* cases, on the other hand, the *sādhyā* "is not established anywhere else",<sup>50</sup> i.e.:

$$(26) \quad \sim(Ex)((x \neq p) \wedge A(s, x)).$$

Since (26) contradicts (25), *kevalavyatireka* is excluded.

A Mādhva author of the XIVth or XVth century rejected the entire classification of inference in three types,<sup>51</sup> but the famous dualist Jayatirtha

<sup>46</sup>*sarvasyāpi dharmasyāsmiṃ mate brāhma-niṣṭhāyantiābhāva-pratīyogitvena . . . Vedāntaparibhāṣā*, ed. and transl. by S. S. Suryanarayana Sastri, Madras, 1942, pp. 55-6.

<sup>47</sup>*Loc. cit.* (*vaṅśyāmah*).

<sup>48</sup>*Op. cit.*, p. 93 (*prāg eva nirastam*).

<sup>49</sup>*kvacit prasiddham anyatra sādhyate hy anumānataḥ: Mānameyodaya*, p. 56.

<sup>50</sup>*siddho nānyatra kutracit: loc. cit.*

<sup>51</sup>*Pramāna-candrikā*, ed. and transl. by S. K. Maitra under the title *Mādhva logic*, Calcutta, 1936, pp. 77, 149.

(XIIth century) agreed in his *Pramāṇa-paddhati* with the logicians in considering *arthāpatti* as a kind of inference.<sup>52</sup>

## V

It is hardly possible to sketch in a few lines the context and background of Indian thought, in which Indian logic plays its part. Such a survey could show how Indian logic constitutes an autochthonous development within Indian culture and cannot be convincingly derived from hypothetical contacts with the West.<sup>53</sup> Presently it may only be attempted to give in conclusion an idea of the context and background of the main terms which have been discussed.

The word *pakṣa* means primarily "wing" and has come to denote "alternative" and hence in general "view", "hypothesis". Frequent uses are *pakṣe*, "on the other hand" ("on the other wing"), *pakṣāntara*, "in the other case".<sup>54</sup> In Sanskrit scholarly literature it has become the commentator's custom to interpret any given text in three successive stages: the first interpretation is the *pūrva-pakṣa*, "*prima facie* view"; the second is the reply to this: *uttara-pakṣa*; and the third and final interpretation is the *siddhānta*, "final and established view". This practice is found since Śabara (Vth century A.D.) but may be older.

The terms *pūrva-pakṣa* and *uttara-pakṣa* can also mean Eastern and Northern wing, respectively, or first and second half of the lunar month. It seems probable that the earliest technical or semi-technical use of the term *pakṣa* originated in the Vedic ritual. The *Taittiriya-saṃhitā* had identified the sacrificial fire with a bird, *vayas* or *pakṣin* (i.e., "winged").<sup>55</sup> In some sacrifices the fire altar is accordingly built in the shape of a bird with two wings.<sup>56</sup> The right wing (*dakṣiṇa pakṣa*) points to the South (*dakṣiṇa*) and the left wing (*uttara pakṣa*) points to the North (*uttara*). Moreover, each wing is further enlarged into the direction it points to,<sup>57</sup> for the longer the wings, the stronger the bird and the wider its flight.<sup>58</sup> The idea is, that the bird

<sup>52</sup>See S. N. Dasgupta, *A history of Indian philosophy IV*, Cambridge, 1955, p. 202.

<sup>53</sup>As actual contacts between European Antiquity and India have been frequent from Alexander onwards, it is impossible to prove that no Indian had ever heard of Aristotelian logic. Actual influence on Indian logic is a different matter: it is *a priori* unlikely because of the intimate relationship between logic and language, and moreover seems to become the more improbable, the more our knowledge of Indian systematic and scientific thought (i.e., primarily of ritual, grammar, and logic) increases.

<sup>54</sup>Monier-Williams' Sanskrit-English dictionary, s.v.

<sup>55</sup>5.7.6.1.

<sup>56</sup>See A. Weber in: *Indische Studien*, Vol. 13 (1873), pp. 233 sq.; cf. A. Hillebrandt, *Rituallitteratur*, Strassburg, 1897, p. 164; L. Renou-J. Filliozat, *L'Inde classique*, Vol. I, Paris, 1947, p. 351; L. Renou, *Vocabulaire du rituel védique*, Paris, 1954, p. 88.

<sup>57</sup>*Āpastamba Śrautasūtra* 16.17.10,12,13. Transl. by W. Caland, Amsterdam, 1928, p. 35.

<sup>58</sup>Cf. *Taittiriya-saṃhitā* 5.2.5.1.

thus carries the sacrificer to heaven.<sup>59</sup> Similarly do the strong wings of *pūrva* and *uttara pakṣa* carry the philosopher to his final view, while *sapakṣa* and *vīpakṣa* cases lead the logician to his final proof.

The Sanskrit grammarians use *pakṣa* for “alternative, view, hypothesis”. The terms *vyakti-pakṣa* and *jāti-pakṣa* are utilised for referring to the use of a word as denoting, respectively, the individual (*vyakti*) and the class (*jāti*) to which it belongs.<sup>60</sup>

The compound *anvaya-vyatireka* seems to have originated among the grammarians and occurs probably for the first time in a *vyārtikā* of Kātyāyana (ca. 200 B.C.) mentioned by Patañjali (ca. 50 B.C.) in his discussion of a *sūtra* of Pāṇini.<sup>61</sup> In this *sūtra* Pāṇini defines a nominal stem as what is meaningful but is neither a verbal root, nor a suffix,<sup>62</sup> nor what ends in a suffix.<sup>63</sup> One of the sections of the commentary deals with the problem of whether a nominal stem in itself can be meaningful. The question arises how to distinguish between meanings of stems and of suffixes. The *vārtikā* is then quoted: “it is established by means of *anvaya* and *vyatireka*”.<sup>64</sup> Patañjali elucidates this further in the following terms: “If in this case *vrkṣaḥ* (“tree”) is pronounced, a certain sound is heard which consists of the sound *vrkṣa* ending in *a* and of a termination *s*.<sup>65</sup> A certain meaning is also conveyed, which consists of what has roots, branches, fruits, and leaves, and of singularity. If then *vrkṣau* (“two trees”) is pronounced, some sound disappears (*hīyate*), something comes into being (*upajāyate*) and something remains (*anvayī*): *s* disappears, *au* comes into being, and *vrkṣa* ending in *a* remains.<sup>66</sup> Also with regard to meaning, something disappears, something comes into being, and something remains: singularity disappears, duality comes into being, and what has roots, branches, fruits, and leaves remains”.<sup>67</sup>

It is clear that here *anvayī* denotes “what remains”, while it is but natural to infer that *vyatirekī*, which is not mentioned by Patañjali, denotes what changes (either comes into being or disappears). The terms *anvayī* and

<sup>59</sup>Cf. *Śatapatha-brāhmaṇa* 6.1.2.35, also: H. Hubert-M. Mauss, *Essai sur la nature et la fonction du sacrifice*, in: *Mélanges d'histoire des religions*, Paris, 1909, p. 38 note 4.

<sup>60</sup>L. Renou, *Terminologie grammaticale du Sanskrit*, Paris, 1957, p. 148, 193.

<sup>61</sup>*Mahābhāṣya* 1.2.2 (Kielhorn I 219). Cf. Renou, *Terminologie*, p. 43, and P. S. Subrahmanya Sastri, *Lectures on Patañjali's Mahābhāṣya*, Tiruchirapalli, 1956, p. 90.

<sup>62</sup>*pratyaya*: Renou specifies (*La Grammaire de Pāṇini*, Paris, 1948, p. 21) “un affix en général ou un mot terminé par ces affixes particuliers que sont les désinences casuelles”.

<sup>63</sup>*arthavad adhātur apratyayaḥ prātipadikam*: Pāṇini 1.2.45.

<sup>64</sup>*siddham tv anvayavyatirekābhyām*.

<sup>65</sup>This *-s* becomes *-h*.

<sup>66</sup>The *a* disappears in the process: *vrkṣa-au* > *vrkṣau*.

<sup>67</sup>*iha vrkṣa ity ukte kaścīd chabdaḥ śrūyate vrkṣaśabdo'kāraṇāntaḥ, sakāraś ca pratyayaḥ/ artho'pi kaścīd gamyate mūla-skandha-phala-palāśāvān ekatvam ca/vrkṣāv ity ukte kaścīd chabdo hīyate kaścīd upajāyate kaścīd anvayī/sakāro hīyate aukāra upajāyate vrkṣaśabdo'kāraṇānto'nvayī/artho'pi kaścīd dhīyate kaścīd upajāyate kaścīd anvayī/ ekatvam hīyate dvītvam upajāyate mūla-skandha-phala-palāśāvān anvayī/|*

*vyatirekī* refer to what in modern linguistics R. S. Wells called “environment” and “focus”, respectively.<sup>68</sup> Applications of the method of “replacement” go back to Pāṇini.

The term *vyatireka* is used in a similar sense in later philosophical literature. The *Nyāyasūtra* (IIIrd century A.D.) discusses transformation (*vikāra*) of gold in earrings and necklaces, and states that under such transformations “there is no *vyatireka* of the essence of gold”.<sup>69</sup> The commentator Vātsyāyana (ca. IVth century) explains this further in a similar terminology as that used earlier by Patañjali: “gold is the continuing (*avasthita*) substratum (*dharmin*) with properties (*dharma*) which are disappearing (*hīyamāna*) and coming into being (*upajāyamāna*)”.<sup>70</sup>

Here *vyatireka* denotes change or difference. This seems to be widespread by the VIIIth century, when it is characteristically interpreted by Śāṅkara in a reference to “the theory of the non-difference (*avyatireka*) of the material cause and its transformations”.<sup>71</sup> In the present paper we have seen how in logic, from the VIIth century onwards, *anvaya* denotes the occurrence of the *sādhya* in similar instances and *vyatireka* the absence of the *sādhya* from dissimilar instances. Accordingly, the term *anvaya-vyatireka* can refer both to an inference which possesses similar (positive) and dissimilar (negative) instances and to the contraposition of an inference.<sup>72</sup>

<sup>68</sup>Immediate constituents, *Language*, Vol. 23 (1947), p. 86. Cf. E. Haugen, Directions in modern linguistics, *Language*, Vol. 27 (1951), p. 214.

<sup>69</sup>*suvarṇabhāvavyatireka: Nyāyasūtra 2.2.49.*

<sup>70</sup>*avasthitaṃ suvarṇaṃ hīyamānena dharmāṇopajāyamānena ca dharmeṇa dharmi bhavati.*

<sup>71</sup>*prakṛti-vikāraṇavyatireka-nyāya: Brahma-sūtra-bhāṣya ad. 2.3.6.*

<sup>72</sup>I should like to express here my gratitude to Dr. D. L. Friedman (London) and to Professor Benson Mates (Berkeley) for their valuable suggestions in connection with the present article. The use of the term *anvayavyatireka* by both grammarians and logicians was noted by Ruegg, who supposed that also the reasoning process was the same in both cases: D. S. Ruegg, *Contributions à l'histoire de la philosophie linguistique indienne*, Paris, 1951, pp. 32, 37. Cf. the present author in: *Philosophy East and West*, Vol. 10 (1960), pp. 53–57. While it is of course possible that the later grammarians made use of the logical method concerned, it is certain that they knew Kātyāyana's use of the term (e.g., *Paramalaghu-maṅjūsā*, ed. by S. S. Shastri, Banaras, 1935, p. 2).

## NEGATION AND THE LAW OF CONTRADICTION IN INDIAN THOUGHT : A COMPARATIVE STUDY

IN the fourth chapter of book *Γ* of the *Metaphysica* Aristotle deals with the principle of contradiction. This law is formulated as follows : 'it is impossible for anything at the same time to be and not to be'.<sup>1</sup> Let us imagine, says Aristotle, that somebody wished to oppose this view. Our opponent cannot hold a view which contradicts the law of contradiction without assuming the validity of this law itself : for otherwise he is not even denying what we are saying. The only alternative for him, then, will be to say nothing. But this is absurd : 'for such a man, as such, is from the start no better than a vegetable'.<sup>2</sup>

We are often told that Indian philosophers do not accept the law of contradiction. This may well be one of the causes of the neglect of Indian thought by Western philosophers : for nobody desires to study a body of propositions when he is at the same time told that their contradictories may hold as well. Is it perhaps a new—and according to some superior—kind of logic to which we are invited to accustom ourselves? Many may not feel the need for what would amount to a radical conversion. Nevertheless such a requirement seems implied in statements like the following, where a Western scholar speaks about principles such as the law of contradiction : 'But the Eastern mind is convinced that, taking together all the circumstances in which we need our thinking to give us adequate guidance, it would be fatal to allow ourselves to be enslaved by these principles'.<sup>3</sup> After stating this the author goes on to quote a Buddhist principle which indeed contradicts the law of contradiction.<sup>4</sup> The question arises whether 'the Eastern mind' has in its entirety thrown the law of contradiction overboard.

The aim of the present study is to see how Indian thinkers treat contradictions and what explicit rules are given for the treatment of contradictions. If

<sup>1</sup> *Met.*, *Γ* 4, 1006 a 3–4. Other statements of this law have been formalized by I. M. Bochenski, *Ancient formal logic*, Amsterdam, 1951, 38–40 ; cf. also J. L. Aekrill, *Mind*, LXXI, 1953, 110–12.

<sup>2</sup> *ibid.*, 14–15 ; cf. translation of W. D. Ross, Oxford, 1908.

<sup>3</sup> E. A. Burtt, 'What can Western philosophy learn from India?', *Philosophy East and West*, v, 1955–6, 202.

<sup>4</sup> The principle called *catuskoṭi*. See P. T. Raju, 'The principle of four-cornered negation in Indian philosophy', *Review of Metaphysics*, VII, 1954, 694–713 ; T. R. V. Murti, *The central philosophy of Buddhism*, London, 1955, 129–31, 146–8. The earliest occurrence perhaps in the Pali canon is *Majjhima-nikāya, sutta* 63 (transl. H. C. Warren, *Buddhism in translations*, Cambridge, Mass., 1946, 117–22). For a formalization see : H. Nakamura, 'Buddhist logic expounded by means of symbolic logic', *Journal of Indian and Buddhist Studies*, VII, 1958, 384–5. The same formalization in an earlier Japanese version (in *Indogaku Bukkyōgaku kenkyū*, III, 1954, 223–31) was criticized by R. H. Robinson, 'Some logical aspects of Nāgārjuna's system', *Philosophy East and West*, VI, 1957, 302. The validity of this criticism may be questioned.

Indian philosophers are either unaware of contradiction or deny the validity of the principle of contradiction, the structure of Indian logic would seem to be so fundamentally different from the structure of Western logic, that the possibility of mutual understanding may become questionable. If, on the other hand, the principle of contradiction is either applied implicitly or laid down explicitly, this may be variously interpreted : there may be historical connexions between India and the West which account for certain influences ; there may be a connexion between the logical law and the structure of Indo-European, manifest in both Greek and Sanskrit ; there may be a general linguistic background to the law of contradiction ; or finally, the law may be a universal logical law—whatever that may mean. A discussion of the question, whether Indian philosophers are to be considered vegetables, may therefore constitute a chapter of comparative logic.

The Sanskrit term which seems to correspond most closely to the Western term contradiction is derived from a root *sidh-* meaning 'to keep away'. The noun *sedha* derived from this root denotes 'keeping away', and the nouns *niśedha* and *pratiśedha* have a similar meaning, which can be sometimes further specified as 'prohibition, negation'. The latter term leads to the formation of the noun *vipratiśedha* meaning 'mutual prohibition' or 'contradiction'.

An early technical or semi-technical use of the term *vipratiśedha* occurs in Āpastamba's *Śrauta-sūtra* (between 400 and 200 B.C.), where during the offering the Vedas are assigned to the various sacrificers in the following manner<sup>1</sup>: 'The *hotṛ* sacrifices with the Ṛgveda ; the *udgātṛ* with the Sāmaveda ; the *adhvaryu* with the Yajurveda ; the *brahman* with all. When it is expressly said, or in case of contradiction (*vipratiśedha*), another (priest) may sacrifice'. Instead of *vipratiśedhāt* the partly parallel text of Hiranyakeśin's *Śrauta-sūtra* has *asambhavāt* 'if it is impossible'. Both passages refer to cases where a priest is unable to sacrifice in accordance with what is prescribed for him on account of one rule or one established practice, because he is already engaged on account of another rule or other rules.<sup>2</sup> The last *sūtra*, therefore, formulates a principle which holds in cases of mutual contradiction between two rules applicable to the same situation. It is a rule for the manipulation of other rules, a *paribhāṣā* 'meta-rule', and it is accordingly placed in the last part of the *Āpastamba-śrauta-sūtra*, which consists of the *yajña-paribhāṣā-sūtrāṇi* 'meta-rules regarding the sacrifice'. The term *vipratiśedha* is a term which applies to the contradictions between formulated rules, and not for instance between entities.

It is characteristic for Indian thought that at an early stage of development the distinction between language and meta-language was made. This is connected with the fact that the subject-matter for various kinds of investigations

<sup>1</sup> *Āpastamba-śrauta-sūtra*, 24.1.16–20.

<sup>2</sup> W. Caland in his translation *ad loc.* specifies this by the following example: 'Z. B. muss ein anderer als der Adhvaryu, da dieser beschäftigt ist, das Opfertier losbinden'.

was a large body of linguistic material, the Vedic texts which were considered transcendent and revealed (*śruti*). The *sūtra* literature on the one hand embodies Vedic passages and continues to prescribe forms of activity in a Vedic fashion, and on the other hand interprets Vedic passages. This literature therefore does not only deal with ritual activity, but deals also with statements regarding ritual activity. In the first respect it constitutes a language dealing with the ritual as object material, in the second respect it constitutes a meta-language dealing with the language which deals with the ritual. The *paribhāṣā* rules, more specifically, were explicit meta-linguistic rules for the manipulation of Vedic passages.

With the development of grammar there is a shift in two respects. On the one hand the object material is new : it is neither the ritual, nor Vedic passages dealing with the ritual, but it is the spoken language (*bhāṣā*) and, to a limited extent, the Vedic language (*chandās*).<sup>1</sup> On the other hand it is not Vedic passages, but the rules of grammar themselves which are the subject of a meta-linguistic investigation. The same term, *paribhāṣā*, applies to rules of this meta-language. The importance of these meta-rules increases when the rules of the grammar of Pāṇini are accepted as authoritative.

In dealing with linguistic problems the grammatical description is mainly confined to the analysis of words (*pada*) and grammar has therefore been called *pada-mīmāṃsā* 'investigation into words'. While it is sometimes rightly stressed that the Sanskrit grammarians neglected or paid little attention to syntax, it is not always realized that grammar was in this respect supplemented by the *vākya-mīmāṃsā* 'investigation into sentences', another name for the most orthodox among the systems of philosophy, more generally known as Pūrva-mīmāṃsā, Karma-mīmāṃsā, or merely Mīmāṃsā.<sup>2</sup> Some syntactical principles of Mīmāṃsā will occupy us here. This philosophical system is the direct successor to the *sūtra* literature, for it systematizes the interpretation of Vedic sentences and evolves general canons of interpretation.<sup>3</sup> Also here general rules of a meta-linguistic nature, dealing with the interpretation of Vedic statements, are given. Such meta-rules are often called *nyāya*. These rules are further used and developed in *dharmaśāstra*. The term *nyāya* is later commonly used to denote logic and the system of logic. We have emphasized elsewhere that the early use of the term *nyāya* as synonymous with *paribhāṣā* may be an indication that elements and laws of Indian logic can be traced back to the discussion on problems of language and meta-language in earlier systems and especially in grammar.<sup>4</sup>

<sup>1</sup> See especially P. Thieme, *Pāṇini and the Veda*, Allahabad, 1935, 67 sq. ; L. Renou, *La Durghatavṛtti de Śaraṇadeva*, I, 1 (introduction), Paris, 1940, 7-8.

<sup>2</sup> cf. L. Renou, *Études védiques et pāṇinéennes*, VI (Le destin du Vēda dans l'Inde), Paris, 1960, 66.

<sup>3</sup> Mīmāṃsā is closest to the *paribhāṣā* sections of the *sūtras*. See D. V. Garge, *Citations in Śabara-bhāṣya*, Poona, 1952, 50 sq.

<sup>4</sup> J. F. Staal, 'The theory of definition in Indian logic', *JAOs*, LXXXI, 2, 1961, 124.

Problems of contradiction are dealt with in this meta-linguistic context in grammar as well as in Mimāṃsā.<sup>1</sup> Pāṇini (c. 350 B.C.) gives the following meta-rule: 'in case of contradiction (between two rules) the later (rule) is to be applied'.<sup>2</sup> Here 'later rule' denotes a rule which occurs later in the sequence of rules in Pāṇini's grammar.<sup>3</sup> Patañjali (? second century B.C.) discusses this *sūtra* in the *Mahābhāṣya*<sup>4</sup> and pays special attention to the term *vipratishedha*. After giving the etymology he explains it by *itaretara-pratishedha* and *anyo'nya-pratishedha* 'mutual prohibition'. He proceeds to quote the following *vārttika*: *dvaṃ prasaṅgāv anyārthāv ekasmin sa vipratishedhaḥ* 'if two rules with different meaning apply to one (word) this is *vipratishedha*'.<sup>5</sup> He adds that this application should be possible 'at the same time' (*yugapad*) and gives the following example. According to Pāṇini, 7.3.102, long *ā* is substituted for final *a* of a nominal stem before terminations beginning with *y* or *bh*.<sup>6</sup> This enables us to form *vrkṣāya* 'to the tree', *vrkṣābhyām* 'to both trees', and apparently *\*vrkṣābhyaḥ* 'to trees', from the nominal stem *vrkṣa-* 'tree'. The following *sūtra*, 7.3.103, prescribes the substitution of *e* for this *a* before a plural termination beginning with *bh* or *s*. This accounts for the formation of *vrkṣeṣu* 'among trees'. But as *-bhyaḥ* is a plural termination, the second *sūtra* applies also to this case and enables us to form *vrkṣebhyaḥ* 'to trees'. The two rules form in this respect a *vipratishedha* 'contradiction'. Then, according to the *paribhāṣā*, as the second rule comes after the first, the second prevails and *vrkṣebhyaḥ* is explained while *\*vrkṣābhyaḥ* is not. We are perhaps inclined to regard the second rule as an exception (*apavāda*) to the first, but this does also produce the same result: for there is a *paribhāṣā* which states that the exception is stronger than the general rule.<sup>7</sup>

The above discussion can be formalized along the following lines. Let the *sūtras*, which are propositions, be ordered (as they are in Pāṇini) and be denoted by  $\phi_1, \phi_2, \dots$ , and let  $\phi_1 < \phi_j$  denote that  $\phi_1$  precedes  $\phi_j$  in this list.<sup>8</sup> In addition let  $\phi_1 \circ \phi_j$  denote that  $\phi_1$  and  $\phi_j$  are contradictory propositions. Now Pāṇini's propositions generally deal with linguistic predicates  $F(x), G(x), \dots$  which have individual words or other sounds  $x, \dots$  as their values.

<sup>1</sup> See L. Renou, 'Connexions entre le rituel et la grammaire en sanskrit', *JA*, CCXXXIII, 1941-2, 116-17, also for what follows.

<sup>2</sup> Pāṇini, 1.4.2: *vipratishedhe param kāryam*. This principle is not valid for the *Tripādī* (Pāṇini, 8.2-4): cf. H. E. Buiskool, *Pūrvatrāsiddham*, Amsterdam, 1934.

<sup>3</sup> It is irrelevant in the present context that the term *param* in the *sūtra* may mean desirable (*iṣṭam*), as it is interpreted by Patañjali (see Buiskool, op. cit., 66-7, 71-6).

<sup>4</sup> *ad loc.*; ed. Kielhorn, I, 304.

<sup>5</sup> See L. Renou, *Terminologie grammaticale du sanskrit*, Paris, 1957, 280 (s.v. *vipratishedha*).

<sup>6</sup> This rule is discussed in another context by the present author in 'A method of linguistic description: the order of consonants according to Pāṇini', *Language* [to be published, 1962].

<sup>7</sup> cf. *paribhāṣā* 38 of Nāgojī Bhaṭṭa's *Paribhāṣeṇḍuśekhara* ed. and transl. F. Kielhorn, Bombay, 1868, 34, 320; cf. also L. Renou, *Études védiques et pāṇinéennes*, II, Paris, 1956, 143.

<sup>8</sup> If *param* means 'desirable',  $\phi_1 < \phi_j$  can be interpreted to mean that  $\phi_j$  is more desirable than  $\phi_1$ .

As most rules state that a certain predicate holds under certain conditions, we may consider all and only those  $\phi_i$  which can be written in the form :

$$(x) [F_1(x) \rightarrow G_1(x)]. \quad (1)$$

According to the definition of *vipratishedha*,  $\phi_i \circ \phi_j$  if and only if :

$$\sim (Ex) [G_1(x) \wedge G_j(x)]. \quad (2)$$

The *paribhāṣā* states (in the following formula connectives and parentheses are printed in bold face in order to denote that they are used in the formalization of the meta-theory) :

$$[ (\phi_i < \phi_j) \wedge (\phi_i \circ \phi_j) ] \rightarrow \phi_j. \quad (3)$$

This *paribhāṣā* presupposes an actual contradiction. For (3) is only meaningful if  $\phi_i$  and  $\phi_j$  apply to the same case, i.e. if :

$$(Ex) [F_1(x) \wedge F_j(x)]. \quad (4)$$

However, (4) and (1) yield :

$$(Ex) [G_1(x) \wedge G_j(x)], \quad (5)$$

which contradicts (2). Then, if  $\phi_i < \phi_j$ , we derive from (3) the proposition  $\phi_j$  or :  $(x) [F_j(x) \rightarrow G_j(x)]$ . As there is an  $x$  such that  $F_j(x)$  according to (4), we can derive  $G_j(x)$  for that  $x$ .

This argument depends on a contradiction which is assumed to hold between (2) and (5), i.e. on a law of contradiction for propositions of the form :

$$\sim (\phi \wedge \sim \phi). \quad (6)$$

The analysis therefore shows what the formulation of the *paribhāṣā* already suggests, namely that the argument presupposes the validity of the principle of contradiction.

The *Kāśikā*,<sup>1</sup> a later commentary (seventh century A.D.) on Pāṇini's grammar defines *vipratishedha* in the same way, having first characterized it as *tulya-bala-virodhaḥ* 'opposition (between two propositions) of equal force'. The same definition is given by Uvāta in his commentary on the *Vājasaneyi-prātiśākhya*.<sup>2</sup> The term *virodha*, which according to the above definition has a wider denotation than *vipratishedha*, is also utilized for contradictions between two possible interpretations in the *sūtra* literature.<sup>3</sup>

In *Mīmāṃsā*, where the same *paribhāṣā* holds,<sup>4</sup> both *niśedha* and *pratishedha* denote 'prohibition'. As in India the science of grammar is primarily descriptive and not prescriptive,<sup>5</sup> the grammatical *sūtras* are propositions where the verb occurs in the *indicative* mood or the construction is purely nominal. *Mīmāṃsā*, on the other hand, is primarily concerned with Vedic injunctions which contain a verb in the *optative* mood. The principal part of such an injunction is the optative verb form, and the principal part of the optative

<sup>1</sup> *ad* Pāṇini, 1.4.2.

<sup>2</sup> Renou, *Terminologie*, 495.

<sup>3</sup> See Renou, in 'Connexions', *JA*, CCXXXIII, 1941-2, p. 117, n. 2.

<sup>4</sup> Although here *param* has a different meaning : see above, p. 55, n. 1.

<sup>5</sup> cf. the article quoted above, p. 54, n. 4.

verb form is the ending, not the verbal root. This doctrine is first formulated for positive injunctions (*vidhi*), but is subsequently transferred to negative injunctions or prohibitions. Such a view can be arrived at on account of the use of prohibitions such as *na bhakṣayet* 'he shall not eat', which in English should be interpreted as 'he shall-not eat', not as 'he shall not-eat'. Such a negative injunction does not enjoin an action which is not-eating, i.e. any definite action different from eating, but it prohibits eating.<sup>1</sup>

This distinction is interesting and seems to go a step beyond the Aristotelian doctrine of negation which is at the base of the modern logical notation. Aristotle evolved a theory of negation for propositions which will be called 'the original doctrine of negation'. This theory was extended in one direction to quantified sentences, thus arriving at the distinction between contrary and contradictory, and in another direction to modal propositions. The philosophers of the *Mīmāṃsā* system extended a similar underlying doctrine of negation to a class of non-indicative propositions, namely injunctions. To express this precisely and formally we shall need a special notation, which may be arrived at by first considering Aristotle's extensions.

In the *De interpretatione* Aristotle gives the original doctrine of negation. He says that the negation of 'man walks' ( $\tau\acute{o}$  *ἀνθρώπος βαδίζει*) is not 'not-man walks' ( $\tau\acute{o}$  *οὐκ ἄνθρωπος βαδίζει*), but 'man does not walk' ( $\tau\acute{o}$  *οὐ βαδίζει ἄνθρωπος*).<sup>2</sup> This is reflected in the modern notation, where the negation of  $F(x)$  is not defined as  $F(\sim x)$ , but as  $\sim F(x)$ .<sup>3</sup> The latter expression is in modern logic formed according to rules of formation so that it means  $\sim [F(x)]$ . Other expressions such as  $(\sim F)(x)$  are not defined and can only be expressed adequately when use is made of Church's  $\lambda$ -operator (e.g.  $[\sim \lambda x.F(x)](x)$ , provided the negation sign is defined in this context).

In the theory of quantified sentences we arrive at contradictories (*ἀντικειμένα*) if the quantifier is negated, and at contraries (*ἐναντία*) if the copula, i.e. the verb, is negated.<sup>4</sup> This can be formalized along Aristotelian lines by considering the four kinds of quantified sentences as follows :

$$\text{SaP} \quad (x)F(x) \quad (7)$$

$$\text{SeP} \quad (x) \sim F(x) \quad \text{or} : \quad \sim (Ex)F(x) \quad (8)$$

$$\text{SiP} \quad (Ex)F(x) \quad (9)$$

$$\text{SoP} \quad (Ex) \sim F(x) \quad \text{or} : \quad \sim (x)F(x) \quad (10)$$

Here (7) and (10) are contradictories, (8) and (9) are contradictories, (7) and (8) are contraries, and (9) and (10) are contraries (traditionally, subcontraries).

In Aristotle's modal logic a detailed theory is given for the relationship between negation and the four modal functors.<sup>5</sup> It is not explicitly stated what

<sup>1</sup> *Mīmāṃsā-nyāya-prakāśa*, ed. and transl. F. Edgerton, New Haven, 1921, sections 320-8.

<sup>2</sup> *De int.* 12, 21 b 1-8.

<sup>3</sup> See J. F. Staal, 'The construction of formal definitions of subject and predicate', *TPS*, 1960, 89-103.

<sup>4</sup> See, e.g., Bocheński, *Ancient formal logic*, 37.

<sup>5</sup> See, e.g., Bocheński, *op. cit.*, 59.

is the negation of the individual modal functors, for example the negation of the necessity functor  $N[F(x)]$  'it is necessary that  $F(x)$ '. Hence it is uncertain whether this negation should be interpreted as  $\sim N[F(x)]$  'it is not necessary that  $F(x)$ ' or as  $N[\sim F(x)]$  'it is necessary that  $\sim F(x)$ '. Aristotle implicitly accepts the first alternative, so that  $\sim N[F(x)]$  actually seems to denote  $(\sim N)[F(x)]$ . It is obvious that special formation rules would be needed if this situation were to be fully expressed.

The Mīmāṃsā thinkers consider exactly this question. Let the injunction 'he shall eat', which can be considered derived from the proposition 'he eats', be denoted by  $N[F(x)]$ , where  $F(x)$  denotes 'he eats'. This formalization is suggested by the fact that an injunction 'he shall eat' can under certain circumstances be interpreted as a necessary sentence 'it is necessary that he eats'. The Mīmāṃsā doctrine then states that a prohibition (*niṣedha*) or the negation of an injunction  $N[F(x)]$  is not  $N[\sim F(x)]$  but  $\sim N[F(x)]$ , just as the negation of 'he shall eat' is not 'he shall not-eat' but 'he shall-not eat'. Here also  $\sim N[F(x)]$  denotes  $(\sim N)[F(x)]$ . The formation rules required here presuppose that along with the injunctive functor  $N$  a prohibitive functor  $(\sim N)$  has been introduced at the start. This requirement is fulfilled in Mīmāṃsā, where the Vedic utterances are initially subdivided into five groups, including along with *vidhi* 'injunction' also *pratiṣedha* or *niṣedha* 'prohibition'.<sup>1</sup>

The question arises whether any meaning is to be attached to the expression  $N[\sim F(x)]$ , just as it is not impossible to conceive that a meaning could be attached to the expression  $F(\sim x)$ , which in modern logic is not a well-formed expression. The Mīmāṃsakas denote all cases of a negation of an injunction which are not *pratiṣedha* or *niṣedha* by the term *paryudāsa* which is generally translated as 'exclusion'. Their main doctrine in this respect is succinctly expressed in the following verse, where the negative is denoted by *nañ* :

*paryudāsah sa vijñeyo yatro'uttarapadena nañ*  
*pratiṣedhaḥ sa vijñeyaḥ kriyayā saha yatra nañ*

'exclusion (*paryudāsa*) is to be understood where the negative (is connected) with the next word<sup>2</sup>; prohibition (*pratiṣedha*) is to be understood where the negative (is connected) with the verb(al ending)'.<sup>3</sup>

There are two kinds of 'next words' with which the negative can be connected: (1) a verbal root; (2) a noun. We shall consider each of these two shortly.

(1) There are sentences<sup>4</sup> such as *nekṣeta* 'he shall not look' where nothing

<sup>1</sup> See, e.g., *Mīmāṃsā-nyāya-prakāśa*, section 10.

<sup>2</sup> The term *uttarapada* literally denotes the second member of a compound. According to the Sanskrit grammarians the negative particle combines with a following noun into a nominal compound (generally either *tatpuruṣa* or *bahuvrīhi*), so that the following noun is appropriately designated by the term *uttarapada*. Since *paryudāsa* applies also to verbs, *uttarapada* has been here translated by 'next word'.

<sup>3</sup> *Mīmāṃsā-nyāya-prakāśa*, section 330.

<sup>4</sup> *Mīmāṃsā-nyāya-prakāśa*, sections 332-40.

can be prohibited because the sentence is introduced as a positive injunction by a kind of injunctive functor, e.g. because the words are preceded by the phrase *tasya vratam . . .* 'his vows are . . .'. Such a sentence positively enjoins something opposed to looking (*īkṣāna-virodhī*), which can be formulated by means of the expression 'not-look' (*nekṣe*, i.e. in Edgerton's words 'the combination *nekṣeta* "he shall not look" minus the optative ending').<sup>1</sup> This is expressed by the formula  $N[\sim F(x)]$ , where  $\sim F(x)$  denotes 'not-look'.

(2) There are also sentences of quite a different type, where a noun (or, in logic, a term) is negated,<sup>2</sup> for example *nānuyājeṣu yeyajāmahaṃ karoti* 'not at the after-sacrifices does he say *ye-yajāmahe*'. Here the context shows that this does not mean *anuyājeṣu yeyajāmahaṃ na kuryāt* 'at the after-sacrifices he shall not say *ye-yajāmahe*', but it means *anuyājavyatirikteṣu yeyajāmahaṃ kuryāt* 'at sacrifices other than the after-sacrifices he shall say *ye-yajāmahe*'. This can only be expressed formally if the injunction is not derived from a predicate but from a relation, for instance  $F(x, y)$  denoting: 'at the after-sacrifices (x) he says *ye-yajāmahe* (y)'. Hence the injunction can be written as  $N[F(x, y)]$  and this kind of negation or *paryudāsa* as  $N[F(\sim x, y)]$ , which again is a positive injunction.

In grammar an analogous distinction is made.<sup>3</sup> Exclusion and prohibition are respectively called *paryudāsa-pratiṣedha* and *prasajya-pratiṣedha*,<sup>4</sup> and an almost identical verse is quoted :

*paryudāsaḥ sa vijñeyo yatrottarapadena nāñ  
prasajyapratīṣedhas tu kriyayā saha yatra nāñ.*

Renou, who mentions this verse, quotes the following example from the *Nyāsa* of Jinendrabuddhi (? eighth century A.D.).<sup>5</sup> The interpretation of the negative prefix *a-* in the word *a-kartari* occurring in Pāṇini, 3.3.19, *akartari ca kārake samjñāyām*, is under discussion. If this negation is interpreted as *paryudāsa* the meaning would be ' (The suffix *ghañ* is applied) to case relationships different from the nominative '. If the negation is interpreted as *prasajya* the meaning would be ' (The suffix *ghañ* is) not (applied) to the nominative (but is applied) to case relationships '. The first interpretation should be rejected, for the nominative is a case relationship and the word *kāraka* 'case relationship'

<sup>1</sup> Edgerton, op. cit., p. 170, n. 222.

<sup>2</sup> *Mīmāṃsā-nyāya-prakāśa*, sections 341-50. These sentences reflect Vedic sentences where the second object follows the verb but is preceded by *na*, e.g. *Aitareya-Brahmana*, 1.17.13, *prayājān evātra yajanti nānuyājān* 'in this case they offer the fore-sacrifices, not the after-sacrifices'. See J. Gonda, *Four studies in the language of the Veda*, 's-Gravenhage, 1959, 7-70 ('Amplified sentences and similar structures', 60).

<sup>3</sup> The *Mīmāṃsā* distinction was also adopted in *dharmaśāstra*. It occurs in a medieval work on *gotra* and *pravara*: see J. Brough, *The early Brahmanical system of Gotra and Pravara: a translation of the Gotra-pravara-mañjarī of Puruṣottama-panḍita*, Cambridge, 1953, p. 70, n. 3.

<sup>4</sup> cf. Edgerton, op. cit., p. 167, n. 219.

<sup>5</sup> Renou, *Durgatavrtti*, 114; also Renou, *JA*, CCXLV, 2, 1957, p. 131, n. 9, and *Terminologie*, 202, 230.

would hence be superfluous, which conflicts with a well-known economy criterion.<sup>1</sup>

The difference between on the one hand the grammatical distinction between two types of negation and on the other hand the Mīmāṃsā distinction between three types reflects the fact that grammar is descriptive and deals with propositions, while Mīmāṃsā is prescriptive and deals with injunctive functors. In formalizing the grammatical propositions, expressions of the type  $F(x)$  are sufficient and there is no need for an injunctive functor  $N$ . The above distinction could for instance be formalized by expressing the proposition 'the suffix *ghāñ* ( $a$ ) is applied to  $x$ ' by  $F(a, x)$  and by abbreviating the nominative as  $n$  and a case relationship as  $c$ . Then the *pariyudāsa* interpretation is given by :

$$F(a, c \wedge \sim n), \quad (11)$$

while the *prasajya* interpretation is given by :

$$F(a, c) \wedge \sim F(a, n). \quad (12)$$

The second type of *pariyudāsa* in Mīmāṃsā comes into being on account of the fact, that the negation can be attached to either  $N$  or  $F$ . This is impossible in grammar. The relation between Mīmāṃsā and grammar can be summarized in the following table, where the characteristic structure of each operation is given by means of a formal expression :

MĪMĀMSĀ	GRAMMAR
<i>vidhi</i> : $N[F(x)]$	<i>sūtra</i> : $F(x)$
<i>niṣedha</i> : $(\sim N)[F(x)]$	<i>prasajya-pratiṣedha</i> : $\sim F(x)$
<i>pariyudāsa</i> II : $N[F(\sim x)]$	<i>pariyudāsa-pratiṣedha</i> : $F(\sim x)$
<i>pariyudāsa</i> I : $N[(\sim F)(x)]$	—

The Mīmāṃsā concepts of *niṣedha* and of two types of *pariyudāsa* constitute a logical system of negations which is quite different from the Aristotelian doctrines of negation which are at the base of modern formulations and also of the modern symbolical notation. We have seen that the formation rules which express the structure of the modern notation and determine whether an expression is well-formed or not, depend on Aristotle's analysis of negation. It is important to realize that these differences in logical structure between India and the West do not imply and are not implied by similar differences in the *total* linguistic structure of the languages in which the logical doctrines were evolved. The linguistic background of the Indian concepts in Sanskrit is quite expressible and intelligible in, e.g., Greek or English. This may go far to show that logical doctrines may have been evolved along the lines suggested by various *partial* structures of ordinary language. The limitations of logical systems, Western as well as Indian, can be studied when it is precisely known which linguistic structure of a language system is at the background of a particular logical system.

In the present context this may be illustrated by writing in a table : A, the

<sup>1</sup> cf. the article quoted above, p. 55, n. 6.

name of the operation concerned ; B, the formal expression for it ; and C, the linguistic structure to which it is related, as exemplified by a simple English sentence :

A	B	C
	F(x)	the door is locked
<i>vidhi</i>	N[F(x)]	the door should be locked
<i>niṣedha</i>	(∼ N) [F(x)]	the door should not be locked
<i>paryudāsa</i> I	N[(∼ F) (x)]	the door should be unlocked
<i>paryudāsa</i> II	N[F(∼ x)]	another door should be locked

While we find in the Mīmāṃsā doctrines a very refined theory of the negation of injunctions, there is no discussion, apparently, of a principle of contradiction. At first sight the reason for this may seem to be that all injunctions which Mīmāṃsā discusses are Vedic statements which are considered revealed texts (*śruti*), so that contradictions are *a priori* excluded. The Mīmāṃsā position is similar to that of any other hermeneutic system of thought which attempts to reconcile a number of statements which are not necessarily compatible, such as some theological systems in the Western monotheistic religions. Historically this fact may partly explain the origination of certain logical doctrines concerning negation. But the inner structure of the logical concepts themselves explains the absence of a law of contradiction. This can be shown for each of the negations considered earlier. In terms of Western logic we should be prepared to find :

$$\sim (N[F(x)] \wedge \sim \{N[F(x)]\}), \quad (13)$$

for this is nothing but a substitution result of the law of contradiction  $\sim (A \wedge \sim A)$ . This is no compelling reason, however, to be certain that each of the following principles should hold :

$$\textit{niṣedha} : \quad \sim \{N[F(x)] \wedge (\sim N) [F(x)]\} \quad (14)$$

$$\textit{paryudāsa}$$
 I :  $\quad \sim \{N[F(x)] \wedge N[(\sim F)(x)]\} \quad (15)$

$$\textit{paryudāsa}$$
 II :  $\quad \sim \{N[F(x)] \wedge N [F(\sim x)]\}, \quad (16)$

for these cannot be all considered direct substitution results from  $\sim (A \wedge \sim A)$ . That (16) need not be valid is also intuitively clear. This may illustrate the special character of the negations considered here. For the proof of (14), (15), and (16) would depend respectively on proofs of the following three equations :

$$(\sim N) [F(x)] = \sim \{N[F(x)]\} \quad (17)$$

$$N[(\sim F)(x)] = \sim \{N[F(x)]\} \quad (18)$$

$$N[F(\sim x)] = \sim \{N[F(x)]\}. \quad (19)$$

If these could be proved, however, each of the three negations considered here would be identical with the Aristotelian negation.

While the law of contradiction need not necessarily hold for injunctions which are always injunctions enjoining action, the same law does hold for the results of the activities based upon these injunctions. In the Sanskrit terminology this can be expressed by saying that the law of contradiction need not

hold in the realm of what is to be established (*sādhya*), but holds in the realm of what is established (*siddha*). The philosophers of the Uttara-Mīmāṃsā or Vedānta of the Advaita school rejected the Mīmāṃsā interpretation of all Vedic sentences as injunctions dealing with *sādhya*. These Vedāntins considered either all or at least some of the Vedic utterances as dealing with *siddha*. The law of contradiction, therefore, is accepted by the Advaitins for Vedic sentences. Within Indian philosophy this is an extremely important development: the consideration of certain authoritative statements, especially the 'great statements' (*mahāvākya*) in which the Advaitins are especially interested, as propositions, leads to the problem of how such propositions are related to the reality which they describe, and hence to the problem of truth. This is expressed in the Advaita doctrine that the truth regarding an object is 'dependent upon the object' (*vastutantra*).

In the following two passages of the *Brahmasūtrabhāṣya* of the famous Advaita philosopher Śaṅkara (eighth century A.D.) this topic is lucidly expounded. In the first<sup>1</sup> Śaṅkara stresses the fact that option (*vikalpa*) is possible with respect to injunctions only. Paradoxically these injunctions are exemplified by sentences where the verb occurs in the indicative; but this is not essential, for philosophy may free itself from language and Śaṅkara explicitly speaks about *vidhi* and *pratiśedha*. There is option, then, according to Śaṅkara, when we meet with positive and negative injunctions such as 'at the *atirātra* sacrifice he takes the *ṣoḍaśin* cup' (*atirātre ṣoḍaśinaṃ grhṇāti*) and 'at the *atirātra* he does not take the *ṣoḍaśin* cup' (*nātirātre ṣoḍaśinaṃ grhṇāti*); 'he sacrifices after the sun has risen' (*udite juhōti*) and 'he sacrifices when the sun has not yet risen' (*anudite juhōti*).<sup>2</sup> All such injunctions pertain to what is *sādhya*. He then continues: 'But there is no option as to whether a thing is thus or thus, is or is not. Option depends on human notions. Knowledge of the nature of a thing does not depend on human notions. It depends only on the thing itself. /To say with regard to a pillar "it is a pillar or it is a man or it is something else" does not result from correct knowledge. To say that it is a man or something else does result from false knowledge. To say that it is a pillar results from correct knowledge, because it depends on the thing itself. Therefore the means of knowing objects, that are existent things, depend on the things themselves'.<sup>3</sup>

In a later passage the concept of knowledge (*jñāna*) is dealt with in a slightly different context, where Śaṅkara speaks about meditation (*dhyāna*), a mental

<sup>1</sup> *Brahmasūtrabhāṣya*, 1.1.3. Ed. Nirṇaya Śāgara, Bombay, 1934, 52-3. Śaṅkara always assumes that no two Vedic utterances are contradictories. See e.g. R. de Smet, 'The theological method of Śaṅkara', *Revue Philosophique de Louvain*, LII, 1954, 31-74, and n. 47.

<sup>2</sup> In Mīmāṃsā this is called *ṣoḍaśi-nyāya* 'principle of the *ṣoḍaśin* cup': see, e.g., Garge, op. cit., 265.

<sup>3</sup> *na tu vas'tv evaṃ naivam asti nāstīti vā vikalpyate / vikalpanāḥ tu puruṣabuddhyapekṣāḥ / na vastuyāthātmyajñānaṃ puruṣabuddhyapekṣam / kiṃ tarhi vastutantram eva tat / nahi sthānāv ekasmin sthānur vā puruṣo'nyo veti tattvajñānaṃ bhavati / tatra puruṣo'nyo veti mithyājñānam / sthānur eveti tattvajñānaṃ vastutantratvāt / evaṃ bhūtavastuviśayajñānaṃ prāmāṇyaṃ vastutantram //*

activity which is based upon a special kind of *vidhi*. Knowledge which is expressed in propositions, for which the law of contradiction holds, is not contrasted with injunctions in general, but with meditation. In Advaita *jñāna* acquires the meaning of intellectual knowledge and Śaṅkara thus goes beyond the magical atmosphere of the Brāhmaṇa literature, where an important aspect of *jñāna* can be described as magical identification.<sup>1</sup> In the following passage<sup>2</sup> Śaṅkara has first quoted a *vidhi* which enjoins meditation and then continues: 'If meditation and reflection are mental activities, they can be performed, not performed, or performed differently by man, because they depend on man. But knowledge originates from the means of knowledge, which have as their objects things as they exist. Therefore knowledge cannot be performed, not performed, or performed differently, for it solely depends on the object. It does neither depend on authoritative statements, nor on man'.<sup>3</sup>

These passages show how Advaita constitutes in this respect a return from Mīmāṃsā to the grammatical doctrines.<sup>4</sup> The difference is, that in grammar the law of contradiction is immediately applied to propositions, while in Advaita the law of contradiction is said to hold for propositions because it holds in reality. Śaṅkara's logic is founded on his ontology. This difference partly reflects the different preoccupations of grammarians and philosophers. It shows incidentally that the Advaita position in philosophy is firmly realistic.<sup>5</sup> The linguistic background for both views is that grammarians as well as Advaitins are mainly interested in description and in propositions where the verb occurs in the indicative mood or where the sentence is purely nominal. The Mīmāṃsakas, on the other hand, are interested in prescription and in injunctions where the verb occurs in the optative mood.<sup>6</sup>

<sup>1</sup> See, for instance, H. Oldenberg, *Die Weltanschauung der Brahmana-Texte*, Göttingen, 1919, 110-23.

<sup>2</sup> *Brahmasūtrabhāṣya*, 1.1.4. Ed. Nirṇaya Sāgara, 83; discussed by the present author in *Advaita and neo-Platonism: a critical study in comparative philosophy*, Madras, 1961, 101-2, and cf. 80-1.

<sup>3</sup> *dhyānaṃ cintanaṃ yady api mānasam tathāpi puruṣeṇa kartumakartumanyathā vā kartum śakyam puruṣatantratvāt / jñānaṃ tu pramāṇajanyam / pramāṇaṃ ca yathābhūtavastuviśayam / ato jñānaṃ kartumakartumanyathā vā kartumaśakyam kevalam vastutantram eva tat / na codanā-tantram / nāpi puruṣatantram //*

<sup>4</sup> cf. Renou, *JA*, CCXXXIII, 1941-2, 115: 'le style nominal, représenté de façon rigoureuse par les sūtra grammaticaux et que reprendront les sūtra philosophiques, cède la place, dans le parer, à un style verbal caractérisé par l'indicatif descriptif, l'optatif prescriptif, l'absolutif d'enchaînement temporel . . .'

<sup>5</sup> This holds on the *vyāvahārika* level. Śaṅkara's explicit statements are somewhat obscured in O. Lacombe, *L'Absolu selon le Védānta*, Paris, 1937, 124.

<sup>6</sup> Brough, who rightly stressed this Mīmāṃsā preoccupation with injunctions, has also drawn attention to the fact that the terms *vidhi* and *pratiśedha* came to refer to indicative sentences as well: 'although later Indian logic deals largely in indicative sentences, the linguistic thought of philosophers in India was not so strictly confined to indicative propositions as that of logicians in the west. This influence can be traced in the terms *vidhi* and *pratiśedha*, originally meaning injunction and prohibition, but in later texts occasionally used to apply simply to positive and negative statements' (J. Brough, 'Some Indian theories of meaning', *TPS*, 1953, 162).

The grammarians, accordingly, do not use *vidhi* to denote positive linguistic propositions, but generally employ the term *sūtra* 'rule' and sometimes, more specifically, *utsarga* 'general rule'. The term used for negative linguistic propositions is, as we have seen, either *pratiśedha* or *nīśedha*. These terms seem to have similar, but not identical functions to the Mimāṃsā term for 'prohibition', reflecting a trend among the grammarians which deviates from their customary stress on description and emphasis on empirically tested usage (*loka*). There are occasions where the grammarians actually prohibit the use of certain words, which may have been colloquial and which are referred to as *apaśabda* 'incorrect word'. In this connexion Patañjali propounds an interesting analysis of the relationship between positive and negative linguistic rules, which reflects the similar relationship between positive and negative ritualistic rules. The text is as follows<sup>1</sup>:

'Now words have to be examined. How is this to be done? Are (correct) words to be taught, or perhaps incorrect words, or perhaps both? Our purpose will be served by the teaching of either. Thus by a restrictive condition (*niyama*) on what food is fit to be eaten is implied a prohibition (*pratiśedha*) of what food is not fit to be eaten. For example when we say "Five five-toed animals are fit to be eaten" it is implied that (five-toed animals) different from these are not fit to be eaten. Or alternatively, by a prohibition of what is not fit to be eaten is implied a restrictive condition on what is fit to be eaten. For example when we say "the domestic fowl is not fit to be eaten, the domestic pig is not fit to be eaten" it is implied that the wild variety (of these animals) is fit to be eaten. This applies also in the present context. If the correct words are taught, for instance when the word *gauḥ* has been taught, it is implied that *gāvī*, etc., are incorrect words. If on the other hand incorrect words are taught, for instance when the words *gāvī*, etc., have been taught, it is implied that *gauḥ* is the correct word'.<sup>2</sup>

This passage gives interesting rules for *pratiśedha* negations which are closely related to the law of double negation. If  $N[F(x)]$  denotes that  $F(x)$  is proscribed by means of a restrictive condition (*niyama*), the ritualistic rule first quoted can be written as:

$$N[F(x)] \longleftrightarrow (\sim N)F(\sim x), \quad (20)$$

where  $F(x)$  denotes: *x bhakṣyam* 'x is fit to be eaten'. The examples which follow correspond to this rule and the grammatical rule can be given in the same form. In all cases Patañjali expresses equivalence by means of biconditionals.

<sup>1</sup> Ed. F. Kielhorn, 1, 5; cf. ed. and transl. K. C. Chatterji, Calcutta, 1957, 34-5.

<sup>2</sup> *śabdānuśāsanam idānīm kartavyam / tat katham kartavyam / kim śabdopadeśah kartavyah āhovid apaśabdopadeśah āhovid ubhayopadeśah iti / anyataropadeśena kṛtam syāt / tad yathā bhakṣyamīyamenābhakṣyapratīśedho gamyate / pañca pañcanakhā bhakṣyāḥ ity ukte gamyate etad ato'nye'bhakṣyā iti / abhakṣyapratīśedhena vā bhakṣyamīyamah / tad yathā abhakṣyo grāmyakuk-kuṭah abhakṣyo grāmyasūkarah ity ukte gamyate etad āraṇyo bhakṣya iti / evam ihāpi / yadi tāvac chabdopadeśah kriyate gaur ity etasminn upadiṣṭe gamyate etad gāvīdayo'paśabda iti / yathāpy apaśabdopadeśah kriyate gāvīdiṣupadiṣṭeṣu gamyate etad gaur ity eṣa śabda iti //*

That the grammarian concept of *pratiṣedha*, however, is not the same as the Mīmāṃsā concept is shown by Patañjali in another passage, where the law of contradiction is explicitly formulated in the following terms : *vidhipratiṣedhayor yugapad vacanānupapattiḥ* ' it is impossible for a statement to express simultaneously a *vidhi* and a *pratiṣedha* '.<sup>1</sup> This corresponds to :

$$\sim \{N[F(x)] \wedge (\sim N) [F(x)]\} \quad (21)$$

and confirms the view that the grammarians, like the Advaitins, deal with indicative sentences, where the law of contradiction holds, while Mīmāṃsakas deal with injunctions, where the law of contradiction need not hold. In grammar, then, *vidhi* means rule or positive statement, and not injunction.

These passages enable us to see what is the precise structure of the negations concerned. If numerous texts of this type could be found and analysed, it would enable us to construct complete lists of formation rules. In the present context our purpose is merely to draw attention to logical possibilities which are different from those generally considered and which show that both Western and Indian concepts of negation are the product of a particular development.

The law of double negation is explicitly given in the *Pradīpa* of Kaiyaṭa, a grammatical commentary of the eleventh century. It is formulated as follows : *pratiṣedhapratiṣedhād vidhir bhavati* ' the prohibition of a prohibition gives an injunction '.<sup>2</sup> This could probably be expressed by :

$$\sim \sim N[F(x)] \rightarrow N[F(x)] \quad (22)$$

since :  $(\sim N) [(\sim N) F(x)]$  is not defined. The Sanskrit dictionary of V. S. Apte quotes a grammatical rule regarding the use of the particle of negation *na* which is also called *niṣedha*. The source is not given. The rule is : *dvau niṣedhau prakṛtārthaṃ gamayataḥ* ' two particles of negation give the meaning of the original '.<sup>3</sup> These formulations of the law of double negation precede by centuries the purely logical laws of double negation propounded in *navya-nyāya* and studied by Ingalls.<sup>4</sup>

We have seen that the grammarian and Advaitin doctrines of negation have a linguistic background in the structure of the indicative mood of the verb, while the Mīmāṃsā doctrine of negation has a linguistic background in the structure of the optative mood of the verb, which is reflected also in the imperative. The grammatical rule about two consecutive negative particles cancelling each other draws our attention to the linguistic background of the law of double negation. It is well known that in Indo-European two negatives can either cancel each other or supplement each other. This is a linguistic

<sup>1</sup> *Mahābhāṣya ad Pāṇini*, 1.1.44, ed. Kielhorn, I, 103 ; cf. V. G. Paranjpe, *Le vārtika de Kātyāyana*, Paris, 1922, 30-1.

<sup>2</sup> Renou, *Terminologie*, 219.

<sup>3</sup> A similar quotation (*dvau nañau prakṛtyarthaṃ gamayataḥ*) is found in P. C. Chakravarti, *The linguistic speculations of the Hindus*, Calcutta, 1933, p. 436, n. 3.

<sup>4</sup> D. H. H. Ingalls, *Materials for the study of Navya-nyāya logic*, Cambridge, Mass., 1951, 68-72 ; cf. J. F. Staal, *Indo-Iranian Journal*, IV, 1, 1960, 70-1.

fact which can be observed in Sanskrit as well as in Greek. It is not very hazardous to assume that the cancelling particles of negation in ordinary language are at the background of the logical law of double negation.

Delbrück called the accumulative use of more than one negative particle 'Ergänzungsnegation' and gave examples from Sanskrit, Lithuanian, Slavonic, and Greek.<sup>1</sup> Accumulative negation is also often found in modern languages.<sup>2</sup> It seems again likely that this use is at the background of the logical systems which reject the law of double negation, either in Navya-nyāya or in Intuitionism.<sup>3</sup>

The distinction between the negation of terms and the negation of sentences or predicates is also related to certain facts of language. It is well known that in Indo-European there are two forms of negation : sentence negation, expressed by *ne* (*nē*) and *mē*, and word negation, expressed by the so-called privative prefix.<sup>4</sup> Chinese has the analogous distinction between respectively *pu* and *fei*,<sup>5</sup> and A. C. Graham has quoted a passage where a law of contradiction is formulated in terms of the term negation *fei*.<sup>6</sup> Inspection of the Sanskrit passages discussed in the above shows that the sentence negation *na* is used in all cases. There is one case, however, where the privative syllable *a(n)*-could have been used. In this case negation of a term was used in the formalization despite the fact that this is not permitted in the customary notation of modern logic. Here, when the ambiguous use of the sentence negation *na* in *nānuyājeṣu* . . . 'not at the after-sacrifices . . .' is explained by *anuyājavya-tirikteṣu* 'at sacrifices other than the after-sacrifices', it would have been possible to explain this merely as *an-anuyājeṣu* 'at the "un"-after-sacrifices'. Though this form is not given in the text, the explanation given there confirms this view : 'because the negative (*nañ*) is combined with the word "after-sacrifices" (*anuyāja*) we have a case of *paryudāsa*. For the negative and the word after-sacrifices together denote what is other than the after-sacrifices'.<sup>7</sup>

Whenever the two negatives remain clearly distinct there is no danger of confusion and logicians can clearly distinguish between negation of terms and of predicates and study the relationships between both. Logical problems arise whenever the sentence negation is used or functions as word negation, and vice versa. In Indo-European both possibilities are realized, though infrequently. In Vedic *nākis* can mean 'nobody' ('not anybody'), but more often

<sup>1</sup> B. Delbrück, *Vergleichende Syntax der indogermanischen Sprachen*, Strassburg, I-III, 1893-1900, II, 535-6.

<sup>2</sup> See, e.g., O. Jespersen, *The philosophy of grammar*, London, 1948, 331-4.

<sup>3</sup> See Ingalls, loc. cit.

<sup>4</sup> Delbrück, II, 533.

<sup>5</sup> See A. C. Graham, *BSOAS*, XXII, 3, 1959, 567, and *Asia Major*, NS, VII, 1-2, 1959, 88.

<sup>6</sup> *Asia Major*, NS, VII, 1-2, 1959, 91, 'One saying that it is an ox, the other that it is not, is "contesting the other's case". Their claims will not both fit'. The first negative particle translates the term negation *fei*, the second the sentence negation *pu*. Cf. also J. F. Staal, *TPS*, 1960, 93.

<sup>7</sup> *nañō'nuyājaśabdena sambandham āśritya paryudāsa āśrīyate, nañānuyājaśabdābhyām anuyājavyatiriktalakṣaṇāt : Mīmāṃsā-nyāya-prakāśa*, section 349.

it merely means 'not' or 'never' and is applicable to the whole sentence.<sup>1</sup> A similar transference of the sentence negative *na* to a term is found in the adverb *naciram* 'not long', which can be explained as a development from sentences of the type *na ciram vasati* 'he does not stay long'.<sup>2</sup>

The Sanskrit grammarians, on the other hand, mention a case where the privative syllable is attached to a verbal form, which thereby acquires a special meaning. This forms an exception to Pāṇini, 2.2.6, *nañ* 'the negative particle (combines with a noun)', and is given by the *Kāśīkā* when commenting upon this *sūtra*. The form given is: *apacasi* 'you are a bad cook, you cook badly', from *pacasi* 'you cook'.<sup>3</sup> Pāṇini describes the word negation *a(n)*- as derived from the sentence negation *na* which loses its initial *n*.<sup>4</sup> While this is for Pāṇini a matter of descriptive grammar, it also holds historically.<sup>5</sup>

Aristotle in the sentences discussed earlier uses the Greek sentence negations *μή* and *οὐκ* (for Indo-European *ne*)<sup>6</sup> also for the negation of terms and nouns. Not only *τὸ μή εἶναι λευκόν*, but also *τὸ εἶναι μή λευκόν*, and not only *τὸ οὐ βιάζει ἄνθρωπος*, but also *τὸ οὐκ ἄνθρωπος βιάζει*. It is not surprising, therefore, that Bocheński finds that Aristotle 'struggled hard with the initial confusion of the negation of a sentence with the negation of its terms'.<sup>7</sup> The Indian thinkers were used to the grammarian's distinctions and do not seem to have been similarly confused, as the *Mīmāṃsā* doctrines quoted above show. In addition Indian philosophers have always been used to dealing skilfully with negations. On the one hand a rich and ramified negative theology developed round the use of the sentence negative in passages such as the *mahāvākya* ('great statement') *neti neti* 'not thus, not so'.<sup>8</sup> On the other hand the negative prefix is used in many cases where Western thought seems inclined to employ a positive terminology.<sup>9</sup>

In Indian logic, lastly, extensive use is made of the prefix *vi-* which further specifies the function of the privative syllable *a(n)*-. The Buddhist logicians called *sapakṣa* any locus where the *hetu* 'reason' occurs. A locus where the *hetu* does not occur is called *asapakṣa*, later *vipakṣa*. Dharmakīrti (seventh century A.D.) formulates the relationship between these two terms by means of the quantifier *eva* 'only'. In his formulation 'occurrence of the reason in *sapakṣa* only' (*sapakṣa eva sattvam*) is equivalent to 'non-occurrence in *asapakṣa* of the reason only' (*asapakṣe cāsattvam eva*).<sup>10</sup> This leads to the discovery of the law of contraposition and to further developments.<sup>11</sup>

<sup>1</sup> Delbrück, II, 524.

<sup>2</sup> Delbrück, II, 534.

<sup>3</sup> *Kāśīkā* to Pāṇini, 2.2.6, and 6.3.73.

<sup>4</sup> Pāṇini, 6.3.73.

<sup>5</sup> Delbrück, II, 529-33.

<sup>6</sup> Delbrück, II, 533.

<sup>7</sup> Bocheński, *Ancient formal logic*, 38.

<sup>8</sup> *Bṛhadāraṇyakopaniṣat*, 2.3.6, 3.9.26, 4.2.4, 4.2.3, 5.15.

<sup>9</sup> See, e.g., J. Gonda, *Four studies in the language of the Veda*, 's-Gravenhage, 1959, 95-117 ('Why are *ahimsā* and similar concepts often expressed in a negative form?'); H. Nakamura, *The ways of thinking of Eastern peoples*, Tokyo, 1960, 23-32.

<sup>10</sup> *Nyāyabindu*, ed. T. I. Stecherbatskoy (Bibliotheca Buddhica, VII), Petrograd, 1918, 19; ed. Candraśekhara Śāstri, Banaras, 1954, 23.

<sup>11</sup> See J. F. Staal, 'Contraposition in Indian logic', to be published in *Proceedings of the 1960 International Congress for Logic, Methodology and Philosophy of Science*, Stanford, Calif. (Chapter 5 in this volume.)

The Indian logicians are mainly interested in contradiction in connexion with the doctrine of inference. Instead of the term *vipratīṣedha* or related terms, use is made of the terms *viruddha* and *virodhī*, which we have seen used as general terms applied to cases of opposition in grammar as well as in Mīmāṃsā. In the *Nyāya-sūtra* (? second century A.D.) a *hetu* 'reason' is called *viruddha* 'opposed', when it opposes a conclusion (*siddhānta*) which has been established.<sup>1</sup> Later, *viruddha* occurs among the fallacious reasons (*hetvābhāsa*) and a *hetu* is called *viruddha* if it is the opposite of the *sādhya* 'what is to be proved'. For example, the inference *śabdo nityaḥ kṛtakatvāt* 'sound is eternal because it is created' is invalid, for creation and eternity are each other's opposite.<sup>2</sup>

In Indian logic the law of contradiction is widely utilized, explicitly as well as implicitly. One of the most explicit formulations is due to Udayana (tenth century A.D.) and is quoted by D. M. Datta.<sup>3</sup> It runs as follows: *paraspara-virodhe na prakārantarasthitih* 'when two are mutually opposed there is no occurrence (of both) within the same class'.<sup>4</sup> Implicitly the principle of contradiction is presupposed in most of the logical discussions of *Navya-nyāya*. It becomes explicit when it is attempted to give a detailed formalization of logical arguments.<sup>5</sup>

The doctrines which have been discussed in this paper belong to numerous systems of thought and it would require much more space to study them fully within their own contexts. However, from the general logical viewpoint adopted here, the various interconnexions between the Indian systems on the one hand, and Aristotle on the other hand, have been more apparent than their divergent backgrounds. We have seen that in India, the law of contradiction is formulated and strictly adhered to in grammar, in Advaita Vedānta, and in logic. In Mīmāṃsā the law of contradiction is discarded.<sup>6</sup> The reason

<sup>1</sup> *Nyāya-sūtra*, 1.2.6.

<sup>2</sup> See, e.g., *Tarka-saṅgraha*, 54, ed. Y. V. Athalye, Poona, 1930, 45-6, 302.

<sup>3</sup> D. M. Datta, 'Epistemological methods in Indian philosophy', in C. A. Moore (ed.), *Essays in East-West philosophy*, Honolulu, 1951, 73-88.

<sup>4</sup> Udayana, *Nyāya-kusumāñjali*, 3.8, *ibid.*, p. 88, n. 17.

<sup>5</sup> See, e.g., J. F. Staal, 'Correlations between language and logic in Indian thought'. *BSOAS*, XLIII, 1, 1960, especially 116-17. (Chapter 1 in this volume.)

<sup>6</sup> Since only the doctrines mentioned are dealt with in this paper, these conclusions do not imply that there are not also other systems where the law of contradiction is denied. Such systems exist in Buddhism (see, e.g., p. 52, n. 4, above, and A. Kunst, 'The concept of the principle of excluded middle in Buddhism', *Rocznik Orientalistyczny*, XXI, 1957, 141-7) and in Jainism (especially in the opportunist *syādvāda* 'let-it-be doctrine'). In Hinduism such thought appears in particular in the Advaita doctrine of the indeterminability (*anirvacanīyatva*) of the world-illusion, which is specified as *sadasadbhyām anirvacanīya* 'indeterminable either as real or as unreal' (similarly: *sadasadbhyām vilakṣaṇa*). If this is interpreted as  $\sim (aV \sim a)$ , it violates the law of contradiction. In Advaita the world-illusion is therefore *sarvanyāyavirodhinī* 'opposed to all logic' (*Naiṣkarmyasiddhi*, 3.66). These doctrines have been vehemently criticized in the name of logic and the law of contradiction both by Viśiṣṭādvaitins (see, e.g., S. N. Dasgupta, *A history of Indian philosophy*, III, Cambridge, 1952, 177), and by Dvaitins (see, e.g., Dasgupta, IV, Cambridge, 1955, 204; A. B. Shastri, *Studies in post-Sāṃkhya dialectics*, Calcutta, 1936, 180, 195-6). The law of

for this is not a mystical tendency, which rejects or merely neglects all intellectual distinctions, but a consistent unfolding of the implications inherent in a particular kind of negation. The law of contradiction is then seen to be dependent on the kind of negation which is used in its formulation. Directing our attention to the different types of underlying negations, we have seen that these can be related to various negations used in ordinary language. These negations occur in Sanskrit as well as in Greek and in other Indo-European languages. The principal distinction is that between the negation of nouns and the negation of verbs, a distinction which also obtains in Chinese and probably in numerous other language types. In as far as syntax, logic, and philosophy are mainly interested in propositions or sentences, the negation of the verb, which generally constitutes the negation of the sentence itself,<sup>1</sup> is the most important negation in the present context. However, the verb possesses several moods and negative particles do not function in the same way in each of these. In particular, the negative particle is combined with the indicative (and therefore also with purely nominal sentences, which mostly have an indicative character<sup>2</sup>) in such a manner, that sentences with and without the negative particle exclude each other. This leads to the law of contradiction formulated in terms of indicative sentences. When the particle of negation is used with other moods, however, in particular with the optative or the imperative, sentences with and without negation do not function in this exclusive fashion. For such sentences, therefore, the law of contradiction need not hold. This is fully recognized in *Mimāṃsā*, a system of thought which is mainly interested in injunctions generally formulated with the help of the optative mood of the verb.

We may now return to our point of departure. Aristotle's defence of the law of contradiction reflects the emphasis he lays upon indicative sentences. That Western logic has found it useful to develop further along similar lines is not surprising, for indicative sentences are those which are used in all descriptive systems, in particular in science. However, if the logical structure of prescriptive

the excluded middle on the other hand seems to be accepted by the Advaitin Toṭaka who maintained that there exists no intermediary between *sat* 'being' and *asat* 'non-being' (see P. Hacker, *Untersuchungen über Texte des frühen Advaitavāda*, Mainz, 1950, 163). Whereas we have seen that Śāṅkara himself uses and accepts the law of contradiction, he can also be seen to argue in accordance with the law of double negation. An example occurs in *Brahmasūtrabhāṣya*, 4.1.3, where the conclusion, that the worshipper is not different from the deity, is drawn from a scriptural statement which opposes the negation of this view, i.e. 'Now if someone worships a deity as other, saying "the deity is one and I am another", he does not know' (*Bṛhadāraṇyakopaniṣat*, 1.4.10). Elsewhere Śāṅkara appears to refer implicitly to the law of contradiction when saying that different Vedānta texts cannot teach different cognitions of Brahman, for it is certain that, if at all they differ, 'only one of them is the right one, the others are erroneous' (*teṣāṃ ekam abhrāntaṃ bhrāntānītarāṇī : Brahmasūtrabhāṣya*, 3.3.1 ; see J. F. Staal in *Atti del XII Congresso Internazionale di Filosofia*, 1958, x, Firenze, 1960, 227).

<sup>1</sup> cf. the article quoted above, p. 57, n. 3.

<sup>2</sup> It has been pointed out, however, that purely nominal sentences of the type *Pax vobiscum!* have an optative character.

sentences had been studied in greater detail (a study which Aristotle had undertaken from a particular point of view in his modal logic), Western logic might have arrived at different doctrines of negation and contradiction in addition to the traditional Aristotelian doctrines. This was done in India in *Mīmāṃsā* and in the legal literature of *dharmaśāstra*, and to a limited extent in grammar. A similar development in Western logic could have been equally useful in Western prescriptive sciences, for instance in law, in ethics, and, to a limited extent and from a particular point of view, in grammar.<sup>1</sup> The fact that this has not been done may account for the characteristically Western situation, where the sciences have evolved a logical structure which is far superior to the logical structure of, e.g., law or ethics. In India, on the other hand, *Mīmāṃsā* possesses in principle as scientific a structure as grammar or logic have. It could be said that in the West in general there is an over-emphasis on indicative sentences, which is similar to the often-stressed over-emphasis on the subject-predicate structure. In fact, the latter is mainly available within the framework of the former.

There is no need to explain the law of contradiction in Indian thought as the result of a possible Western influence. On the contrary, it reflects a use of negation which can be found in all Indo-European languages, and probably in many other languages. The last problem which may therefore be raised in this connexion is the Wittgensteinian problem of the relation between language and logic or between language and thought in general. Does the material discussed in the present paper give support, or provide counter-examples, to the thesis that thought depends on language?

In its original form this thesis, either in the manner in which it was analysed by Wittgenstein, or in the manner in which it was suggested by Whorf and by other linguists,<sup>2</sup> is almost certainly unverifiable. In addition Wittgenstein's own method, confined as it was to German and to English, could never attain a satisfactory degree of universality. It has therefore rightly been suggested that the thesis could be tested by the empirical study of linguistic material from divergent sources.<sup>3</sup> The present study, which was partly undertaken in a similar spirit, may show that we are only at the beginning of possibly extensive investigations into the relationships of particular logical, philosophical, and grammatical doctrines with particular linguistic structures of the languages in which they are expressed. These investigations may be particularly illuminating when comparative studies are made of the linguistic structure of

<sup>1</sup> Though modern linguistics aims at being-descriptive and not prescriptive, it is possible to formulate general rules *prescribing* how to arrive at a set of rules which together constitute a *description* of a language. This is for example done by Chomsky when he gives transformation rules in the form: 'rewrite X as Y', i.e. in the form of an injunction (e.g. N. Chomsky, *Syntactic structures*, 's-Gravenhage, 1957, 26 sq.).

<sup>2</sup> See, e.g., B. L. Whorf, *Language, thought and reality*, New York, 1956; H. Hoijer (ed.), *Language in culture*, Chicago, 1954.

<sup>3</sup> See A. H. Basson and D. J. O'Connor, 'Language and philosophy: some suggestions for an empirical approach', *Philosophy*, xxii, 1947, 49-65.

a language, in which relatively highly developed logical, philosophical, and grammatical doctrines are available. This requirement is obviously fulfilled in the case of Sanskrit and Indian thought, and it may to a smaller extent be fulfilled with respect to Chinese.

From this point of view the provisory results of the present paper can perhaps be formulated as follows. It could be shown, perhaps somewhat unexpectedly, that neither the linguistic structure of Sanskrit, nor that of Greek, leads to a particular logical structure. In a detailed investigation it makes no sense to speak of *the* structure of a language. It has been seen, however, that certain structures of language, which are available in Sanskrit, in Greek, and in other languages, are related to particular logical doctrines. The problem then becomes, to see which particular linguistic structure is related to which particular logical doctrine. In the present study some examples of such relationships have been given. On the basis of similar investigations it may be possible to reach more general conclusions.

Does this finally imply that logic or thought in general can be derived from language? The answer must, for the time being, remain uncertain, but we have no good reason as yet to believe that it might turn out to be in the affirmative. For language itself, in order to function, has to obey certain rules, some of which may be made explicit as syntactical rules. Such rules may also be reflected in logical rules and in rules of thought. But while logical rules cannot be reached and formulated without a linguistic background and without a linguistic framework, no language can function without a logical structure which is implicit in it.<sup>1</sup>

<sup>1</sup> I am grateful to Dr. D. L. Friedman who read this paper and made valuable observations.

THE CONCEPT OF *PAKṢA* IN INDIAN LOGIC

In studying a civilization different from our own we are prone to impose the conceptual framework and prejudices of our own tradition. The study of Indian logic by Western scholars, including Indian scholars who accepted certain tenets of Western logic, forms no exception. S. C. Vidyabhusana, the first historian of Indian logic, looked at his subject through eyes so colored by what he regarded as Aristotelian logic, that he talked of the 'Indian syllogism' and saw in it traces of the influence of Aristotle – a historical claim no serious student of Indian logic would nowadays wish to make his own. Moreover, like many other scholars of his generation, Vidyabhusana was not really familiar with Aristotle, but rather with what is generally called 'traditional logic', a mixture extracted from Aristotle, but enriched with the left-overs of numerous other dishes. A decade later, the great Russian pioneer of the study of Buddhist logic, Th. Stcherbatsky, adopted a Kantian framework and introduced thereby even greater confusion. For unlike Aristotle, who doubtless continues to be the greatest logician in the Western tradition, Kant was no logician, and the greatest weaknesses of his philosophy are due precisely to his ignorance of logic.

I present these remarks by way of introduction, but they should not be regarded merely as historical anecdotes. For the prejudices of Vidyabhusana and Stcherbatsky continue to affect our understanding of Indian logic. This is clear from the literature in Western languages even on such elementary notions as the concept of *pakṣa*. This particular notion is furthermore obscured by the fact, that the term *pakṣa* is within Indian logic itself not used unambiguously. And so we witness the growth of a dense jungle of scholarship – first in Sanskrit, and next in Western languages – due to confusions around a notion that is basic and quite elementary. This paper attempts to clear some parts of that jungle.

There is no point in criticizing theories unless it is from the perspective of what one regards as the correct theory. Similarly, in order to appreciate precisely where and how some interpretations have gone wrong, we have

to know the right interpretation first. I shall therefore begin by elucidating the most important and most technical meaning of the term *pakṣa* in Indian logic, which occurs throughout its development, Hindu as well as Buddhist, and which is in fact straightforward and simple. In Indian logic, entities are never considered as if they were hanging in the air, but always as occurring in a locus (*āśraya*; *ādihāra*; *adhikaraṇa*). In this sense, Indian logic has its feet firmly on the ground. This ground, in the case of any specific case of inference, is called its *pakṣa*.

In order to explain this a little more precisely I shall make use of formal expressions, thereby perhaps inviting the accusation that I am imposing on the Indian material a framework of modern mathematical logic, thus merely substituting a more fashionable bias for the Aristotelian and Kantian prejudices of my predecessors. But this in fact is not the case. I use symbols merely because they are more precise and unambiguous than ordinary English. By introducing them I do not import any notions, theorems, or theories of contemporary Western logic.

In Indian logic, an entity, say  $x$ , is never regarded in isolation, but always considered as occurring in a locus, say  $y$ . The fundamental relation which underlies all expressions is therefore the relation which obtains between each entity and its locus. Since such a relation relates  $x$  to  $y$ , it is a two-place relation, which may therefore be written as:

$$(1) \quad A(x, y).$$

This may be read as: ' $x$  occurs in  $y$ ', or, alternatively, as ' $y$  is locus of  $x$ '. The relation  $A$  may be called an occurrence relation.

Given such relations, an inference is not merely a relationship between two entities, but a relationship between two entities as occurring in a locus. It would therefore be incorrect, or at least a rather rough approximation, to express the relationship between  $h$ , the *hetu* 'reason' and  $s$ , the *sādhya* "thing-to-be-inferred, inferendum"<sup>1</sup>, as:

$$(2) \quad h \rightarrow s.$$

It is more appropriate to express the relation of inference as a relation between two occurrence relations of the type (1), i.e., as:

$$(3) \quad A(h, p) \rightarrow A(s, p).$$

This may be read as follows: 'if the *hetu* occurs in  $p$ , then the *sādhya* occurs

in  $p$ '. Here the particular locus  $p$  in which the particular inference between  $h$  and  $s$  occurs, is what is called the *pakṣa*. In the stock example, the *hetu* is smoke, the *sādhya* is fire, and the *pakṣa* is a mountain. On this interpretation, (3) may be read as: 'if smoke occurs on a mountain, then fire occurs on that mountain'.

Part of the later history of Indian logic is the attempt to generalize expressions of this form in such a way, that they hold not only for a specific *pakṣa*, but for all loci. What is then attempted is to arrive at expressions equivalent to:

$$(4) \quad (x) (A(h, x) \rightarrow A(s, x)),$$

which may be read as: 'for all  $x$ , if  $h$  occurs in  $x$ , then  $s$  occurs in  $x$ '; or: ' $s$  occurs wherever  $h$  occurs' (cf. Staal, 1962). I am not concerned with these later developments (treated in *navya-nyāya* in some of the commentaries upon the section called *pakṣatā* of the *Tattvacintāmaṇi*), but shall confine myself to the original notion of *pakṣa* itself.

Vidyabhusana interpreted the notions of *pakṣa*, *hetu* and *sādhya* in terms of traditional logic as minor term, middle term and major term respectively (e.g., Vidyabhusana, 1921, pp. 176, 312). In order to see to what extent this is correct we shall have to go back to Aristotle (cf. Bocheński, 1951, pp. 42–46). Aristotle considered primarily sentences of the form ' $x$  belongs to  $y$ ', which may be symbolized as:

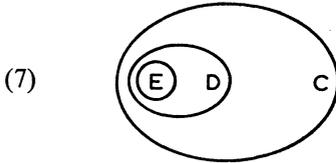
$$(5) \quad B(x, y).$$

A syllogism then consists of two premisses of the form (5) from which one conclusion again of the form (5) is derived. Different types of syllogism are obtained by substituting for ' $x$ ' and ' $y$ ' in the premisses and in the conclusion three terms: one in one of the premisses and in the conclusion; another in the other premiss and in the conclusion; and a third in both premisses. The third term is called 'the middle'; the other two, 'the extremities'. The following is an example, where  $D$  is the middle term, and  $C$  and  $E$  are the extremities:

$$(6) \quad \begin{array}{l} B(C, \text{all } D) \\ B(D, \text{all } E) \\ \hline B(C, \text{all } E) \end{array}$$

or: 'if  $C$  belongs to all  $D$ , and  $D$  belongs to all  $E$ , then  $C$  belongs to all  $E$ '.

Of the extremities, one is called the minor term and the other is called the major term. These terms, however, are not defined according to their formal position in syllogisms of the form (6); rather, they are defined, at least for the first figure<sup>2</sup>, according to their *extension*. What this means is best seen when (6) is illustrated as follows:



Here, *E*, which has the smallest extension, is the minor term; and *C*, which has the largest extension, is the major term.

We are now in a position to return to Indian logic. It is obvious why Vidyabhusana thought that the *hetu* corresponds to the middle term: neither the *hetu*, nor the middle term occurs in the conclusion of the inference. That is to say, the *hetu* does not occur on the right side of (3), i.e., in:

$$(8) \quad A(s, p),$$

and the middle term does not occur in the conclusion of (6), i.e., in:

$$(9) \quad B(C, \text{all } E).$$

It is also clear why he thought that the *pakṣa* and the *sādhya* correspond to the minor term and the major term, respectively: for (8) or 'the *sādhya* occurs in the *pakṣa*' looks somewhat like (9) which seems to express: 'the major term belongs to all the minor term'.

But if we look a little more closely here, we discover that these interpretations are nothing but the results of a confused muddle. In the Aristotelian syllogism, there are two premisses and one conclusion, and all three are of the same form, i.e., (5). In the Indian inference, on the other hand, there is only one premiss, viz.,  $A(h, p)$ , and the two relations from the *hetu* to the *pakṣa* and from the *sādhya* to the *pakṣa* are always the same relation, namely the occurrence relation *A*. There is an entirely superficial and fortuitous similarity between the particular syllogism illustrated in (6), and the general form of the Indian inference, viz., (3). The three terms in Aristotelian logic, and in the traditional logic which is

derived from it, are all of the same category. The Indian *pakṣa*, on the other hand, is an entirely different kind of thing from the *hetu* and the *sādhya*: it is the particular locus in which both the *hetu* and the *sādhya* happen to occur; it plays no part in the inference itself, though it is inseparable from each of its two terms. The *pakṣa* is the locus where the *hetu* occurs, and where the occurrence of the *sādhya* is doubted and sought to be established.

The difference may also be formulated thus. In Aristotle's syllogism the three terms are always related to each other through a relation of the form  $B(x, y)$  or 'x belongs to y'. In the Indian syllogism there are two terms, *h* and *s*, which are always related to each other through the relation of *pervasion*; both *h* and *s* are in addition related to the locus *p* but only through the occurrence relation *A*.

Vidyabhusana's erroneous identifications and comparisons have been repeated or relied on by almost all later interpreters of Indian logic. Stcherbatsky entangles them even further in his *Buddhist Logic*, but this deserves separate treatment, and I shall return to it. Some scholars confine themselves to using the terms 'minor', 'middle' and 'major' for the three Indian terms, without referring to the function which these former terms have in Western logic (e.g., Athalye, 1930, p. 281; Kuppuswami Sastri, 1932, pp. 188–189; Bocheński, 1956, pp. 497, 501; Goekoop, 1967, pp. 11–12, 56; Matilal, 1971, pp. 128–129). Others go out of their way to relate the Indian and the Aristotelian systems, and get wrapped up in greater confusions (e.g., Foucher, 1949, pp. 117–118; Barlingay, 1965, p. 109). The only scholar I found who seems to have had an inkling that these comparisons do not hold water, is Ingalls (1951, p. 35): though he uses the Western terms when introducing the Sanskrit ones, he remarks in a footnote: "Since the relation between *s* and *h* is one of pervasion, *h* may be equal to *s*.... In such cases, it is only by an extension of the literal meanings that one can speak of 'major', 'middle' and 'minor' terms. This is one reason why I prefer to keep the Sanskrit names, '*s*', '*h*', and '*p*'" (*ibid.*, note 28). But the assumption underlying this observation is incorrect: Aristotle does consider cases where the middle and major terms are equal<sup>3</sup>.

Turning now to Stcherbatsky we meet with a weird mixture that seems to have nothing whatsoever to do with logic. Though Stcherbatsky's translations are on the whole quite literal and reliable, his interpretations and

explanations are often extremely confused and almost always unclear. The Indian distinction between *svārthānumāna* 'inference for one's own sake' and *parārthānumāna* 'inference for the sake of others' is expressed and interpreted by Stcherbatsky by calling the second kind of inference, but not the first, 'syllogism'. Stcherbatsky is of course well aware of the Aristotelian and traditional connotations of that term. In fact, he says: "We have given the name of Syllogism to inference 'for others' because of its outward similarity with Aristotle's First Figure" (I 278; the other figures need not detain us, he says, they are 'false subtlety' anyway: I 309). As a result, chapter II of part III, which explains inference, is relatively free from Aristotelian (though not from Kantian) bias. But chapter III of part III, which deals with 'syllogism', though clearly enunciating one of its basic characteristics ("It thus consists of a general rule and its application to an individual case": I 279), uses the expressions minor, middle and major in a very confusing manner, applying 'minor' and 'major', as in traditional logic, to premisses as well as to terms.

The second volume of Stcherbatsky's *Buddhist Logic* contains the translation of Dharmakīrti's *Nyāyabindu*. In the translation of the third *pariccheda*, which deals with the *parārthānumāna* 'syllogism', many inferences are interpreted in terms of a major premiss, an example, a minor premiss and a conclusion. Though this forces the meanings of the original expressions into a very badly fitting straight-jacket, I shall confine myself to the treatment given to the concept of *pakṣa*. When this concept is introduced in the second *pariccheda*, Dharmakīrti does not refer to it by the term *pakṣa*, but by its predecessor, the term *anumeya*. Stcherbatsky translates the latter term as 'the object cognized by inference' and identifies it with the minor. In the section entitled 'Minor term', he translates the definition of *anumeya* (*anumeyo'tra jijñāsitaviśeṣo dharmī*) correctly as: "The object cognized in inference is here the substratum whose property it is desired to cognize." The term 'substratum' (*dharmīn*) refers here to the locus, which has for its property (*dharma*) or specification (*viśeṣa*) the *sādhya* which occurs in it, so that the relation:

$$(8) \quad A (sādhya, anumeya)$$

holds.

Now it is clear that the expression (8) refers itself to the conclusion of the inference. The commentary of Dharmottara on this definition reveals

a certain ambiguity of the term *anumeya* as used in this context. In Stcherbatsky's translation: "The word *here* means that the object of inference appears as a substance (a substratum) when the definition of its mark is considered (the mark being an attribute of this substance). But from another standpoint, when the deduced (conclusion) is realized, the subject of the inference would be a complex (idea of the substratum together with its property). And when the invariable concomitance (between the middle and the major terms) is considered, then the inferred fact appears as an attribute (of this substance, as the major term). In order to point out (these differences) the word *here* has been used. We call *object of inference* an object whose property, or specification, it is desired to cognize" (II 58) (*atra hetulakṣaṇe niścetavye dharmy anumeyaḥ | anyatra tu sādhyapratipattikāle samudāyo'numeyaḥ | vyāptiniścayakāle tu dharmo'numeya iti darśayitum atra grahaṇam | jijñāsito jñātum iṣṭo viśeṣo dharmo yasya dharmināḥ sa tathoktaḥ*).

Stcherbatsky attaches a footnote explaining *anumeya*: "In a general sense it may mean an object which possesses the united properties of the major, the minor and the middle terms, e.g., 'the mortal man Socrates'; it is then *ekam vijñānam*. It may also mean the major term or the conclusion separately, as well as the thesis which is also the conclusion (= *pakṣa* = *sādhyā*). In a special sense it means the minor term, the subject of the conclusion, and even more precisely, the underlying substratum (*dharmin*), the efficient point-instant, that underlying point of reality upon which any amount of interconnected qualities may be assembled as a superstructure" (*ibid.*, note 1).

Such muddles do not help the understanding of Indian logic. In fact, no logician who reads them can fail to lose whatever regard he might ever have had for that subject. And yet, Stcherbatsky was partly correct in his interpretations. The confusion is partly due to the Indian logicians themselves, and the rest results from imposing upon their expressions an Aristotelian framework that has nothing to do with it. The ensuing jumble may be sorted out and clarified along the following lines.

As we have seen, the conclusion of an Indian inference may be expressed by  $A(s, p)$  (8). Now Dharmottara referred to three things that may be called *anumeya* in this conclusion because each of them may be used to express what it is that is concluded: the *dharmin*  $p$ , which has  $s$  for its property; the *dharma*  $s$ , which  $p$  has for its property; and the complex

(*samudāya*) of the *dharmin* together with its *dharma*. The reason for this undecidedness is that the Buddhist logicians failed in this context to properly express that the conclusion is the *relation*  $A(s, p)$ : they confined themselves to expressions denoting terms and ‘complexes’ of terms. Stcherbatsky compounded this error by imposing the Aristotelian framework he had adopted, and so came to speak of a term in the inference which may be either the major or the minor – as if such a terminology should ever be introduced for any other reason than in order to distinguish between the two. The two facts that the terms *sādhya* and *pakṣa* are used by Indian logicians in such a way that either of them can be used to state the conclusion, and that this was done without the entire edifice falling to pieces, should have alerted Stcherbatsky that these terms could not possibly denote the same concepts as the Aristotelian major and minor terms.

Though the Buddhist logicians did not clearly express the relational character of the conclusion of the inference in the contexts in which they introduced and discussed terms such as *hetu*, *sādhya* and *pakṣa*, they did so in other contexts. But to confuse matters again, they there refer to this relation by the term *pakṣa*. Accordingly, *pakṣa* refers sometimes to  $p$ , and sometimes to  $A(s, p)$ . This is not as strange as it may seem, though it is certainly bad logic. The occasional confusion between a term and a sentence is not uncommon in Indian logic, and it is undoubtedly related to the structure of the Sanskrit language, where a certain type of expression can either refer to a noun or to a sentence (cf. Staal, 1965, p. 181; 1971, p. 200). Thus, *parvato vahnimān* can mean either ‘the mountain possesses fire’ or ‘the mountain which possesses fire’; and this may depend on the context. A more accurate way of expressing the fact that the term *pakṣa* is ambiguous in this way, is by saying that it refers either to  $A(s, p)$ , or to  $\text{p}A(s, p)$ , which may be read as: ‘that  $p$  such that  $A(s, p)$ ’ (rather than just  $p$ ).

Stcherbatsky does not explicitly refer to this distinction, but he implicitly distinguished between the two meanings by translating *pakṣa* sometimes as *subject* (or similar terms, in the contexts already referred to) and sometimes as *thesis*. The latter translation is adopted in II 153 and following, where he deals with the definition of *pakṣa* in the *Nyāyabindu*. Later scholars have felt this same ambiguity. For example, Tachikawa writes in the notes to his excellent translation of the *Nyāyapraveśa*:

“According to the definition given by Śāṅkarasvāmin, the *pakṣa* is an object which the arguer wishes to prove to be qualified by a property, not the statement of the form: A property-possessor is qualified by a property. The statements, however, are here taken as examples of the *pakṣa*, which seems to indicate some laxity in the usage of the term *pakṣa*” (Tachikawa, 1971, 132, note 9)<sup>4</sup>.

The *Nyāyapraveśa* of Śāṅkarasvāmin contains yet another use of *pakṣa*, interesting especially because the ambiguity of the Sanskrit original is resolved in the Chinese translation. Tachikawa renders the relevant passages correctly but has to resort to rather free translations. In these contexts, the term *pakṣa* refers to what the *sapakṣa* and the *vipakṣa* have in common. This can of course be said to be *pakṣa* from a morphological point of view, if the terms are taken to be mentioned, not used. But the meaning of the term which is thus constructed does not correspond to the more common meanings of the term *pakṣa* in Indian logic. In fact, what *sapakṣa* and *vipakṣa* have in common excludes what is ordinarily called the *pakṣa*. For *sapakṣa* is defined as any locus, *different from the pakṣa*, where the *sādhya* occurs; and *vipakṣa* as any locus where the *sādhya* does not occur (see Staal, 1962, 634–635).

This special sense of *pakṣa* occurs first in the discussion of a reason (*hetu*) which is fallacious because it is common to both the *sapakṣa* and the *vipakṣa*. An example is: ‘sound is permanent because it is an object of valid cognition’ (*śabdaḥ prameyatvān nityaḥ*: Tachikawa, 1971, 124, 142). Here the *hetu* is fallacious ‘because the property of being an object of valid cognition is common to both permanent and impermanent things’ (*nityānityapakṣayoḥ sādharmaṇatvād*). In other words, since both permanent and impermanent things can be known (everything can be known in the Nyāya view), it is not proper to conclude that sound is permanent from the fact that it can be known.

The Chinese translators render *sapakṣa* and *vipakṣa* each by a combination of two characters. The two pairs have one character in common, which is however different from the character used to translate *pakṣa*. This particular structure enables the Chinese translators to avoid the ambiguity of the Sanskrit original. Thus in Hsüan Tsang’s translation of the *Nyāyapraveśa*, entitled 因明入正理論 (*Yin ming ju cheng li lun*: T, 1630, Vol. 32, 11a ff.), *pakṣa* is rendered by 宗, *sapakṣa* by

同品 and *vipakṣa* by 異品 . But in the above passage, translated as 常無常品皆共此因 (T.1630, 11c: 20), the term *pakṣa* is rendered by 品 , the character which the expressions for the *sapakṣa* and the *vipakṣa* have in common.

Thus the Chinese translation avoids the confusing ambiguity of the original and expresses its logical structure more adequately. This should come as a surprise to those who maintain that the Chinese translators did not appreciate the subtler points of the Sanskrit originals, but merely replaced words by characters in a mechanical manner, the results being fit for recitation and perhaps meditation, but not susceptible to further rational analysis. In this instance, at least, Hsüan Tsang's translation is decidedly superior to many of the modern translations I have been concerned with in the above attempt to clear some of the interpretative jungle that surrounds the concept of *pakṣa* in Indian logic.<sup>5</sup>

#### NOTES

- <sup>1</sup> The translation 'inferendum' was suggested to me by Mr. Paul Z. Panish.
- <sup>2</sup> Aristotle gives different definitions in different contexts, and the definitions given for the first figure do not apply to the others. See Łukasiewicz (1957) pp. 28–82 and Patzig (1959) = Patzig (1968), Chapter IV.
- <sup>3</sup> In *An. pr.* B5, 75b 35 ff. and in *An. post.* A3, 73a 6ff., Aristotle considers arguments where all three terms are convertible with each other, which is at least as strong a claim as that of extensional equality.
- <sup>4</sup> The same 'laxity' is present when Tachikawa takes *pakṣa* to be the sense of *sādhya* in *anitye śabde sādhye* (Tachikawa, 1971, p. 132, note 9).
- <sup>5</sup> I am very grateful to Professor Lewis R. Lancaster with whom I taught a seminar on the *Nyāyapraveśa* and who initiated me into the intricacies of its Chinese translation, and to Professor Michael Frede who improved my account of Aristotle's logic and provided the references in note 3.

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**Note added in proof.** After sending this paper to the press I read an unpublished manuscript by Professor M. Tachikawa, entitled 'On *pakṣa*', in which very similar conclusions are reached.



*Part Two*

# Indian Linguistics



# Euclid and Pāṇini

## INTRODUCTION

THE PRIMARY AIM of this paper is to draw a comparison between the methods employed by the Greek mathematician Euclid (third century B.C.) and the Indian linguist Pāṇini (fourth century B.C.).<sup>1</sup> At the same time a comparison will be drawn between the influences which these two scientific methods have exerted within their respective philosophic traditions, i.e., the Western and the Indian. One thesis defended in this paper is that the mathematical method is characteristic of much of Western philosophy, whereas the grammatical method is characteristic of much of Indian philosophy.<sup>2</sup> Another thesis, partly implicit, is that the recent occupation of modern Western philosophy, not only with linguistic analysis, but also with the science of language,<sup>3</sup> can be expected to yield results of philosophic value. For this reason a comparison between linguistic and mathematical methods may be of interest to modern philosophers. Moreover, a consideration of linguistic method throws light on the relationship between the language of philosophy and philosophy itself. Some illustrations of this relationship will be considered.

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<sup>1</sup> This article is a free translation and revision of about two thirds of an Inaugural Lecture delivered on appointment to the Chair of General and Comparative Philosophy at the University of Amsterdam. I am grateful to Professor D. H. H. Ingalls and the editor of this Journal for reading an earlier version of the English translation and for making valuable suggestions concerning both contents and presentation.

For Pāṇini's date, see P. Thieme's review of T. Burrow, *The Sanskrit Language* (London: Faber and Faber, Ltd., 1955), in *Language*, 31 (1955), 428-448, which quotes further literature. The facts hardly justify Jaspers' theory of an *Achsenzeit* (in *Vom Ursprung und Ziel der Geschichte* (München: R. Piper & Co. Verlag, 1949), though a simultaneous flourishing of philosophy could to some extent explain a subsequent flourishing of scientific method.

Editor's note: Because of the highly technical and specialized nature of this paper and the notes—and the fear of errors and/or inconsistency due to editing—the author's style in the notes has been largely retained. Some minor inconsistencies of style and some apparently incomplete items are present, but clarity is not affected.

<sup>2</sup> It is probable that this was first pointed out by D. H. H. Ingalls in "The Comparison of Indian and Western Philosophy," *Journal of Oriental Research*, 22 (1954), 4. See also my review of D. S. Ruegg, *Contributions à l'histoire de la philosophie linguistique indienne* (Paris: E. de Boccard, 1959), in *Philosophy East and West*, 10 (1960), 53-57.

<sup>3</sup> See, e.g., J. A. Fodor and J. J. Katz, *The Structure of Language. Readings in the Philosophy of Language* (Englewood Cliffs: Prentice-Hall, Inc., 1964). Cf. J. F. Staal, "Generative Syntax and Semantics," *Foundations of Language* 1 (1965), 133-154.

Though linguistic and mathematical methods will be described mainly in a historical setting, they exemplify differences which are of theoretical interest and which pose problems which remain, on the whole, unsolved. Some parts of contemporary Western philosophy and logic can be said to deal with the problems that arise from the confrontation of these two different approaches. Outside of philosophy and logic, the two approaches meet in the recently developed science of mathematical linguistics.<sup>4</sup> This paper may also present material to justify the expectation that mathematical linguistics will continue to be inspired by Indian grammatical methods.<sup>5</sup> It will be seen that these methods often possess a degree of systematization, formalization, and conceptualization which in the West is generally associated with mathematics and the mathematical sciences only. Lastly, it may not be superfluous to add that the conclusions to which these comparisons lead can help to dispel the naïve but persistent view that only the Western tradition has produced and employed rational and scientific views.

## I

The classic example of mathematical method occurs in the thirteen books of the *Elements* (Στοιχεῖα) of Euclid. The classic example of linguistic method occurs in the eight chapters of the Sanskrit grammar (*aṣṭādhyāyī*) of Pāṇini. Both had precursors, whose results are largely lost,<sup>6</sup> but each constructed an almost complete system by means of a precise method, strictly maintained. Though there are, as we shall see, certain parallels between the two, the object material also has in one respect a similar structure. This similarity is indicated by the Greek term “στοιχεῖον,” which denotes the elements as well as the letters of which words consist. Proclus declares in his commentary on the *Elements*: “Just as an expression in language consists of, first, most simple and undivided principles, which we call elements, and just as each word and each discourse is constructed from these, there are also certain theories, called elements, which precede the whole of geometry, function as principles for following theorems, extend over all theorems and provide proofs for many particular cases.”<sup>7</sup> It is true that the linguist’s activity is

<sup>4</sup> In due course, this science may be called merely theoretical linguistics: Y. Bar-Hillel, *Language and Information* (Reading, Mass.: Addison-Wesley, 1964), p. 185.

<sup>5</sup> Bloomfield acknowledged indebtedness to Pāṇini. For a direct comparison with methods of mathematical linguistics, see J. F. Staal, “Context-sensitive Rules in Pāṇini,” *Foundations of Language*, 1 (1965), 63-72.

<sup>6</sup> On the one hand, Hippocrates of Chios, Leon, Eudoxos, Theudios, etc.; on the other hand, Āpiśali, Kāśyapa, Gārgya, and the Northern and the Eastern school.

<sup>7</sup> See trans. P. Ver Eecke (Bruges: Desclée de Brouwer, 1948), 65; E. J. Dijksterhuis, *De elementen van Euclides I-II* (Groningen: P. Noordhoff, 1929-1930), I, 107: *hōs gār tēs*

only very approximately described as the composition of words from letters. Moreover, linguists also deal with spoken languages, where there is no use for pencil and paper and where a tape recorder is required. Proclus and the ancient grammarians following Dionysius Thrax constructed words from letters instead of from phonemes, while neglecting morphemes altogether. In these respects Pāṇini and the Indian grammarians made a unique contribution.<sup>8</sup> But let us first confine our attention to Euclid's *Elements*.

Euclid starts the first book of his work with three groups of statements, and these are followed by numerous propositions or theorems. The other books of the *Elements* contain other theories, sometimes preceded by new definitions which circumscribe new topics. Propositions are derived from the initial statements by means of logical derivations, which sometimes refer to propositions that have been derived earlier. The main structure which holds the *Elements* together, therefore, is logical derivation. In general, each such derivation consists of several steps, of which the most important ones are construction (κατασκευή) and/or proof (ἀπόδειξις). The three groups of initial statements are definitions (ὅροι), postulates (αἰτήματα), and common insights (κοινὰ ἔννοιαι). Since the time of Proclus the last have been called axioms (ἀξιώματα). The mutual relations between these three types of principles pose several problems, more so because there can be disagreement with regard to the status of some statements. For the present purpose they can be roughly characterized as follows: Definitions introduce concepts which are the object of geometry. Postulates either introduce constructions or assume the existence of constructions and their results. Axioms, according to Aristotle, are basic to all sciences. Hence the name "common insights."

Let us consider an example of each. The fifteenth definition runs as follows: "A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another."<sup>9</sup> This definition explains how the term "circle" will be used, without asserting that circles exist and without providing a construction for circles. This is done in the third postulate, which says: "To describe a circle

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*eggramátou phonēs eisin archai protai kai haploustatatai kai adiairetoi, hais tò ónoma ton stoicheiōn epiphēmizomen, kai pasa léxis ek toutōn hyphéstēken kai pas lógos, houtō de kai tēs hólēs geometrias estí tina theōrēmata proēgoumena kai archēs lógon échonta pròs tò ephexes kai diekonta dià pántōn kai parechόμενα pollon apodeixeis symptōmátōn, hā de stoicheia prosagoreúousi: Proclus, In primum Euclidis elementorum librum commentariū, ed., G. Friedlein, Leipzig: B. G. Teubner, 1873, 72, 6. 13.*

<sup>8</sup> See P. Thieme, in B. Shefts, *Grammatical Method in Pāṇini: His Treatment of Sanskrit Present Stems* (New Haven: American Oriental Society, 1961), p. ix.

<sup>9</sup> *Kúklos estí schēma epípedon hupò mias grammēs periechόμεnon, pròs hēn aph' hēnōs sēmeiou ton entòs tou schēmatis keiménōn pasai hai prospíptousai eutheiai ísai allelais eisin.*

with any center and distance."<sup>10</sup> This can be taken as postulating the construction or the existence of circles.<sup>11</sup>

The first axiom is quite different. It states: "Things which are equal to the same thing are also equal to one another."<sup>12</sup> Though this axiom can be interpreted as valid for line segments, it allows many other interpretations as well. For example, it can be expressed in terms of algebra by "if  $b = a$ , and  $c = a$ , then  $b = c$ ."<sup>13</sup> One might feel inclined to consider such an apparently general axiom as belonging to logic, but the concept of equality (*ισότης*) has a mathematical character. Proclus asserts in this connection that the axioms are valid not only for magnitudes, but also for numbers, movements, and time intervals.<sup>14</sup>

Euclid proceeds to derive numerous propositions from these definitions, postulates, and axioms with the help (implicitly) of further axioms and logical rules and metarules. This is so similar to the familiar proofs of school geometry that it need not be illustrated. Moreover, a detailed examination of a complete proof would take up more space than is available here, especially on account of the characteristic references to results established earlier.

In how far do the five axioms of Euclid (and, correspondingly, the five postulates and first twenty-three definitions) meet the geometrician's need? In classical antiquity other axioms were current, too, e.g., "Things which are halves of the same thing are equal to one another."<sup>15</sup> Proclus notes that axioms should not be needlessly increased: such axioms "follow from preceding axioms and are justly omitted in most copies."<sup>16</sup> This principle is of fundamental importance for the entire Euclidean system. In modern terminology it could be expressed by stating that each axiom must be independent from the others. Proofs of independence are given much later by the use of models. The construction of such models has, in turn, led to the discovery of non-Euclidean, discontinuous, non-Archimedean and multi-dimensional geometrics.<sup>17</sup>

The requirement of independence is a simplicity criterion. In philosophy

<sup>10</sup> *Kaì pantà kéntrōi kaì diastemati kúklon gráphēsthai.*

<sup>11</sup> Dijksterhuis, *op. cit.*, I, 124. Compare T. L. Heath, *Euclid's Elements I-III* (reprint) (New York: Dover Publications, Inc., 1956), I, 195.

<sup>12</sup> *tà toi autoi isa kaì allēlois estin isa:* Euclides, *Elementa I*, ed. I. L. Heiberg (Leipzig: B. G. Teubner, 1883), 10, 2.

<sup>13</sup> The commutativity of identity need not be presupposed here.

<sup>14</sup> Proclus, *op. cit.*, 195.25-196.1.

<sup>15</sup> Euclides, *op. cit.*, I.10.9. Compare Heath, *op. cit.*, I.223.

<sup>16</sup> *Tauta oun hēpetai tois proeirēmēnois axiomasi kaì eikōtōs en tois pleistois anti-gráphois paraleipetai:* ed. Friedlein, 198.2-4. Compare trans. P. Ver Eecke, 174.

<sup>17</sup> See, e.g., E. W. Beth, *The Foundations of Mathematics* (Amsterdam: North-Holland Publishing Co., 1959), 153-155; B. L. van der Waerden, *De logische Grondslagen der Euklidische Meetkunde* (Groningen: P. Noordhoff, 1937), 39; H. Weyl, *Philosophie der Mathematik und Naturwissenschaft* (München: Leibniz Verlag, 1928), 18 *et seq.*

it is known as Ockham's razor: Entities should not be multiplied beyond necessity (*entia non sunt multiplicanda praeter necessitatem*). In another version it runs: What can be done by fewer [principles] is done in vain by more (*frustra fit per plura quod potest fieri per pauciora*).<sup>18</sup> This principle is not a desideratum of philosophers alone and merely of aesthetic nature; it is a motive of consequence in the development of science. During the Middle Ages Ockham's razor led to the doctrine that terrestrial and celestial bodies consist of the same matter: the phenomena can be explained by assuming either one or two kinds of matter, and hence there is no reason to postulate more than one.<sup>19</sup> Such speculations paved the way for Newton's discovery of the law of gravity, which applies to planets as well as to apples. On the other hand, a complicated theory is less probable than a simple one. When astronomical observations became increasingly accurate, an increasing number of epicycles had to be constructed to explain the phenomena in terms of Ptolemy's theory. However, these laws could be derived from Newton's law, through the intermediary of Kepler's laws.<sup>20</sup> In modern physics, attempts at unification appear regularly, for example, in unified field theory or in derivations of elementary particles from other and fewer elementary particles.

In philosophy, the criterion of simplicity gave rise to the concept of monism, variously developed and exploited by metaphysicians. Another philosophic result provides a further example of Euclid's method. This is the doctrine of categories as it occurs, e.g., in Aristotle. Aristotle suggests that the categories of his list are independent by enumerating them as a disjunction in terms of "or."<sup>21</sup> In the *Topica*,<sup>22</sup> he notes that there are just as many categories as he has mentioned. In other contexts it appears that the independence of each category is presupposed. In the *Topica*, a relatively early work, two categories occur which are later omitted.<sup>23</sup> This may point to the fact that Aristotle had come to consider these as derivable. Throughout the history of philosophy lists of categories presuppose the independence of each category. In Kant, the categories constitute a system which is analogous to a structure of axioms and theorems. In the *Critique of Pure Reason* categories are called the true root-concepts of pure understanding (*die wahren Stammbegriffe des reinen Verstandes*). They possess, in turn, derived concepts (*reine abgeleitete*

<sup>18</sup> Weyl, *op. cit.*, 117.

<sup>19</sup> E. Gilson, *La philosophie au moyen-âge* (Paris: Payot, 1925), 283.

<sup>20</sup> Cf. Weyl, *op. cit.*, 117-118.

<sup>21</sup> *Categoriae* 4, 1 b 25—2 a 4. However, see *ibid.*, 8, 11 a 37, quoted in W. and M. Kneale, *The Development of Logic* (Oxford: Clarendon Press, 1962), 30.

<sup>22</sup> *Topica*, A 9, 103 b 38.

<sup>23</sup> W. D. Ross, *Aristotle* (London: Methuen & Co., Ltd., 1953), 21-22; I. M. Bocheński, *Ancient Formal Logic* (Amsterdam: North-Holland Publishing Co., 1951), 33-34.

*Begriffe*), regarding which Kant says that they belong to transcendental philosophy but that he "can rest content merely to mention these" (*mit deren blosser Erwähnung . . . zufrieden sein kann*).<sup>24</sup> Kant, then, provides axioms without deriving any theorem and without asking how such derivation should be carried out.

In natural language, categories are not expressed by sentences but by words or terms. In mathematics, the corresponding structure is therefore a system of definitions. In such systems, too, independence is a meaningful concept. Alessandro Padoa derived a method for establishing the independence of a concept, i.e., its undefinability, in terms of earlier postulated concepts. It is natural to apply this also to systems of categories. As an exercise in traditional philosophy an attempt could be made to describe the world solely in terms of the first three categories of Aristotle. If any possible world can be described in terms of all the categories but one, the independence of this latter category has been thereby established.

In modern logic, corresponding investigations aim at the construction of formal systems without using particular principles or concepts. Intuitionistic logic thus dispenses with the principle of the excluded third; negationless logic dispenses with the concept of negation. The resulting systems appear more or less seriously curtailed from the point of view of ordinary propositional calculus. Other kinds of restrictions are brought about within propositional calculus, when it is shown that the connectors "and," "or," "not," "if . . . then" can to some extent be derived from each other or from "neither . . . nor . . .," symbolized by Sheffer's stroke. Similarly, the number of axioms can be reduced. These transformations do not affect the results which can be obtained within the system. They merely reflect what Quine has called economy in grammar and vocabulary. This sort of economy leads to longer statements and derivations. On the other hand, brevity in statements and derivations calls for a wealth of basic idioms.<sup>25</sup> In general, within artificial languages, new categories are introduced by new notations and are therefore easily recognized. The categories presupposed by natural languages are not so easily recognized.

The analysis of traditional systems of categories assumes new importance when the language in which such systems were first formulated is taken into account. The dependence on natural language is generally implicit and goes unnoticed. Still, it has often been assumed that there is some connection between Aristotle's categories and grammatical distinctions. In the nineteenth

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<sup>24</sup> *Kritik der reinen Vernunft*, 2. Auflage, *Transcendentale Elementarlehre II*, *Transcendentale Logik* I, § 10, Akademie-Ausgabe. (Berlin: Georg Reimer, 1911) III, 94.

<sup>25</sup> W. Van O. Quine, *From a Logical Point of View* (Cambridge: Harvard University Press, 1961), 26-27.

century, Trendelenburg defended the view that Aristotle's logical categories can be derived from grammatical categories. However, his arguments were not cogent, and later scholars have rejected not only the reasoning but also the result. As late as 1958, Benveniste convincingly established that the categories of Aristotle can be understood against the background of Greek syntax.<sup>26</sup> For example, the two categories *κείσθαι* and *ἔχειν*, which were later deleted by Aristotle himself, reflect the verbal functions of the Greek middle voice and perfect, respectively. Faddegon established as early as 1918 that the categories of the Indian Vaiśeṣika system also were based upon grammatical distinctions;<sup>27</sup> this had been known to Indian scholars. Given the fact that Greek and Sanskrit are cognate languages, this explains the striking similarity between Western and Indian categories.

Discoveries of this kind open up new vistas. They emphasize the desirability that philosophers should take full account of linguistics. With the help of linguistics, philosophy is in a position to enter a fruitful area of research. In this respect, philosophers are in a more favorable position than mathematicians—for the latter investigate what they have first created, while the former face the richness and variety of natural languages, where reality surpasses the boldest imagination. Mathematicians can never enter other spaces than the one in which they were born, not even with the advancement of space travel. At most, they can propose to physicists that they should describe physical space with the help of another geometry. Philosophers, on the other hand, can learn a language and thereby enter a new world of experience: the linguistic categories of a newly learned language may not segment reality in the same way as do the categories Western philosophers are accustomed to. Philosophers obtain passports for non-Aristotelian worlds as soon as they begin to study the syntax of a language which is sufficiently different from Greek. Of course, the fruitfulness of such research increases if the language studied belongs to a civilization which has also produced philosophy, logic, and linguistics or related fields of study.

Important results of this kind can be expected from the study of the so-called exotic languages. But, since the language of philosophy is even more conservative than colloquial language, similar information can be obtained by studying the language of philosophy in this spirit. In Chinese, no verb corresponds to "to be," which both functions as copula and expresses existence. Kant's criticism of the view that existence is a predicate can therefore never

<sup>26</sup> E. Benveniste, "Catégories de pensée et catégories de langue," *Les Études Philosophiques*, New Series, 13 (1958), 419-429.

<sup>27</sup> B. Faddegon, *The Vaiśeṣika System Described with the Help of the Oldest Texts* (Amsterdam: Johannes Müller, 1918).

be meaningfully translated into Chinese. A. C. Graham has shown that a Chinese translation of relevant passages of Kant's *Critique of Pure Reason* (itself based upon the English translation of the German original) is quite unintelligible to persons who know Chinese but no German.<sup>28</sup> Similarly, English translations of the categories *κείσθαι* and *ἔχειν*, for instance, as "position" and "possession," respectively, are hardly meaningful in English, since the relevant functions expressed by the Greek terms do not straightly correspond to similar structures in English. Even if *ἔχειν* is translated as "to have," and attention is paid to the fact that in English also a tense somewhat similar to the perfect tense can be expressed by means of "to have," it remains unintelligible what could have motivated Aristotle to assume this concept as a category. This provides a good illustration for Wittgenstein's slogan, that philosophic problems appear when language goes on a holiday—on a holiday in Greece, in this case.

Through the intermediary of the categories linguistic usage has exerted profound influence on thought. Philosophic innovation often springs from a reaction against traditional usage of speech and thought. A contemporary example of such innovation occurs in existentialism, as well as in analytical philosophy. According to Heidegger, the "essence" of the human being (*Dasein*) lies in its existence. It is possible to say that *Dasein* is, but not what it is. It is not a substance that can be further specified with the help of categories, for categories are especially adapted to another kind of being than human being ("*Seinbestimmungen des nichtdaseinsmässigen Seienden*"). Heidegger therefore introduces existentials, and thus avoids the error of substantialization. From a linguistic point of view this means that it is possible for a noun to refer to human being without functioning in the same way as other nouns which refer to things. In practice, Heidegger often replaces nouns by pronouns and more often by verbs. This might indicate a Semitic rather than a Greek inspiration. This impression is confirmed by other points of resemblance between existentialism and Biblical thought.

As mentioned earlier, Wittgenstein in a similar vein warns against substantialization of the words of a sentence. This may lead us astray, both in speech and in thought. Wittgenstein constructs a language where all words refer to things by imagining a system of communication between a builder and his assistant.<sup>29</sup> This language consists of the words "block," "pillar," "slab," and "beam." Each time the builder utters one of these words, the assistant hands him the relevant thing. Wittgenstein's artificial language is based upon

<sup>28</sup> A. C. Graham, "Being' in Western Philosophy Compared with *shih/fei* and *yu/wu* in Chinese Philosophy," *Asia Major*, New Series 7 (1959), 107-108.

<sup>29</sup> *Philosophical Investigations* (Oxford: Basil Blackwell, 1958) (I 2), p. 3.

a particular analysis of the use of language. Ryle, in *The Concept of Mind* and elsewhere, considers different types of category mistakes. They are basically of the question-answer type: "Where is Socrates?"—"Yesterday." Heidegger applies this by confining himself for methodological reasons (as he puts it) to *Dasein*. Strange as it may seem, *The Concept of Mind* and *Sein und Zeit* are similar in this respect. Indian thinkers of the Advaita Vedānta—in this respect comparable to Neo-Platonism, negative theology, and apophatic mysticism—specify the same in a different manner by maintaining that the Absolute is not a substance (*dravya*), because it has no qualities (*nirguṇa*). To think otherwise would be the result of a category mistake.<sup>30</sup>

The principles from which categories are derived and the simplicity or economy criterion play important roles in modern linguistics. Syntactic form-classes and morpheme-classes are defined by means of substitution in identical environments. Though morphemes, as was noted before, were not known in Western classical antiquity, these methods lead to results which are closely related to the parts of speech (*μερῆ λόγου*) put forward by Greek grammarians and logicians of the Stoa. These, again, are connected with Aristotle's categories. Pāṇini and the Indian grammarians use substitution for similar purposes. They draw explicit attention to its fundamental importance. The Indian method of substitution (*ādeśa*) corresponds to Aristotle's method for deriving categories. Characteristic for Aristotle is the fact that he starts from questions.<sup>31</sup> In modern linguistics, substitution is used for establishing a morpheme-class, for example, by studying the environment ( )-(ly). In this environment, "slow," "near," and "quaint" fit, and these therefore belong to the same morpheme-class. Though "dead" can occur in this environment, too, it functions differently elsewhere; it does not, therefore, belong to the same class.<sup>32</sup> "Deadly" functions in a "deadly blow" in the same way as "terrific" does in a "terrific blow." But a "slowly blow" is never given.

It is hardly necessary to emphasize the importance of Euclid's methods for the history of Western science. According to Beth,<sup>33</sup> Euclid's geometry is the classical example for Aristotle's theory of science. In this theory, postulates, shown by history to be hardly compatible, have been combined. The postulate of reality (i.e., "each proposition refers to a particular area of reality") could

<sup>30</sup> For the linguistic background of categories and category mistakes, see J. F. Staal, "Some Semantic Relations between Sentoids," *Foundations of Language*, 3. (In press.)

<sup>31</sup> For a modern development, see H. Hiz, "Questions and Answers," *The Journal of Philosophy*, 59 (1962), 253-265.

<sup>32</sup> R. S. Wells, "Immediate Constituents," *Language* 23 (1947), 81; B. Bloch and G. L. Trager, *Outline of Linguistic Analysis* (Baltimore: Linguistic Society of America, 1942), 60-61, 77-79.

<sup>33</sup> See, e.g., Beth, *The Foundations of Mathematics*, 31-51.

be maintained in the empirical sciences only by dispensing with the other postulates: the postulate of deduction, which incorporates the deductive structure referred to earlier; and the postulate of evidence, which requires that the meaning of the first principles and the validity of the axioms be self-evident.<sup>34</sup> The deductive sciences, on the other hand, have dispensed with the postulate of reality and adhere only to the postulate of deduction. Since the development of non-Euclidian geometry, the postulate of evidence has been rejected. In modern science, theories are established with the help of many criteria, but mainly with adequacy and simplicity in view.

Since Euclid, philosophy has made repeated attempts to become or present itself as a deductive science. In this development Proclus is of historical importance. He was a well-informed but not an original scholar who was not only commentator of Euclid's *Elements* but also systematizer of Neo-Platonic metaphysics. Though some of Proclus' comments on Euclid are astute, the applications of his learning to philosophy are less than illuminating. In his *Elements of Theology* (*Στοιχείωσις Θεολογική*) Proclus confines himself to enumerating and explaining propositions. He appears to assume that propositions can function as propositions without being derived from axioms. It is true that the explanation of a proposition may contain logical remarks. This is not surprising, since in this theology there are propositions such as number 147: "In any divine rank the highest term is assimilated to the last term of the supra-jacent rank."<sup>35</sup> But considerably stronger methods of proof would be required to establish such a proposition as number 187: "Every soul is indestructible and imperishable."<sup>36</sup> By the irony of history, Euclid's diligent scientific method entered metaphysics through the series of unproven propositions of his commentator. One has to agree with Martin when he says that Proclus attached value to the *Elements* of Euclid, mainly because they contradict neither the Chaldean oracles nor the speculations of the Pythagoreans.<sup>37</sup>

Endeavors to imitate Euclid's method are found in numerous medieval treatises.<sup>38</sup> Such attempts, which may simultaneously have undergone the

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<sup>34</sup> Formulated clearly by Proclus, ed. Friedlein, *op. cit.*, 195.17-19: *pánta axiómata hōs ámesa kai autophanē paradóteon, gnorima aph' heauton ónta kai pistá* (All axioms must be given as immediate and self-evident, being known from themselves and reliable).

<sup>35</sup> *Pántōn ton thēiōn diakósmōn tà akrótata tois pērasin homoioutai ton huperkeiménōn*. E. R. Dodds, *Proclus: The Elements of Theology* (Oxford: Clarendon Press, 1933), 128.

<sup>36</sup> *Pasa psuchē anolethrós esti kai áphthartos*: Dodds, *op. cit.*, 162.

<sup>37</sup> Quoted by Heath, *op. cit.* (above, note 11), I 30, note 2. See, e.g., Proclus' commentary on definitions 30-34.

<sup>38</sup> See H. A. Wolfson, *The Philosophy of Spinoza* (New York: Meridian Books, 1960), 40 *et seq.*

influence of Aristotle's syllogistic, occur in the works of Duns Scotus, Burgersdyck, Maimonides, Avicenna, Averroes, and many others. This tendency culminates in Spinoza's *Ethica more geometrico demonstrata*. One might argue that Spinoza's proofs are rarely convincing, or that the geometrical exterior has nothing to do with the metaphysical content, which closely follows medieval treatises. Wolfson has maintained the view that the real Spinoza appears only when an *Ethica more scholastico rabbinicoque demonstrata* is discovered behind the *Ethica more geometrico demonstrata*. Be this as it may, Spinoza intended to prove his propositions and paid greater attention (as appears, for example, from his letters) to the demonstrative value of his proofs than his modern readers, who are often disposed to irrationalism.

## II

Let us now turn to Pāṇini's system of Sanskrit grammar. This grammar opens with a list of the sounds which, according to Pāṇini, occur in Sanskrit and which function as elements for later rules.<sup>39</sup> This list starts *a, i, u, e, o, ai, au*, etc. Consonants are always followed by a short *a*, for example, *ja, ba, ga, ḍa, da*. Then follow the *sūtras*, or rules, which can be divided into three types in a way roughly parallel to Euclid: (1) The *sūtras* proper, or theorems (*vidhi*), which describe linguistic facts while emphasizing word formation. (2) The defining *sūtras* (*saṃjñā-sūtra*), which introduce technical terms, e.g., "homogeneous" or "homorganic" (*savarṇa*) for sounds which "are pronounced in the same place and with the same tension of the mouth."<sup>40</sup> Examples of such sounds are *a* and *ā* or *pa, pha, ba, bha*, and *ma*, also called labials. Finally (3) the metatheorems (*paribhāṣā-sūtra*), which explain how rules have to be treated and applied in particular cases. Examples of these are given later.

Pāṇini's rules are exclusively derived from ordinary usage, and his principal problem is to give an adequate description of ordinary usage. Linguistics is not prescriptive, but descriptive. The description is based upon forms which everybody has at his disposal. He who wants pots goes to the shop of a potter, says Pāṇini's commentator Patañjali, but he who wants words does not go to the shop of a grammarian.<sup>41</sup> In practice, the problem of adequate description is a problem of correct formulation. An attempt is made to establish this aim by

<sup>39</sup> See J. F. Staal, "A Method of Linguistic Description: the Order of Consonants According to Pāṇini," *Language* 38 (1962), 1-10.

<sup>40</sup> *tulyāsyaḥ prayatnam svarṇam*: Pāṇini 1.1.9.

<sup>41</sup> *yathā ghaṭena kāryam kariṣyan kumbhakāraḥ kulaṃ gatvāha-kuru ghaṭam kāryam anena kariṣyāmiti na tadvac chabdān prayuyukṣamāno vaiyākaraṇakulaṃ gatvāha-kuru śabdān prayokṣya iti*: Patañjali, *Mahābhāṣya*, Bhārgava Sāstrī, ed. (Bombay: Nirṇaya-sāgara, 1951) I, 64 b.

making predominant use of a principle of concision (*lāghava*), i.e., by a criterion of simplicity. This is illustrated by the famous saying that grammarians rejoice over the saving of the length of half a short vowel as over the birth of a son.<sup>42</sup> The first consequence of this simplicity criterion is that repetitions are eschewed. This requires the material to be ordered in a particular manner, which at first sight appears artificial to Westerners, who are used to the Latin “grammar of ideas,” which, for example, completes the treatment of the noun before embarking upon the verb. But Pāṇini needs only one rule to explain the long *ā* in the second syllable of the nominal form *śivāya* (to Śiva), in the second syllable of the verbal form *pacāmi* (I am cooking), and other long *ā*'s.

One rule of this grammar will be dealt with in greater detail. This rule lays down that certain vowels are replaced by semi-vowels if a heterogeneous vowel follows—for example, *dadhi atra* (the milk here) is replaced in uninterrupted speech by *dadhyatra*. In customary transliteration, *i* is replaced by *y*. Similarly, we say, instead of *madhu atra* (the honey here), *madhvatra*, where *u* is replaced by *v*.

In formulating this theorem, Pāṇini makes use of several abbreviations which are basic to the structure of his grammar. Since the above-mentioned substitution also applies to the specifically Sanskrit vowels *ṛ* and *ḷ*, Pāṇini has to describe four linguistic facts which obtain if a vowel is followed by a heterogeneous vowel: *i* is replaced by *y*, *u* by *v*, *ṛ* by *r*, and *ḷ* by *l*. Pāṇini could have expressed this as follows: “*i*, *u*, *ṛ*, and *ḷ* are replaced by *y*, *v*, *r*, and *l*, respectively, in uninterrupted speech, if a heterogeneous vowel follows.” But this is too verbose and has to be subjected to the economy criterion. In the beginning of the grammar, sounds are ordered in such a way that what has later to be combined is placed together and can be referred to by means of a particular device. In the initial series of sounds, the sequence *i u ṛ ḷ* is followed by an indicatory sound, *k*, and the sequences *ya va ra la* are followed by an indicatory sound, *ṇ*. The elements *k* and *ṇ* belong to the metalanguage. The convention is to abbreviate as follows: if in the list a sound, *A*, is followed by an indicatory sound, *B*, *CB* denotes *C* as well as the following sounds up to and including *A*. For example, *ik* denotes *i u ṛ ḷ*; *uk* denotes *u ṛ ḷ*; *yaṇ* denotes *ya va ra la*; *vaṇ* denotes *va ra la*, etc. Following this convention, Pāṇini could have formulated the rule as follows: “*ik* is replaced by *yaṇ* in uninterrupted speech, if a heterogeneous vowel follows.” However, it is obvious that the phrase “in uninterrupted speech” occurs in other rules as well. Pāṇini groups all such rules to-

<sup>42</sup> *ardhamātrālāghavena putrotsavaṃ manyante vaiyākaraṇāḥ: Paribhāṣenduśekhara* 122, ed. F. Kielhorn (Bombay: Indu-Prakash Press, 1868), 115.

gether and places before the first rule a separate one, i.e., “in uninterrupted speech.”<sup>43</sup> This applies to what follows, up to a particular rule, which was originally marked by an accent.

We are still handicapped by the clumsy expression “if a heterogeneous vowel follows.” Here Pāṇini applies a simple argument. If there is a rule which states what happens if a particular condition is not fulfilled, there must also be a rule which tells us what happens if this condition is actually fulfilled. For example, we may wish to state: *dadhi atra* → *dadhyatra*, where the following vowel is not homogeneous. But what happens to *dadhi indra* when the following vowel is actually homogeneous? If the occurrence of a condition is explicitly formulated in one rule, its absence in another rule is thereby implied. Now, there actually is a rule which states that a certain phenomenon takes place “if a homogeneous vowel follows.”<sup>44</sup> It is therefore sufficient to formulate in the rule under discussion: “if a vowel follows.” Had this vowel been homogeneous, the other rule would be contradicted. Hence it is not homogeneous, and it is superfluous to state this.

This application of the simplicity criterion hardly differs from a type of conclusion we draw daily. If we see a notice, “No admittance for children and dogs,” we do not need much self-knowledge to conclude that we are allowed to enter. We do not look for another notice which says, “Adults may enter.” In later systems of Indian philosophy, similar arguments are systematically studied. A textbook example is the following. Fat Devadatta never eats by day. Since fatness is caused by eating, and eating by day is denied, it follows that Devadatta eats by night.<sup>45</sup>

In the preceding paragraphs we have encountered the principle of contradiction. In order to make his system consistent, Pāṇini applies in particular cases the following *paribhāṣā*, or metatheorem: “In case of contradiction [between two rules] the latter rule prevails.”<sup>46</sup> Apparently, the rules of grammar are ordered in such a way that this metatheorem is valid. Only the final portion of Pāṇini’s grammar begins with a *paribhāṣā* which states that the following *sūtras* are not ordered in this way.<sup>47</sup> Later grammarians have applied this

<sup>43</sup> *saṃhitāyām*: Pāṇini 6.1.72. Cf. 8.2.108.

<sup>44</sup> *akāḥ savarne dīrghaḥ*: Pāṇini 6.i. 101.

<sup>45</sup> See J. F. Staal, “Contraposition in Indian Logic,” in E. Nagel, P. Suppes, and A. Tarski, eds., *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress* (Stanford: Stanford University Press, 1962), p. 645.

<sup>46</sup> *vipratīśedhe paraṃ kāryam*: Pāṇini 1.4.2. See J. F. Staal, “Negation and the Law of Contradiction in Indian Thought: A Comparative Study,” *Bulletin of the School of Oriental and African Studies* 25 (1962) 52-71.

<sup>47</sup> *pūrvatrāsiddham*: Pāṇini 8.2.1. See H. E. Buiscool, *Pūrvatrāsiddham* (Amsterdam: H. J. Paris, 1934).

metatheorem of consistency to a greater number of cases than Pāṇini did himself.

The theorem under discussion can now be formulated as: “*ik* is replaced by *yaṅ* if a vowel follows.” But the same method of abbreviation can be used to refer to vowels. Pāṇini avoids the introduction of special categories, such as vowels, since he needs only to group the required sounds together in his initial list and apply the above method of abbreviation. This is far more empirical, since simplicity of description is an objective criterion and does not commit grammarians to theories regarding different kinds of sounds, such as vowels and consonants. Pāṇini dispenses with such theories, since he constructs a descriptive grammar and not a “grammar of ideas.” His initial list of sounds begins with what other grammarians call vowels (a distinction which was known in India before and after Pāṇini). The first vowel is *a*, and the indicator following the last vowel is *c*. Pāṇini can therefore denote vowels by *ac*.

The theorem can now be formulated as: “*ik* is replaced by *yaṅ* if *ac* follows.” Still, we have not reached the most concise formulation. In grammar, expressions such as “*x* is replaced by *y*” and “if *x* follows” are obviously extremely common. These are, as it were, fundamental metalinguistic relations. Pāṇini expresses these relations by using the Sanskrit cases in a pregnant manner. Since cases are distinguished in Sanskrit by their terminations, Pāṇini says: “*ik* + genitive termination, *yaṅ* + nominative termination, *ac* + locative termination.” In Sanskrit, this becomes: “*iko yaṅ aci*.” This is the rule as it actually occurs in Pāṇini’s grammar.<sup>48</sup> Incidentally, one should not suppose that expressions like “*iko yaṅ aci*” are intelligible to Sanskrit speakers or scholars who have not studied Pāṇini—not any more than an algebraic formula in an English textbook is intelligible to English speakers who have no knowledge of algebra. *Sūtras* in Sanskrit scientific literature correspond to formulas in Western scientific literature.

The rule under discussion is primarily a rule for pronunciation and not for orthography. It is probable that Pāṇini constructed his grammar without using writing—an achievement hard to visualize anywhere but in India, where large bodies of knowledge are orally composed and transmitted. However, this rule could also be regarded as descriptive of a historical process, in the manner of the sound laws of comparative philology. Pāṇini, however, never says that *i be-*

<sup>48</sup> Pāṇini 6.1.77. Many of Pāṇini’s rules are stated in accordance with this scheme:

a + *abl. term.* b + *gen. term.* c + *nom. term.* d + *loc. term.*

In modern terminology this can be expressed by substitution rules as:

a b d → a c d or:

b → c in context a—d.

In the latter expression and in Pāṇini’s expressions, repetition of a and d is avoided. (See the article quoted above, note 5.)

comes  $y$ . He speaks of *replacing  $i$  by  $y$* , in the manner indicated above. His commentators use in this connection the term "substitution" (*ādeśa*). It is of some importance to realize that laws of development such as " *$i$  becomes  $y$ ,*" which are postulated without corresponding to perceptible change, are based upon purely descriptive statements of the type " *$i$  is replaced by  $y$ .*" As in the case of natural science, the most concise formulation yields the most probable theory. The scientific value of a diachronic hypothesis depends on the simplicity of a system of synchronic descriptions. This applies not only to linguistics but also to anthropology.<sup>49</sup> Morris Halle has shown this with regard to sound laws of Indo-European, such as the laws of Grimm and Verner: "The acceptance of these laws as historical fact is based wholly on considerations of simplicity."<sup>50</sup>

When comparing Pāṇini's system with Euclid's *Elements*, a characteristic of the latter, i.e., deduction, appears absent from the former. It is true that there is a kind of deduction in Pāṇini's grammar: *dadhyaatra* is deduced from *dadhi atra*, and other forms are similarly deduced with the help of rules. But such deductions do not seem to attain the same degree of generality as Euclid's proofs. However, the difference reflects a distinction of object, not of structure. In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed *ad hoc* (i.e., *sūtras*). Both systems exhibit a structure of logical deduction with the help of rules, and both scholars attempted to arrive at a structural description of facts. In both systems, contradictions and unnecessary complications are avoided. In both cases, the aim is adequate and simple description.

So far, only one example of grammatical method in India has been given. More detailed investigations into the methods of Euclid and Pāṇini would throw light on points of difference as well. Another common characteristic is the above-mentioned desire to shorten principles (where Euclid pays attention to minimum number, Pāṇini to minimum length), while disregarding the length of derivations. In Kleene's mathematical logic, for example, the proposition  $a = a$  is deduced in seventeen steps.<sup>51</sup> Pāṇini derives the aorist *ajāgariṣam* ("I woke up") from the stem *jāgr-* and the affix *-iṣam* by making use of nine

<sup>49</sup> See J. F. Staal, "Sanskrit and Sanskritization," *The Journal of Asian Studies* 22 (1963), 261-275.

<sup>50</sup> M. Halle, "On the Role of Simplicity in Linguistic Descriptions," in R. Jakobson, ed., *Structure of Language and Its Mathematical Aspects* (Providence: American Mathematical Society, 1961), 89-94.

<sup>51</sup> S. C. Kleene, *Introduction to Metamathematics* (Amsterdam: North-Holland Publishing Co.; Groningen: P. Noordhoff, 1952), 84.

different *sūtras*.<sup>52</sup> In modern logic, the underlying problem has come to the forefront. Theoretical investigations on the length of proofs may also be relevant to questions regarding the time needed by electronic computers. Quine, who distinguishes, as we saw, two complementary types of simplicity, has obtained short derivations and avoided a complicated substitution rule by assuming an infinite list of axioms.

Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in Western thought.<sup>53</sup> Scientific developments have therefore taken different directions in India and in the West. Pāṇini's system produced at an early date such logical distinctions as those between language and metalanguage, theorem and metatheorem, use and mention, which were discovered much later in Europe.<sup>54</sup> In other Indian sciences, e.g., in mathematics and astronomy, as well as in later grammatical systems of Sanskrit, Prakrit, and Tamil, systematic abbreviations are used which not only are ingenuous but also constitute new adaptations of the same method.<sup>55</sup> In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. In the West, this corresponds to the belief that mathematics is the most perfect among the sciences. Just as Plato reserved admission to his Academy for geometers, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics. In India, grammar was called the Veda of the Vedas, the science of sciences. Renou declares: "To adhere to Indian thought means first of all to think like a grammarian" (*Adhérer à la pensée indienne, c'est d'abord penser en grammairien*).<sup>56</sup> This has determined the form and method of a large part of Indian philosophy, an important feature which is generally lost when Sanskrit originals are translated into Western languages. It seems almost unavoidable that translations of an accurate original should therefore appear vague.

The simplicity criterion looms large in later Indian logic, where cumbersome solutions are replaced by more perspicuous and elegant formulations. Complicated arguments, theories, and technical concepts, as postulated, for example, by the ritualistic philosophers of the Mīmāṃsā, were rejected by

<sup>52</sup> Quoted in L. Renou-J. Filliozat, *L'Inde Classique II* (Paris: Payot, 1953), 88.

<sup>53</sup> See above, note 2.

<sup>54</sup> See J. Brough, "Theories of General Linguistics in the Sanskrit Grammarians," *Transactions of the Philological Society* (1951), 27-46; J. F. Staal, "The Theory of Definition in Indian Logic," *Journal of the American Oriental Society* 81 (1961), 122-126.

<sup>55</sup> See L. Renou, *Introduction générale* for J. Wackernagel, *Altindische Grammatik I* (Göttingen: Van den Hoeck & Ruprecht), 123 note 614. See also P. S. Subrahmanya Sastri, *History of Grammatical Theories in Tamil* (Madras: The Journal of Oriental Research, 1934), 108.

<sup>56</sup> *L'Inde classique II* 86.

logicians because they could be derived from more elementary expressions.<sup>57</sup> Such rejections are generally accompanied by the expression “*gauravāt*” (on account of heaviness). It is clear that this “Indian razor” could be fruitfully applied also to Western philosophy, where heavy examples are common. While simplicity of language is always a sign of good style, it has become a valued indication of clarity of thought, especially in British philosophy.

The algebraic style of the *sūtras* can assume forms which are hardly intelligible without a commentary.<sup>58</sup> What is clear to insiders becomes thereby abstruse to outsiders, as is often the case in modern science. This is one of the unavoidable implications of the use of an artificial language. A famous Indian example is provided by the definition of *Brahman*, the Absolute, in the *Vedānta-sūtra* as: “On account of which the origin, etc., of this.”<sup>59</sup> In order to understand this it is necessary to know that the last word, “this,” denotes this world, while “origin, etc.” means origin, preservation, and dissolution. Hence, the *sūtra* defines *Brahman* as that on account of which the origin, preservation, and dissolution of this world exist. The term “etc.,” at times ambiguous, which has also led to discussions in the West, was used by Pāṇini to indicate lists of words or roots by attaching it to the first member. Two appendices attached to Pāṇini’s grammar, the *Dhātupāṭha* and the *Gaṇapāṭha*, enumerate such lists.<sup>60</sup> Elsewhere in the grammar, special cases are enumerated within a *sūtra*, provided these are not too many. In the following verse this is ridiculed, and a pun is made on the word “*sūtra*” which signifies both “rule” and “thread”:

No wonder that the girl strings together glass, gems,  
and gold on one thread.

Even Pāṇini, who ought to know better, combined dog,  
youth, and king of the Gods in one rule.<sup>61</sup>

Finally, some *sūtras* will be quoted from the traditional manual of a system of Indian philosophy that more than any other has led in the West to un-

<sup>57</sup> For example, *arthāpatti*, *anuṣalabdhī*.

<sup>58</sup> The definition of *sūtra* by Madhva (13th cent.) is itself not in *sūtra* style: *svalpākṣanam asamāgīdham sāravad viśvatomukham astobham anavadyam ca sūtram sūtravidō viduḥ*. See also L. Renou, *Histoire de la langue sanskrite* (Paris: Editions IAC, 1956), 53-59, and especially L. Renou, “Sur le genre du *sūtra* dans la littérature sanskrite,” *Journal Asiatique*, 251 (1963), 165-216.

<sup>59</sup> *janmādy asya yataḥ: Vedānta-sūtra* I.12.

<sup>60</sup> For a modern treatment of lists, see F. W. Householder, “Lists in Grammars,” *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress*, 567-576.

<sup>61</sup> *kācam maṇim kāñcanam ekasūtre grathnāti bālā na hi tad vicitram viśeṣavīṭ pāṇinir ekasūtre śvānam yuvānam maghavānam āha*, quoted in K. C. Chatterji, *Patañjali’s Mahabhashya: Paspasahnika* (Calcutta: A. Mukherjee and Co., 1957), 126. This refers to Pāṇini 6.4.133.

reliable treatment and mystification: the Yoga system as dealt with in Patañjali's *Yoga-sūtra*. As indicated by the title, this treatise is composed of *sūtras*. In the first four *sūtras* the subject matter is mentioned, defined, and characterized. This is followed by a classification of some specific forms.

The *sūtras* run as follows: "1. Now follows a treatment of *yoga*. 2. *Yoga* is cessation of the functions of the mind. 3. Then the perceiver remains in his own form. 4. Else he assumes the form of the functions."<sup>62</sup> This is followed by an enumeration of these functions. The remaining part of the first chapter explains how the cessation of functions is brought about.

These four *sūtras* lay the foundation for the entire system, which is constructed logically and fulfills the requirements of Indian methodology. Having introduced and defined the subject matter, the *sūtras* continue to explain each technical term. The third *sūtra* states what is philosophically most important in the system. Whoever stops the functions of his mind, i.e., the *yogin*, is the real man, who abides in himself (i.e., his self). The fourth *sūtra* supplements this by declaring that those who act differently, lose themselves (i.e., their selves) and identify themselves with the functions of their mind.

In this system, the attempt at accurate description and construction is present to the same extent as, for instance, in the chapter on affections in Spinoza's *Ethics*. The study of Indian philosophy in general loses much of its importance if the underlying methods are neglected. These methods are not only related to the language in which they are expressed, but they are often directly inspired by studies of this language by Indian grammarians. In the West, the recognition that linguistic structures play a decisive role in philosophy is slowly gaining ground. In India, it has long been explicit. This recognition may be expected increasingly to affect, if not to undermine, our philosophic certainties.

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<sup>62</sup> (1) *atha yogānuśāsanam*, (2) *yogas cittavṛttinirodhaḥ*, (3) *tadā draṣṭuḥ svarūpe 'vasthānam*, (4) *vṛttisārūpyam itaratra*.

A METHOD OF LINGUISTIC DESCRIPTION  
THE ORDER OF CONSONANTS ACCORDING TO PĀṆINI

1. It is well known that the earliest and probably best descriptive grammar in existence is the Sanskrit grammar of Pāṇini (?350 B.C.). The subject of the following essay is Pāṇini's phonological analysis of the Sanskrit consonants. This illustrates some of the more important techniques used by Pāṇini and his colleagues. It also enables us to reconstruct some of the methods by means of which Pāṇini constructed his grammar.<sup>1</sup>

Though the analysis is of general interest, the object language is a particular one, Sanskrit. Some knowledge of the sound system of Sanskrit is therefore necessary to appreciate Pāṇini's treatment. For this purpose I adapt here a part of the chart The Sanskrit Alphabet given by W. S. Allen in his book *Phonetics in ancient India* 20 (London, 1953). This chart explains at the same time the customary Roman transliteration used in the following paper.

Similar enumerations or partial enumerations are given in some of the early phonetic treatises which are the special subject of Allen's monograph. In Pāṇini's grammar itself the matter is stated differently. His grammar consists of rules or sūtras grouped together in eight chapters and therefore often referred to as *Aṣṭādhyāyī* 'the eight chapters'. Each chapter consists of four sections called *pāda* and within each *pāda* the sūtras are numbered from one onward. Each sūtra of the grammar is therefore determined by three numerals and is in general quoted accordingly. For example, 7.3.102 refers to the 102nd sūtra of the 3rd *pāda* of the 7th *adhyāya*.

The *Aṣṭādhyāyī* is preceded by the *Śivasūtra*, a list of the sounds which are used in the grammar. The order of sounds in this list is different from the order of the above chart. Reasons for this difference will be given in detail below, but the principle upon which they are based is accounted for, in Allen's words (*ibid.*), 'by the phonological, as opposed to phonetic, approach there adopted'.

ŚIVASŪTRA

a i u ṛ / r ḷ k / e o ṅ / ai au c / ha ya va ra ṭ / la ṇ / ña ma na ṇa na m /  
jha bha ṅ / gha ḍha dha ṣ / ja ba ga ḍa da ś / kha pha cha ṭha tha ca ṭa ta v /  
ka pa y / śa ṣa sa r / ha ḷ /

The *Śivasūtra* starts with the vowels. Then come the semivowels and the consonants, each one always followed by a short -a. Subdivisions in the list are marked by certain indicatory sounds, always semivowels or consonants, which are not followed by a short -a and therefore easily recognized. Such an indicatory sound can be taken to indicate in the first place the end of a sūtra. Similar devices are not rare in a tradition like the Indian, which is predominantly oral.<sup>2</sup>

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<sup>1</sup> I am grateful to Morris Halle for valuable suggestions.

<sup>2</sup> See for example J. F. Staal, *Nambudiri Veda recitation* 24 ('s-Gravenhage, 1961).

			'glottal'	velar	palatal	retroflex	dental	labial
CONSONANTS								
stops	voiceless	unaspirated		<i>k</i>	<i>c</i>	<i>ṭ</i>	<i>t</i>	<i>p</i>
		aspirated		<i>kh</i>	<i>ch</i>	<i>ṭh</i>	<i>th</i>	<i>ph</i>
	voiced	unaspirated		<i>g</i>	<i>j</i>	<i>ḍ</i>	<i>d</i>	<i>b</i>
		aspirated		<i>gh</i>	<i>jh</i>	<i>ḍh</i>	<i>dh</i>	<i>bh</i>
		nasals		<i>ṅ</i>	<i>ñ</i>	<i>ṇ</i>	<i>n</i>	<i>m</i>
		semivowels			<i>y</i>	<i>r</i>	<i>l</i>	<i>v</i>
fricatives	voiceless		<i>h</i>		<i>ś</i>	<i>ṣ</i>	<i>s</i>	
		voiced	<i>ɦ</i>					
VOWELS								
	short		<i>a</i>		<i>i</i>	<i>r</i>	<i>ḷ</i>	<i>u</i>
	long		<i>ā</i>		<i>ī</i>	<i>ṛ</i>		<i>ū</i>
					<i>e</i>			<i>o</i>
						<i>ai</i>		<i>au</i>

The view has been defended that the *Śivasūtra* and the *Aṣṭādhyāyī* are mutually independent. According to H. Sköld,<sup>3</sup> the *Śivasūtra* cannot be the work of Pāṇini. The arguments he adduces for this are not only inconclusive but also rest on numerous assumptions regarding the authenticity of certain sūtras, all unproven. Moreover Sköld makes no attempt to explain the order as it is. Actually no patient observer can fail to see that the *Śivasūtra* and the *Aṣṭādhyāyī* presuppose each other to a very large extent. This has moreover been convincingly established by Paul Thieme in his book *Pāṇini and the Veda* 108–10 (Allahabad, 1935).

The close interrelationship between *Śivasūtra* and *Aṣṭādhyāyī* is best illustrated by Pāṇini's special use of the indicatory sounds given above, which are called *anubandha*. These indicatory sounds belong to the metalanguage of Pāṇini's grammar;<sup>4</sup> they will from now on be printed in boldface. Each *anubandha* enables Pāṇini to refer in his grammar to groups of sounds by means of a special technique referred to as *pratyāhāra* 'condensation'. This technique consists in referring to a sequence of sounds, the last of which is followed by an *anubandha*, by means of an abbreviation consisting of the first sound of the group and the *anubandha* following the last sound.<sup>5</sup> For instance, the first three sounds *a i u* are

<sup>3</sup> Facts and Conjectures about the Śivasūtras, *Papers on Pāṇini and Indian grammar in general* 8–22 (Lund and Leipzig, 1926).

<sup>4</sup> Cf. Staal, The Theory of Definition in Indian Logic, *JAOS* 81.123–4 (1961).

<sup>5</sup> Pāṇini 1.1.71: *ādir antyena sahetā* 'an initial sound joined to a final (indicatory) sound (denotes the intervening sounds as well)'. This applies not only to the grouping of simple sounds, but also to verbal and nominal terminations, etc.

followed by the anubandha ण. Therefore  $a\bar{n}$  denotes the sounds  $a i u$ ;  $i\bar{n}$  denotes the sounds  $i$  and  $u$ ;  $u\bar{n}$  denotes  $u$  and is hence superfluous as an abbreviation.<sup>6</sup> From this it is obvious that the sounds should be ordered in such a way, and that the indicatory sounds should be inserted at such spots, that it becomes easy to refer to all sounds which have to be combined in groups in the grammatical statements.

The problem can be expressed in mathematical terms. Let a set of  $n$  sounds be ordered as a sequence  $a_1, \dots, a_n$ . Let this sequence be interrupted at certain spots by indicatory sounds  $x$ , such that  $x_i$  is the indicatory sound following the sound  $a_i$ . Then the sequence of sounds  $a_1, \dots, a_i$  can be 'condensed' to  $a_i x_i$ . Now the question is, given a set of combinations of sounds  $a_1, \dots, a_m$  which are needed in the grammar, how to establish an ordered sequence of the above type with appropriate insertions of indicatory sounds. This was Pāṇini's problem when he constructed the *Śivasūtra*. In general, if a combination  $a_1, \dots, a_m$  is needed this requires the insertion of an indicatory sound  $x_m$  in the list; but as we shall see, Pāṇini simplified this requirement. His procedure was probably along the following lines. He gave provisional formulations of several grammatical rules where certain groups of sounds were needed. He then constructed the *Śivasūtra* on the base of these requirements, which had to be carefully combined. Lastly he gave the final formulation to the grammatical rules concerned with the help of 'condensations' based upon the *Śivasūtra*.

We shall now try to reconstruct some of the procedures used by Pāṇini and illustrate the method which led to the construction of the *Śivasūtra*. The only scholar, apparently, who has given a detailed discussion of this matter is Paul Thieme, in the work mentioned. But while his discussion of the relatively transparent order of vowels is exemplary, the discussion of the order of consonants is incomplete and not always correct.

The procedure followed by Pāṇini in the construction of the *Śivasūtra* with the appropriate insertions of indicatory sounds deserves attentive study, for Pāṇini arrived by means of similar procedures at the establishment of the order of other rules and the use of other indicatory sounds by means of which he could refer to linguistic forms (verbal and nominal terminations, etc.).

2. We shall rewrite the list of consonants of the *Śivasūtra* in tabular form, since this will enable us to refer to rows and columns in a simple manner:

$\tilde{n}$	$m$	$\dot{n}$	$\eta$	$n_m$
$j_h$	$bh_{\bar{n}}$	$g_h$	$d_h$	$dh_{\bar{s}}$
$j$	$b$	$g$	$d$	$d_{\bar{s}}$
$kh$	$ph$	$ch$	$t_h$	$th$
$c$	$\ddot{t}$	$t_v$	$k$	$p_y$
$\acute{s}$	$\ddot{s}$	$s_r$		

Here indicatory sounds are written as subscripts. The short  $-a$  which follows each consonant is left out here but will be used when a *pratyāhāra* is formed. Thieme

<sup>6</sup> That  $u$  itself here denotes both  $u$  and  $\bar{u}$  is irrelevant in the present context. Pāṇini refers to short  $u$  by means of  $ut$ . This applies similarly to  $a$  and  $i$  in accordance with 1.1.70: *taparas tatkālasya* '(a vowel) followed by  $t$  denotes (the vowel) of its own length'.

has formulated the general principle with the help of which this order should be explained.<sup>7</sup> 'The arrangement of Pāṇini's list of sounds, which at first looks rather disorderly, is explainable as due to the phonetic catalogue of sounds having been adapted to the practical requirements of the grammar, in which Pāṇini wanted to refer to certain groups of sounds by short expressions'.

The first point which strikes us here is that the horizontal order reflects the manner of articulation of the sounds: the rows give us respectively nasals, aspirated and unaspirated voiced stops, aspirated and unaspirated voiceless stops. Then follow the fricatives, which we will leave out of consideration for the time being. One might accordingly expect the vertical order to reflect the place of articulation of the sounds. If the order of sounds in the other rows were in accordance with the order of sounds in the first row, the columns would give us respectively palatals, labials, velars, retroflexes, and dentals. This order is found in the first three rows, but not in the last two. Actually this poses no direct problem to Pāṇini, as he uses another kind of abbreviation for these groups by denoting them by their unaspirated voiceless member followed by short -u.<sup>8</sup> For example *ku* denotes the velars *k*, *kh*, *g*, *gh*, *ṅ*.

We shall now have to explain the order of rows and the order of sounds within each row. Thieme (102) explains the order of rows as follows: 'Within the groups, there came first the simplest sounds, the surd mutes, and last the most complicated, the aspirant sonant mutes and the nasal consonants.' Here Pāṇini's order is reversed and a standard is introduced—simplicity—which may not be a formally definable characteristic.<sup>9</sup> As we shall see, the order will be reversed later. Thieme proceeds to explain the order of sounds within each row as follows: "The groups themselves followed each other according to the place of the articulating organs, which determine the character of each, in the mouth, proceeding from the back, the throat (*kanṭha*), to the front, the lips.' As this suggested order does not obtain in any of the five rows, which in addition are not parallel to each other, Thieme proceeds to modify this scheme in order to arrive at Pāṇini's order. He first constructs:

<i>ṅ</i>	<i>ṅ̄</i>	<i>ṅ̆</i>	<i>n</i>	<i>m</i>
<i>gh</i>	<i>jh</i>	<i>ḍh</i>	<i>dh</i>	<i>bh</i>
<i>g</i>	<i>j</i>	<i>ḍ</i>	<i>d</i>	<i>b</i>
<i>kh</i>	<i>ch</i>	<i>ṭh</i>	<i>th</i>	<i>ph</i>
<i>k</i>	<i>c</i>	<i>ṭ</i>	<i>t</i>	<i>p</i>

by reading the original table from right to left and upwards. One might ask whether such a table could not equally well serve as a starting point, being, as Thieme himself observes, 'as strictly phonetic as the old one'. Now the irregularities have to be explained and Thieme proceeds by stating (106) that this arrange-

<sup>7</sup> Pāṇini and the Veda 104; cf. also Allen, *Phonetics in ancient India* 20 n.

<sup>8</sup> Panini 1.1.69: *aṅudīt savarṇasya cāpratyaḃāḥ* 'aṅ (vowels and semivowels) and (consonants) marked with u denote the homorganic sounds as well, provided they are not suffixes'. For *savarṇa* 'homorganic' see L. Renou, *Terminologie grammaticale du sanskrit* 331 (Paris, 1957), and Allen, *Phonetics in ancient India* 45.

<sup>9</sup> This criterion is related to N. S. Trubetzkoy's distinction between *merkmaltragend* and *merkmallos*: *Grundzüge der Phonologie* 67 (reprinted Göttingen, 1958).

ment 'could be made still more serviceable by singling out certain sounds which had to be excluded on certain occasions from operations that were applied to, or were conditioned by, their fellow sounds. Such were *jh* and *bh*, and *k* and *p*. In consequence, the arrangement according to the place of articulation, till now observable within the groups, was disturbed in two places.' We are not told, however, why these two pairs are singled out and why the first pair is placed in front as

*jh bh gh dh dh*

while the second is placed behind as

*c ṭ t k p.*

But we are told that in the first row the order of the second is anticipated, so that we arrive at:

*ñ m ñ n n*  
*jh bh gh dh dh.*

Analogously, according to Thieme, in the fourth row the order of the fifth is anticipated, so that we arrive at:

*kh ph ch ṭh th*  
*k p c ṭ t*

which Thieme writes as:

*kh ph ch ṭh th*  
*k p c ṭ t*

But this does not explain why the third row follows the second and does not for example anticipate the last row; and why the fourth row can only be said to anticipate the fifth if the latter is written as Thieme wrote it, i.e. as

*k p c ṭ t,*

which is different from Pāṇini's order. In general we may ask whether such anticipations can be called rational in the beginning of a grammar where few things seem to be without a necessary reason. Why for instance do certain rows anticipate certain others, but not all? Why could certain rows not follow certain others?

There are points which appear to remain unexplained as well in the singling out of certain sounds as in the change of order in consecutive rows. Moreover these attempts at an explanation do not yet take the indicatory sounds into account.

3. In order to arrive at a fuller explanation we should first of all observe the location of the indicatory sounds in the table as Pāṇini gives it. We may for the time being accept without questioning that they are placed at the end of rows: for this, presumably, facilitates reference to the rows as a whole. But three questions arise in this connection: (1) Why is there an *anubandha* after *bh*? (2) Why is there an *anubandha* after *t*? (3) Why is there no *anubandha* after *th*?

The first point seems to confirm Thieme's assertion that *jh* and *bh* had to be singled out. The second point on the other hand appears to refute his view that *k* and *p* had to be singled out. For in Pāṇini's order, *k* and *p* can easily be referred to as *kay* without an extra *anubandha* being introduced. Actually Pāṇini does not need this *pratyāhāra* anywhere. If on the other hand *k* and *p* had been placed at the beginning of the fourth row, an *anubandha* could have been inserted after *p* just as an *anubandha* was inserted after *bh*. The fact that the fifth row starts with *c ṭ t* followed by an *anubandha*, while the fourth is not followed by an *anubandha*, seems to suggest that Pāṇini wished to single out a group of certain sounds in the fourth row as well as *c*, *ṭ*, and *t*.

These questions can only be answered completely by a close investigation into the *pratyāhāra* combinations which Pāṇini actually forms and uses. This in turn can be explained by the groups which Pāṇini needs in the formulation of grammatical rules. These groups can be expected to explain not only the location of the indicatory sounds, but the order of sounds as well. Such a study can be rather easily undertaken with the help of the list of *pratyāhāra* combinations given by Boehlingk in his note on the *Śivasūtra* and by the alphabetical *Erklärung der grammatischen Elemente* of the same scholar, which also enumerates the *sūtras* where a given *pratyāhāra* is utilised.<sup>10</sup>

First a general methodological principle of Pāṇini has to be observed. When Pāṇini uses a *pratyāhāra* to refer to a group of sounds, this group should contain ONLY sounds occurring in the sequence denoted by the *pratyāhāra*, but it need not contain ALL sounds of the sequence. For example the *pratyāhāra yañ* denotes the sequence of sounds *y v r l ñ m ṇ n jh bh*. But Pāṇini uses *yañ* to refer to a group of sounds actually consisting of only *m v y bh*.

Another general principle is also implicitly used by Pāṇini. This is the famous economy criterion, which the later grammarians denoted by the term *lāghava* 'lightness, conciseness'.<sup>11</sup> In accordance with this principle each linguistic rule should be given in the shortest possible form, whereas the number of metalinguistic symbols should be reduced as far as possible. The group of sounds *m v y bh*, therefore, should be referred to by means of *yañ*, rather than by 'pu and yaṭ' or the like. Similarly, as we shall see, the number of *anubandhas* also should be as far as possible reduced.

The *pratyāhāra yañ* is the only *pratyāhāra* which makes use of the *anubandha ñ*. It is used in two grammatical rules, 7.3.101 and 7.3.102. The first says that there is lengthening of a short final *a* of a verbal stem before a verbal ending beginning with *yañ*.<sup>12</sup> Examples: \**paca-mi* → *pacāmi* 'I cook'; \**paca-vaḥ* →

<sup>10</sup> O. Boehlingk, *Pāṇini's Grammatik* 1 and 146\*-92\* (Leipzig, 1887).

<sup>11</sup> This concept was used by the grammarians (see Renou, *La Durghaṭavṛtti de Śaraṇadeva* 1.1.81-2 n. [Paris, 1940]) and by other authors of scientific Sanskrit, in particular the logicians; see D. H. H. Ingalls, *Materials for the study of Navya-nyāya logic* 47 (Cambridge, Mass., 1951). For a comparison between the uses of the economy criterion in logic and in grammar see Staal, *Contra-position in Indian Logic*, to appear in *Proceedings of the 1960 International Congress for Logic, Methodology and Philosophy of Science* (Stanford).

<sup>12</sup> Pāṇini 7.3.101: *ato dīrgho yañi* 'a long vowel is substituted for the final *a* of a verbal stem before (a *sārvadhātuka* ending beginning with) *yañ*'. The *sārvadhātuka* endings comprise most verbal endings (see Renou, *Terminologie grammaticale du sanskrit* 337).

*pacāvah* 'we both cook'. The next rule says that this holds 'also before a nominal ending',<sup>13</sup> i.e. that there is lengthening of a short final *a* of a nominal stem before a nominal ending beginning with *yañ*. Examples: \**vrkṣa-ya* → *vrkṣāya* 'to the tree'; \**vrkṣa-bhyām* → *vrkṣābhyām* 'to two trees'. In other words, of *yañ* we need only *m v y bh*. On the other hand we wish to exclude many other consonants, e.g. *t* and *s* (for we have merely *pacasi*, *pacati* 'you cook, he cooks', without lengthening). The question arises whether *yaś* would not have been as good as *yañ*, which would enable us to do without the *anubandha* ñ. However *yaś* would include *dh*, which is the first sound of the ending of the 2nd plural present middle. Because we actually have in Sanskrit *paca-dhve* 'you cook for yourselves' and not \**pacā-dhve*, the sound *dh* has to be excluded from the rule. Therefore an *anubandha* is needed after either *bh*, *gh* or *ḍh*. It is understandable that Pāṇini placed the *anubandha* after *bh*, the last sound he wished to include in his group.

This shows (1) that the location of *bh* before *dh* and the location of the *anubandha* ñ was established AFTER a provisional formulation of the rules 7.3.101–102, possibly by means of the unabbreviated enumeration 'm, v, y, and bh'; (2) that the final formulation of these two rules with the help of *yañ* took place AFTER the establishment of the order and the location of the *anubandha*. At the same time it becomes imperative, given the location of the semivowels immediately before the group of nasals and stops, to begin the latter group with the nasals and the aspirated voiced stops containing *bh*. This follows from the fact that the fourth and fifth row contain sounds which do not cause lengthening of the *a* in the situation described in 7.3.101–102 (namely *th* and *t*; similarly the following row of the fricatives, containing *s*). The question remains whether the third row could not have been placed elsewhere. But for this there are other reasons. Pāṇini needs *jhaś* for referring to all voiced stops in sūtra 8.4.53, which states<sup>14</sup> that unaspirated voiced stops (*jaś*) are substituted for stops, fricatives, and *h* (*jhal*<sup>15</sup>), when a voiced stop (*jhaś*) follows. Example: \**labh-dhā* (from \**labh-tā*) → *labdhā* 'he will take'.

This gives us all the reasons for the singling out of *bh* and the insertion of an *anubandha* after it. The question remains whether *bh* could not have been placed before *jh*, as *jh* at any rate does not occur in a position which would affect the situation dealt with in 7.3.101–102. The answer to this could be inferred from the use of the *pratyāhāras* *bhaś* as well as *baś* in 8.2.37, which states:<sup>16</sup> '*bhaś* (is to be substituted) for (initial) *baś* of a monosyllabic (verbal stem) ending in *jhaś*, either at the end of a word or followed by a suffix with initial *s* or *dhv*'. Examples:

<sup>13</sup> Pāṇini 7.3.102: *supi ca* 'also before a nominal ending'. The metalinguistic expression *sup* is a *pratyāhāra* formed from *su*, the name of the ending -s of the first (i.e. Nominative) case in the Singular in combination with the *anubandha* p which is placed after the ending of the last (i.e. Locative) case in the Plural. In our system *su* could also be written *su* or even *su* (since the Nominative Singular ending is often zero).

<sup>14</sup> Pāṇini 8.4.53: *jhalām jaś jhaśi* 'jaś is substituted for *jhal* when *jhaś* follows'.

<sup>15</sup> The inclusion of fricatives and *h* in *jhal* is here purely theoretical, as Renou rightly observes, *La grammaire de Pāṇini* 3.143 (Paris, 1954).

<sup>16</sup> Pāṇini 8.2.37: *ekāco baśo bhaś jhaśantasya dhvoḥ*.

*bhotsyate* 'he will perceive' from *budh-*; *godhuk* 'milker of cows' from *dugh-*. It is not entirely clear, however, why *jh* should be excluded here.<sup>17</sup>

The order of sounds in the first row is not merely an anticipation of the order of sounds in the second row, but is partly necessitated by the fact that Pāṇini has to combine velar, retroflex, and dental nasals in one group whilst excluding the labial and palatal nasals. This is needed in *sūtra* 8.3.32, which states:<sup>18</sup> '*ṅam* (has to be placed) before a(n initial) vowel after (final) *ṅam* (respectively) after a short vowel'. Examples: *\*udaṅ āste* → *udaṅṅāste* 'he sits towards the North'; *\*vaṅ āste* → *vaṅṅāste(?)*; *\*kurvaṅ āste* → *kurvaṅṅāste* 'he sits while doing'. This does not hold for *m*, for we have *aham asmi* 'I am', which does not become *\*aham-masmi*. This case shows again, analogously, that the subdivision of the first row in two groups was established after a provisional formulation of 8.3.32, and that the final formulation of this rule took place after the subdivision of the nasals. In addition it may be observed that it was obviously preferable to place the group *ṅ n n* after *ṅ m*, and not before; for otherwise an additional *anubandha* would have had to be introduced after *n*, which would conflict with the economy criterion mentioned earlier. A final *anubandha* was at any rate required, because reference is made to the group of semivowels and nasals, now referred to by *yam*. This is an additional reason which necessitates the location of the semivowels immediately before the nasals.

4. Now let us consider the final row of stops. The *anubandha* *v* is used in only one *pratyāhāra*, namely *chav*. This denotes the sequence of unvoiced 'central' stops,<sup>19</sup> aspirated and unaspirated, needed in *sūtra* 8.3.7, which states<sup>20</sup> that *anusvāra* is substituted for a final *n* (excepting the final *n* of *praśān* 'calm') when *chav* follows. Example: *bhavāṃś cinoti* 'he collects' from *\*bhavān cinoti*. This requirement leads to location of the group *ch ṭh th* at the end of the fourth row and location of the group *c ṭ t* at the beginning of the fifth row. Here again this order

<sup>17</sup> Cf. J. Wackernagel, *Altindische Grammatik* 1. §106 (Göttingen, 1896), where palatals are not mentioned. The sound *jh* is a late development in Sanskrit (Wackernagel §141).

<sup>18</sup> Pāṇini 8.3.32: *ṅamo hrasvād aci ṅamuṅ nityam*. For *nityam* 'necessarily' cf. Renou, *Études védiques et Pāṇinienes* 1.126-7 (Paris, 1955).

<sup>19</sup> The term *central* is used by M. Halle, In Defense of the Number Two, *Studies presented to Joshua Whatmough* 71 ('s-Gravenhage, 1957). This concept could be used in connection with the classification of sounds which do not interfere with the retroflexion caused by *r*, *ṛ*, *ṛ*, and *ṣ*. See Pāṇini 8.4.1: *raṣābhyām no ṅaḥ samānapade* 'ṅ is substituted for *n* after *r* and *ṣ* in the same word'; 8.4.2: *aṭ-ku-pv-āṅ-num-vyavāye 'pi* 'also when *aṭ* (here: vowels, *y*, *v*, *h*), *ku* (velars), *pu* (labials), *āṅ* (the preposition *ā*), or an augment intervenes'. (For 'interfering' articulations the term used in the same context in the *Ṛkprātiśākhya* is *vigh-nakṛt*: Renou, *Terminologie grammaticale du sanskrit* 493.) According to these two rules, *\*arkeṇa* → *arkeṇa* 'by the sun', *\*premnā* → *premnā* 'by love'; but in *arcanam* 'worship', *arṇavana* 'by the ocean' and *ardhena* 'by half' there is no retroflexion of the dental *n*. Another instance where the concept could be used is the rule according to which the dental fricative *s* is substituted for the voiceless 'glottal' *visarjanīya* (*h*) when followed by a suffix beginning with an unvoiced 'noncentral' stop (i.e. *k* and *p*); Pāṇini 8.3.38: *so'padādau* '*s* is substituted (for *h*, when *k* and *p*) not occurring at the beginning of a word, follow'. Examples: *\*payah-pa* → *payaspa* 'milk-drinker', *\*payah-kāma* → *payaskāma* 'desire for milk'. This rule is optional; cf. Pāṇini 8.3.37, 49.

<sup>20</sup> Pāṇini 8.3.7: *naś chavy apraśān*.

follows a provisional formulation of 8.3.7, but precedes the final formulation of this rule with the help of *chav*. This of course does not yet explain the order *c t t* within the group. But that may merely reflect the order of the related sounds in the first three rows—an assumption which seems less artificial than that of anticipation of a certain row in a previous one.

The *anubandha* sounds at the end of the first three rows have by now been explained as well. The last remaining *anubandha* is *y*, which however is obviously required when reference is to be made to all stops (*jhay*) or all unvoiced stops (*khay*). Reference is made to *may* in sūtra 8.3.33.<sup>21</sup> If it were clear why here *ñ* is to be excluded, an explanation would have been given for the order of the first two sounds in the first row (*ñ m*) and perhaps, accordingly, in the second and third rows.

This completes the discussion of the order of nasals and stops and the insertion of indicatory sounds. The addition of the fricatives *ś ṣ s* followed by the *anubandha* *r* does not present a difficult problem. The order of the third, fourth, and fifth rows has been explained already, and Pāṇini wished in addition to combine the fricatives with the unaspirated voiceless stops. This is needed in sūtra 1.1.58 and in sūtras 8.4.54–56 on reduplication. A discussion of these rules will not be undertaken here, since it would necessitate the analysis of numerous other sūtras. Moreover the inclusion of fricatives in the reduplication rules appears to be purely theoretical. The order of fricatives within the group *ś ṣ s* is the same as the order of the three corresponding sounds in the five rows discussed before.

The final position of *hal*, lastly, can be easily explained; Boehlingk's explanation appears to be incorrect. He says (*Pāṇini's Grammatik* 1): '*ha* erscheint zweimal, um die Sigla (*pratyāhāra*) *hal* und *haś* bilden zu können'. This is not clear: for if the final sound *h* with its *anubandha* *l* were not there, *haś* would continue to be denoted by *haś*, while *hal* would be denoted by *har*. The right explanation should be that the double occurrence of *h* enables Pāṇini to combine the sound *h* on the one hand with the semivowels, nasals, and voiced stops (by means of the *pratyāhāra* *haś*), on the other hand with the fricatives (by means of the *pratyāhāra* *śal*). This reflects the phonetic fact that *h* is the only voiced fricative. The above analysis shows that it was impossible to solve this difficulty in any other way, as the rows could not have been ordered differently. At the same time the sound *h* could not have been placed BEFORE the fricatives (by means of which the introduction of the final *anubandha* *l* could have been avoided), for in such *pratyāhāra* combinations as *khar* the sound *h* could not have been included. This condensation *khar* is used in sūtra 8.4.55, where it is stated<sup>22</sup> that (in internal sandhi) an unaspirated voiceless stop is substituted for a stop when followed by *khar*. For example: *\*bhed-tum* → *bhettum* 'to split'. But *h* should not have been included in *khar*, for we have (in external sandhi), from *\*tad hi*, not *\*tat hi* but *tad dhi* 'because that'. Hence *h* has to follow the fricatives.

The analysis above may give a glimpse of the techniques used in Pāṇini's workshop. It also shows that a phonological analysis of the sounds of a language,

<sup>21</sup> Pāṇini 8.3.33: *maya uño vo vā* 'after *may* the consonant *v* may be substituted for the particle *u* (when a vowel follows)'. Example: *kim u ayam* or *kim v ayam* 'who is this?'.  
<sup>22</sup> Pāṇini 8.4.55: *khari ca* 'also when *khar* follows (*jaś* is substituted for *jhal*)'.

expressed in a mere list with subdivisions, can be fully determined by the grammatical structure of the language. Here I have attempted to establish some of the methods underlying the construction of Pāṇini's grammar by a purely internal analysis. The same material could be analysed by studying the commentaries from Patañjali onwards; but this would require an extensive analysis, as is evident from Thieme's study of the treatment of *vārttika* 5 on Pāṇini 1.1.9 in the commentaries.<sup>23</sup>

<sup>23</sup> P. Thieme, *Bhāṣya zu vārttika zu Pāṇini 1.1.9 und seine einheimischen Erklärer, Nachrichten der Gesellschaft der Wissenschaften zu Göttingen, philologisch-historische Klasse NF* 1.5.171-216 (Berlin, 1935).

## CONTEXT-SENSITIVE RULES IN PĀṆINI

Many rules used in grammatical description apply only within certain contexts. For example, the rule that *i* is replaced by *y* applies in Sanskrit only when a vowel follows. A simple and natural way of writing this would be as:

$$[i \rightarrow y] \text{ vowel}^1,$$

where the context is marked by brackets, or as:

$$i \rightarrow y \quad \text{in the context:} \quad -\text{vowel}.$$

In general, when a rule states that *b* is replaced by *c* in the context *a-d*, so that in fact we have:

$$abd \rightarrow acd,$$

we shall write:

$$a[b \rightarrow c]d \quad (1)$$

which expresses the same as:

$$b \rightarrow c \quad \text{in the context} \quad a-d.$$

In what follows it will be assumed that *b* is always a single element. If either *a* or *d* or both are nonnull, such rules are called *context-sensitive*. If both *a* and *d* are null, the rules are called *context-free*. A constituent-structure grammar containing only context-sensitive rules is called a context-sensitive grammar; a constituent-structure grammar containing only context-free rules is called context-free.

Chomsky, from whom this terminology is taken<sup>2</sup>, has shown that context-sensitive grammars are essentially richer than context-free grammars. But even context-sensitive grammars are most likely unable to account for all features of natural languages. In the following we shall be concerned with some rules of Sanskrit grammar as described by Pāṇini, which are context-sensitive. It is neither suggested that such rules suffice for the description of

<sup>1</sup> Elements of the metalanguage will be written in Roman in formulas but sometimes in italics in the text, whereas elements of the object-language will always be written in italics. This paper will also be published in *Festschrift M. B. Emeneau* (Ed. B. Krishnamurti).

<sup>2</sup> See e.g., N. Chomsky-G. A. Miller, 'Introduction to the formal analysis of natural languages' in: *Handbook of Mathematical Psychology*, New York 1963, II, 294.

Sanskrit grammar, nor that Pāṇini thought so. On the other hand, context-free rules (which have been shown to suffice for example for the grammatical description of such programming languages as ALGOL) were clearly insufficient for Pāṇini's purpose. Pāṇini's use of context-sensitive rules and his theoretical preoccupation with their general metalinguistic description, may inspire modern theorists to study the formal properties of context-sensitive grammars in greater detail than has been done so far. Also in automata theory, little appears to be known about the corresponding automata, namely nondeterministic linear-bounded automata. Yet context-sensitive grammars are needed for the description of perhaps most features of natural languages.

Pāṇini's theoretical preoccupation with the general metalinguistic description of context-sensitive rules is found in his grammar mainly in three metarules, viz., rules which state how other rules have to be interpreted and used.<sup>3</sup> In these rules it is presupposed that the main function of a grammatical rule is to express a substitution. The substitute Pāṇini prescribes is always the subject of the rule and therefore occurs in the nominative case. It corresponds to what is denoted by *c* in (1). However, as we shall see, the substituendum is not always a single element, in which case the rule is neither context-sensitive, nor context-free. The three metarules referred to are the following:

- 1.1.49 *ṣaṣṭhī sthāneyogā*  
 1.1.66 *tasmīn iti nirdiṣṭe pūrvasya*  
 1.1.67 *tasmād ity uttarasya.*

The first rule states: "the sixth (i.e., genitive case ending) is used for that in the place of which (something is substituted)", viz., for the substituendum denoted by *b* in (1). The second rule states: "when something is referred to with the locative ending, (the substitute appears) in the place of the preceding element", viz., the preceding element of what is denoted by *d* in (1). The third rule states: "when something is referred to with the ablative ending, (the substitute appears) in the place of the following element", viz., the following element of what is denoted by *a* in (1).

Pāṇini therefore refers to context-sensitive rules by means of expressions of the form:

$$\begin{array}{l} a + \text{ablative ending, } b + \text{genitive ending,} \\ c + \text{nominative ending, } d + \text{locative ending.} \end{array} \quad (2)$$

Some paṇḍits expand this as follows: "after *a*, in the place of *b* (*b* + genitive ending *sthāne*), *c*, when *d* follows (*d* + locative ending *pare*)". Pāṇini's ex-

<sup>3</sup> Cf. J. F. Staal, 'The theory of definition in Indian logic', *Journal of the American Oriental Society* 81 (1961) 123-4. See also B. Shefts, *Grammatical Method in Pāṇini*, New Haven 1961, 4, note 8.

pression (2) corresponds to (1), but it should be noted that the order of elements in (2) is not fixed since their function is fully circumscribed by the case endings. In (1), on the other hand, the order is fixed, since the function of the elements depends only on their place in the formula. The difference between (1) and (2) reflects the fact that in expressions such as (2) which occur in inflected languages word order is immaterial, whereas the order of elements in (1) is as important as word order is in modern uninflected languages in general.<sup>4</sup>

Before exemplifying Pāṇini's use of context-sensitive rules some other methodological devices employed in his grammar will have to be mentioned. Most of these arise from the desire to obtain conciseness and simplicity (of a particular kind). The first and most well-known of these devices is *anuvṛtti* "recurrence". In accordance with this principle the repetition of a recurrent element or string should be avoided. This necessitates that the rules be ordered in a fixed manner.<sup>5</sup> Here the recurrent element will be supplied in parentheses whenever necessary. For example, instead of the rules:

$$\begin{aligned} a_1 [b_1 \rightarrow c_1] d_1 \\ a_1 [b_2 \rightarrow c_2] d_2 \end{aligned}$$

Pāṇini writes:

$$\begin{aligned} a_1 [b_1 \rightarrow c_1] d_1 \\ [b_2 \rightarrow c_2] d_2 \end{aligned}$$

The second rule has *anuvṛtti* of  $a_1$  and may be expanded, whenever necessary, as:

$$(a_1) [b_2 \rightarrow c_2] d_2.$$

The use of *anuvṛtti* may be extended over numerous rules which need not all occur consecutively. Ambiguity is therefore not always excluded, whereas further extension of this practice leads to certain anomalies which it is not necessary to refer to here.<sup>6</sup>

Another useful simplification is arrived at by listing elements and referring to these in their respective order in the list. For example, instead of the set of rules:

<sup>4</sup> Optional word order is not easily described by grammars based upon concatenation: cf. J. F. Staal, 'Reification, quotation and nominalization', *Festschrift I. M. Bocheński* (forthcoming).

<sup>5</sup> Outside the Indian tradition, the first to emphasize this appears to be Morris Halle (e.g., 'On the role of simplicity in linguistic descriptions', *Structure of Language and its Mathematical Aspects*, American Mathematical Society, Providence 1961, 89-94; 'Phonology in Generative Grammar', *Word* 18 (1962) 54-72.

<sup>6</sup> See L. Renou, *La Durghaṭavṛtti de Śaraṇadeva I I*, Paris 1940, 85-6 and *Terminologie grammaticale du sanskrit*, Paris 1957, s.v. *maṇḍūkagatī*, *siṃhāvalokita*.

$$\begin{array}{c} a_1 [b_1 \rightarrow c_1] d_1 \\ \cdot \\ \cdot \\ a_1 [b_n \rightarrow c_n] d_1 \end{array}$$

Pāṇini formulates a single rule as follows:

$$a_1 [b_1 \dots b_n \rightarrow c_1 \dots c_n] d_1.$$

This will be written as:

$$a_1 \left[ \begin{array}{c} \left[ \begin{array}{c} b_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{array} \right] \rightarrow \left[ \begin{array}{c} c_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ c_n \end{array} \right] \end{array} \right] d_1.$$

Lastly Pāṇini uses many artificial metalinguistic expressions to refer to particular elements of the object-language. The condensations (*pratyāhāra*) described elsewhere<sup>7</sup>, which are used in the following examples are:

- 'a' for 'a, ā'; 'i' for 'i, ī'; etc.
- 'at' for '(short) a'
- 'ac' for 'all vowels'
- 'ik' for 'i, u, r, l'
- 'eñ' for 'e, o'
- 'ec' for 'e, o, ai, au'
- 'yaṅ' for 'ya, va, ra, la'

$$\begin{array}{l} \text{'guṇa'} \text{ for } \left\{ \begin{array}{c} a \\ e \\ o \\ ar \\ al \end{array} \right\} \text{ with respect to } \left\{ \begin{array}{c} i \\ u \\ r \\ l \end{array} \right\} \\ \text{'vṛddhi'} \text{ for } \left\{ \begin{array}{c} ā \\ ai \\ au \\ ār \\ āl \end{array} \right\} \text{ with respect to } \left\{ \begin{array}{c} a \\ i \\ u \\ r \\ l \end{array} \right\} \end{array}$$

Additional artificial metalinguistic elements, where indicatory letters perform various functions<sup>8</sup>, are the following:

<sup>7</sup> J. F. Staal, 'A method of linguistic description: the order of consonants according to Pāṇini', *Language* 38 (1962) 1-10.

<sup>8</sup> Cf. *Language* 39 (1963) 483-8.

- 'ruk' for a certain infix 'r'
- 'pit' for an element marked 'p'
- 'kṛt' for a certain class of suffixes, including kvip and lyap
- 'āt' for the verbal augment 'ā'.

Lastly use is made of the following metalinguistic expressions, which gradually merge into the vocabulary of ordinary natural Sanskrit:

- hrasva* 'short'
- dīrgha* 'long'
- saṃhitā* 'continuous (speech)'
- pratyaya* 'suffix'
- pūrva* 'preceding'
- para* 'following'
- pada* 'word'
- anta* 'end' (hence *padānta* 'end of a word')
- savarṇa* 'homorganic'<sup>9</sup>
- eka* 'one'
- ca* 'and'.

The context-sensitive rules which will now be analysed have been selected from the first *pāda* of the sixth *adhyāya* of Pāṇini's grammar. They deal with the more well-known features of vowel *sandhi*. The rules are the following:

- 6.1.71 *hrasvasya piti kṛti tuk*
- 6.1.72 *saṃhitāyām*
- 6.1.73 *che ca*
- 6.1.77 *iko yaṅ aci*
- 6.1.78 *eco'yavāyāvaḥ*
- 6.1.79 *vānto yi pratyaye*
- 6.1.84 *ekaḥ pūrvaparayoḥ*
- 6.1.85 *antādivac ca*
- 6.1.87 *ād guṇaḥ*
- 6.1.88 *vṛddhir eci*
- 6.1.90 *āṭas ca*
- 6.1.94 *eṅi pararūpam*
- 6.1.96 *usy apadāntāt*
- 6.1.97 *ato guṇe*
- 6.1.101 *akaḥ savarṇe dīrghaḥ*
- 6.1.107 *ami pūrvāḥ*
- 6.1.109 *eṅaḥ padāntād ati*

<sup>9</sup> See *Language* 38 (1962) 4 note 8.

Before analysing these rules some remarks must be made. From a certain rule onwards Pāṇini considers rules of the form:

$$[p_1 + p_2 \rightarrow e]. \quad (3)$$

The rule 6.1.84 which introduces these expressions is actually a metarule. The next metarule, 6.1.85, states that the new substitute is both the final element of the preceding word and the initial element of the following word, i.e.:

$$\alpha[p_1 + p_2 \rightarrow e]\beta \text{ generates } \alpha e \beta.$$

Rules of this kind are neither context-sensitive, nor context-free, since the substituendum is not a single element. However, if a null-element  $\varphi$  is introduced, (3), whether or not restricted by a context  $\alpha-\beta$ , can be replaced by two context-sensitive rules and one context-free rule either as:

$$\begin{aligned} & [p_1 \rightarrow e^*] p_2 \\ & e^* [p_2 \rightarrow \varphi] \\ & e^* \rightarrow e, \end{aligned}$$

or as:

$$\begin{aligned} & p_1 [p_2 \rightarrow e^*] \\ & [p_1 \rightarrow \varphi] e^* \\ & e^* \rightarrow e. \end{aligned}$$

The freshly introduced symbol  $e^*$  is needed to prevent the application of  $e[p_2 \rightarrow \varphi]$  to cases where  $ep_2$  does not result from the application of  $[p_1 \rightarrow e] p_2$  (respectively, to prevent the application of  $[p_1 \rightarrow \varphi] e$  to cases where  $p_1 e$  does not result from the application of  $p_1 [p_2 \rightarrow e]$ ). In general, any rule of the form:

$$a_1 \dots a_m \rightarrow b_1 \dots b_n \quad (4)$$

(the symbols + expressing concatenation have been omitted), whether or not restricted by a context  $\alpha-\beta$ , can be replaced by an ordered list of  $m$  context-sensitive rules:

$$\begin{aligned} & [a_1 \rightarrow b^*] a_2 \dots a_m \\ & b^* [a_i \rightarrow \varphi] \quad \text{for } i = 2, \dots, m \end{aligned}$$

followed by one context-free rule:

$$b^* \rightarrow b_1 \dots b_n.$$

The freshly introduced symbol  $b^*$  is needed to prevent the application of  $b_1 \dots b_n [a_i \rightarrow \varphi]$  to cases where  $b_1 \dots b_n a_i$  does not result from the application of  $[a_1 \rightarrow b_1 \dots b_n] a_2 \dots a_m$  or earlier applications of  $b_1 \dots b_n [a_i \rightarrow \varphi]$ .

Also (4) can be replaced by other rules in more than one way. Moreover, the deletions introduced by  $\varphi$  prevent the association of a unique P-marker

with the string  $b_1 \dots b_n$ . A similar difficulty presents itself when the null-element is used to bring into line rules of the form:

$$a[c]d,$$

where a new element is introduced (e.g., 6.1.71), with context-sensitive rules by writing:

$$a[\varphi \rightarrow c]d. \quad (5)$$

Because of these ambiguities rules such as (3)–(5) are generally excluded from a context-sensitive grammar.

#### ANALYSIS

6.1.71 (root ending with) short (vowel) [ruk] pit kṛt

6.1.72 in continuous speech (this rule continues through *anuvṛtti* in the following rules)

6.1.73 (root ending with short vowel) [ruk] *ch*

$$6.1.77 \quad \left[ \begin{array}{c} \left\{ \begin{array}{c} i \\ u \\ r \\ l \end{array} \right\} \rightarrow \left\{ \begin{array}{c} y \\ v \\ r \\ l \end{array} \right\} \end{array} \right] \text{vowel}$$

$$6.1.78 \quad \left[ \begin{array}{c} \left\{ \begin{array}{c} e \\ o \\ ai \\ au \end{array} \right\} \rightarrow \left\{ \begin{array}{c} ay \\ av \\ \bar{a}y \\ \bar{a}v \end{array} \right\} \end{array} \right] \text{(vowel)}$$

$$6.1.79 \quad \left[ \left( \left\{ \begin{array}{c} o \\ au \end{array} \right\} \rightarrow \left\{ \begin{array}{c} a \\ \bar{a} \end{array} \right\} \right) v \right] \text{suffix (beginning with) } y$$

6.1.84 [preceding + following → one] (this rule continues through *anuvṛtti* in the following rules)

6.1.85 ( $\alpha[p_1 + p_2 \rightarrow e]\beta$ ) generates  $\alpha e \beta$

$$6.1.87 \quad \left[ a + \left[ \left[ \begin{array}{c} a \\ i \\ u \\ r \\ l \end{array} \right] \right] \rightarrow \left[ \begin{array}{c} a \\ e \\ o \\ ar \\ al \end{array} \right] \right]$$

$$6.1.88 \quad \left[ (a) + \left\{ \begin{array}{c} e \\ o \\ ai \\ au \end{array} \right\} \rightarrow \left\{ \begin{array}{c} ai \\ au \\ ai \\ au \end{array} \right\} \right]$$

- 6.1.90  $\left[ \tilde{a}t + \left( \text{vowel} \rightarrow \left\{ \begin{array}{l} ai \\ au \end{array} \right\} \right) \right]$
- 6.1.94  $\left[ (\text{preverb ending } a) + \left\{ \begin{array}{l} e \\ o \end{array} \right\} \rightarrow \left\{ \begin{array}{l} e \\ o \end{array} \right\} \right]$
- 6.1.96  $[\text{non-final } (-a-) + us (\rightarrow us)]$
- 6.1.97  $\left[ (\text{non-final } a) + \left\{ \begin{array}{l} a \\ e \\ o \\ ar \\ al \end{array} \right\} \rightarrow \left[ \left[ \begin{array}{l} a \\ e \\ o \\ ar \\ al \end{array} \right] \right] \right]$
- 6.1.101  $\left[ \left[ \begin{array}{l} a \\ i \\ u \\ r \\ l \end{array} \right] + \left[ \begin{array}{l} a \\ i \\ u \\ r \\ l \end{array} \right] \rightarrow \left[ \begin{array}{l} \bar{a} \\ \bar{i} \\ \bar{u} \\ \bar{r} \\ \bar{l} \end{array} \right] \right]$
- 6.1.107  $\left[ \left[ \left[ \begin{array}{l} a \\ i \\ u \\ r \\ l \end{array} \right] \right] + a \rightarrow \left[ \begin{array}{l} a \\ i \\ u \\ r \\ l \end{array} \right] m \right]$
- 6.1.109  $\left[ \text{final} \left\{ \begin{array}{l} e \\ o \end{array} \right\} + a \rightarrow \left( \left\{ \begin{array}{l} e \\ o \end{array} \right\} \right) \right]$

## COMMENTS AND EXAMPLES

- 6.1.71 The elements within parentheses are recurrent elements from earlier rules. Since *lyap* is a *kṛt*-suffix realized as *ya* and marked with a *p*, this rule is exemplified by: *prakṛ-ya* → *prakṛtya*.
- 6.1.72 This rule remains valid until 6.1.157.
- 6.1.73 Example: *ga-chati* → \**gatchati*. This is replaced by *gacchati* in accordance with 8.4.40.
- 6.1.77 Examples: *dadhi atra* → *dadhy atra*; *madhu atra* → *madhv atra*.
- 6.1.78 Examples: *ce-ana* → *cayana*; *lau-ana* → *lāvana*.
- 6.1.79 This rule refers to *av* and *āv* in the previous rule by singling out the common element *v*. Example: *nau-ya* → *nāvya*.
- 6.1.84–5 Discussed before. These rules remain valid until 6.1.111.
- 6.1.87 This rule is formulated for all vowels (*ac*) though in fact it does not hold for *e*, *o*, *ai*, *au* (*ec*). However for these vowels the next

rule is especially formulated and hence they are excluded from the present rule by an application of the well-known metarule *vipratishedhe param* ‘in case of contradiction, the following (rule prevails)’.<sup>10</sup> By means of this Pāṇini is able to use the *anuvṛtti* of *ac* from 6.1.77. The alternative would have been to mention *ak* (for *a, i, u, ṛ, ḷ*). Note also that *a, i, ...* stand for the long and short vowels *a, ā; i, ī; ...*. Examples: *tava idam* → *tavedam; tava udakam* → *tavodakam*.

- 6.1.88 Examples: *brahma eti* → *brahmaiti; brahma odana* → *brahmaudana*.
- 6.1.90 There is *anuvṛtti* of *ac* from 6.1.77 and of *vṛddhi* from 6.1.88. Example: *ā-ikṣata* → *aikṣata*.
- 6.1.94 There is *anuvṛtti* of *a* (denoting *a* and *ā*) from 6.1.87 and of ‘preverb’ from 6.1.91. The expression ‘following’ indicates that [ $p_1 + p_2 \rightarrow p_2$ ]. This can be replaced by a context-sensitive rule by means of: [ $p_1 \rightarrow \varphi$ ]  $p_2$ . Example: *pra-elayati* → *prelayati*.
- 6.1.96 Example: *bhindyā-us* → *bhindyus*.
- 6.1.97 Here the short *a* is expressly mentioned (referred to by *at* in accordance with 1.1.70) and there is *anuvṛtti* of ‘non-final’ from 6.1.96 and of ‘following’ from 6.1.94. Examples: *paca-anti* → *pacanti; paca-e* → *pace*.
- 6.1.101 Note again that *a, i, ...* stand for the long and short vowels. Examples: *daṇḍa agram* → *daṇḍāgram; adhi-itya* → *adhītya*.
- 6.1.107 The expression ‘preceding’ indicates that [ $p_1 + p_2 \rightarrow p_1$ ]. This can be replaced by a context-sensitive rule by means of:  $p_1$  [ $p_2 \rightarrow \varphi$ ]. Examples: *vṛkṣa-am* → *vṛkṣam; agni-am* → *agnim; vāyu-am* → *vāyum*. Note that this rule would be superfluous should *am* be replaced by *m*. However this would lead to difficulties in other parts of the grammar.
- 6.1.109 There is *anuvṛtti* of ‘preceding’ from the preceding rule. The short *a* is again expressly mentioned. Example: *agne atra* → *agne’tra* (the *avagraha* ’ is introduced by another rule).

The structure of Pāṇini’s grammar is such that it is difficult to isolate a set of rules without referring to many other rules, which in turn refer to other rules, etc. The rules dealing with vowel *sandhi*, selected for the above analysis, can be isolated from the system only at the expense of introducing a certain amount of artificiality. Apparent complexities and subtleties disappear however, when it is realized that particular methods and abbrevi-

<sup>10</sup> See J. F. Staal, ‘Negation and the law of contradiction: a comparative study’, *Bulletin of the School of Oriental and African Studies* 25 (1962) 52–71.

ations are frequently and consistently used throughout the system. The more rules analysed, the fewer comments required. A complete analysis of all context-sensitive rules in Pāṇini's grammar with the help of the above formalizations would certainly be a lengthy affair, but it would also present a more unified and consistent picture of what Bloomfield described as 'one of the greatest monuments of human intelligence'.

*University of Amsterdam, Amsterdam*

*Pāṇini Tested by Fowler's Automaton*

In "How ordered are Pāṇini's rules?"<sup>1</sup> Professor Murray Fowler raises a valid question: can the consistency of the *Aṣṭādhyāyī* (abbreviated as *A*) be tested by an automaton? Before answering such a question, one should specify what kind of consistency and what kind of automaton one has in mind. I shall begin by considering these preliminaries (1 and 2) and wind up with a brief discussion of the order of Pāṇini's rules (3), showing thereby that Fowler's "cursory examination" (45) is not only inadequate but also misleading.

**1.1** Consistency is generally defined as a property of a system of rules. Such a system is called consistent when it is impossible to derive with the help of the rules two results which are, in some sense, contradictory or incompatible. *A* is consistent in this sense on account of the rule 1.4.2, which says that in case of contradiction between two rules, the following rule prevails. This requires the rules of *A* to be ordered in a special manner.<sup>2</sup>

**1.2** For Fowler, consistency requires *well-orderedness* of linguistic rules, defined as "an arrangement such that the relation of pre-supposition is maintained in a regular progression" (45). More explicitly, a sequence of rules  $R_1, \dots, R_n$  is called well-ordered if and only if, for no rule  $R_i$  there is a later rule  $R_j$  presupposed by  $R_i$ ; or, if and only if:

$$(R_i) \rightarrow (ER_j)[(1 \leq i < j \leq n) \wedge (R_i \text{ presupposes } R_j)].$$

Fowler rightly says that the whole of *A* is not well-ordered in this sense; this follows from the examples he discusses and it does also follow from many others. He is also right where he maintains that attempts at well-ordering the rules (such as Bhaṭṭoji Dikṣita's in the *Siddhānta Kaumudī*) can only be successful if the phrasing of the rules is altered accordingly; in fact it would have to be altered substantially. Fowler concludes that Bhaṭṭoji's order, "with the needs of an automaton in mind, seems to be much better than the original". This might be true for a particular kind of automaton (see below) provided the phrasing was altered accordingly; however, this has never been done consistently, not even by Bhaṭṭoji

Dikṣita himself, so that, against Fowler, we must agree with Boehlingk ("Spätere Grammatiken, die Pāṇini's Sūtra aus der unverrückbaren Ordnung gebracht haben, um alles dem Stoffe nach Zusammengehörige aneinander zu reihen, sind ohne ausführliche Commentare, die stets auf etwas weit Vorangegangenes oder Folgendes Rücksicht nehmen müssen, ganz unverständlich und als Missgriffe zu betrachten": quoted n.5) and even with Colebrooke ("But the sūtras of Pāṇini, thus detached from their context are wholly unintelligible . . .").<sup>3</sup> In fact, the *Siddhānta Kaumudī* is a commentary upon and an introduction to *A*, not an alternative to it.

It may be noted that the treatment given to Pāṇini's grammar by Bhaṭṭoji Dikṣita is similar to that accorded to Euclid's *Elements* by Petrus Ramus in the XVIth century: Ramus rewrote the *Elements* by ordering the theorems according to topic and omitting the proofs. The logical significance of geometry, i.e., its deductive character, is thereby lost. Bhaṭṭoji Dikṣita and Fowler similarly sacrifice the logical structure of *A*.

**2.** Fowler's automaton seems to make sense only if it is interpreted as a finite automaton; in that case, however, it makes no sense in the present context, as we shall see. A finite automaton may be considered to be on a par with a finite state grammar (FSG), which generates a finite state language (FSL). Referring to tests by such an automaton, Fowler states: "Excellent grammars are now being written with this test in mind" (44). In the explanatory footnote he refers to the grammar of C. G. Zull, "A Formal System for generating French Verb Paradigms" (n.2). But a formal system for generating paradigms can at most be part of a grammar, it can never be a grammar in the usual sense: for a grammar is meant to generate sentences.

Now it is well known that the set of English sentences is not a FSL; i.e., it cannot be generated by a FSG.<sup>4</sup> It requires some skill but little original-

<sup>3</sup> Preface to *Laghukaumudī*, ed. J. R. Ballantyne, Benares 1891, ii.

<sup>4</sup> See e.g. N. Chomsky, "Three models for the description of language", reprinted in: *Readings in mathematical psychology*, II, ed. Luce, Bush & Galanter, New York & London 1965, 105-124.

<sup>1</sup> JAOS 85 (1965) 44-7.

<sup>2</sup> See J. F. Staal, "Negation and the law of contradiction: a comparative study", BSOAS 25 (1962) 53-6.

ity to show that Sanskrit is not a FSL either. Hence, if *A* could be shown to be a FSG, it would follow that *A* would be incapable of describing Sanskrit. Fowler, who indeed suggests that *A* would probably generate "a highly complex" FSL,<sup>5</sup> does not consider adequacy ("conformity with an external object"); but for Pāṇini, as for all grammarians, this is a major concern (formulated in Sanskrit as the requirement of providing rules for all that is *loka* "common usage"). In other words, Fowler's proof that *A* is not a FSG is in no way surprising and merely saves *A* from being seriously inadequate.

This result can also be established directly. Since it has been shown that *A* contains many context-sensitive rules,<sup>6</sup> it follows, not only that *A* is not a FSG, but that it is not a context-free phrase-structure grammar either. Further, it can be shown that *A* is also stronger than a context-sensitive phrase-structure grammar, since it contains rules which cannot be written as context-sensitive rules.<sup>7</sup> In short, *A* contains rules which are strong enough, in principle, to generate Sanskrit sentences; if *A* were to contain only rules of the kind that can be dealt with by Fowler's automaton, it would be inadequate for describing Sanskrit.

**3.1** A slightly more careful perusal of *A* will show that the order of its rules is in many ways crucial; this has been noted by the Indian commentators, from Patañjali onwards, and by many Western scholars. Fowler is aware of the fact that "the great problem of *anurṛtti* remains" (47). In fact, the order of the rules of *A* should be explained in the first place by taking the economy criterion (*lāghava*) and *anurṛtti* into consideration. Any suggestions for altering the order upset the organization of the grammar in ways which are not always immediately apparent. Discussions of this sort, recently by Shefts,<sup>8</sup> for example, have made it quite clear that other features may also influence order. That it is simply a desire to treat certain

grammatical topics together which accounts for some features of order, is especially apparent in the case of *anurṛtti* of *adhikāras* "chapter headings". This has been studied, for example, by Renou.<sup>9</sup>

**3.2** The earlier quoted meta-rule 1.4.2<sup>2</sup> introduces another kind of order into *A*: not connected with the phrasing of rules, but with their actual effect. In these cases altering the order can never be neutralized by re-formulation. If the structure of the grammar is to be saved from disruption, there are two alternatives: either the relative order of all pairs of rules to which 1.4.2 is applied must remain the same; or it must be reversed in every case, whilst "following" in 1.4.2 is being replaced by "preceding".

**3.3** So far the effect of *A* on a given input can be described as follows: *any* rule of *A* applies to *any* part of the input, provided the conditions for its application (often formulated as context restrictions) are fulfilled; if, moreover, two rules should provide incompatible results, the latter prevails. But this description holds good (approximately: see 3.4) for (the largest) part of the grammar only: the first seven *adhyāyas* and the first *pāda* of the eighth *adhyāya* (1.1-8.1). For the last three *pādas* of the eighth *adhyāya* (8.2-4) the situation is different: here the rules apply to a given input one after the other in the given order. This part, which is called the *tripādī* and which contains 295 rules (i.e., less than 8% of the total number of rules) begins with the meta-rule and *adhikāra* 8.2.1: *pūrvatrāsiddham* "(From now on every rule is regarded as) not having taken effect with reference to preceding ones". This means that in 8.2-4 no rule applies until earlier rules (either, unordered, in 1.1-8.1 or, ordered, in 8.2.4 itself) have been applied. This rule and its effect have been the subject of a monograph by Buiskool.<sup>10</sup>

**3.4** However, *within* 1.1-8.1 also arbitrary applicability of rules is restricted by another application of the principle of *asiddhatva* laid down by meta-rule 50: *asiddham bahiraṅgam antaraṅgam* "that which is *bahiraṅga* is (regarded as) not having taken effect when that which is *antaraṅga* is

<sup>5</sup> 44 n.4, where it is also stated that *A* "is not very powerfully generative", "because of the absolute necessity of lists to complete it". Is there any grammar that can operate adequately without a lexicon? There is also some confusion about the "evaluation function" mentioned in n.1 with a reference to Lg 39 (1963) 599 n.8, itself a reference to the relevant source.

<sup>6</sup> J. F. Staal, "Context-sensitive rules in Pāṇini", *Foundations of Language* 1 (1965) 63-72.

<sup>7</sup> *Ibid.* 68-9.

<sup>8</sup> B. Shefts, *Grammatical method in Pāṇini: His treatment of Sanskrit present stems*, New Haven 1961; cf. Lg 39 (1963) 483-8.

<sup>9</sup> L. Renou, *Études védiques et Pāṇinéennes*, I, Paris 1955, 124-6.

<sup>10</sup> H. E. Buiskool, *Pūrvatrāsiddham: Analytisch onderzoek aangaande het systeem der Tripādī van Pāṇini's Aṣṭādhyāyī*, Amsterdam 1934; the same, *The Tripādī, being an abridged English recast of Pūrvatrāsiddham (An analytical-synthetic inquiry into the system of the last three chapters of Pāṇini's Aṣṭādhyāyī)*, Leiden 1939.

to take effect)". This means that among the rules of *A* there are many ordered pairs. In each pair, the rule that is applied first is called *antaraṅga* "inner cause of the operation"; the rule that is applied next is called *bahiraṅga* "outer cause of the operation".

An example is provided by 6.1.77 *iko yaṅ aci* which is *antaraṅga* with respect to 7.3.86 *pu-gantalaghūpadhasya ca*. The first rule requires, among other things, that *i* should be replaced by *y* when followed by a vowel. When applied to *si+ū+na* this would result in *sy+ū+na* (1). The second rule requires, among other things, that *i* and *u* should undergo *guṇa* when certain conditions are fulfilled. When applied to *si+ū+na* this would result in *se+ū+na* (2). Now if (1) is *antaraṅga* with respect to (2), we obtain first *sy+ū+na* and next *sy+o+na*; but if (2) is *antaraṅga* with respect to (1), we obtain *se+ū+na* and there is no scope for (1) to apply. Since, in fact, *syona* is the correct result, (1) is declared *antaraṅga* with respect to (2). This principle has been studied by Kielhorn,<sup>11</sup> Boudon<sup>12</sup> and Renou.<sup>13</sup> The transformational cycle

of phonemic rules, whereby innermost brackets are first erased,<sup>14</sup> is to some extent reminiscent of this principle.

4. These are some of the features of the types of order underlying Pāṇini's grammar *A*. These and other features can be fruitfully studied from many points of view. If we distinguish three stages in the study of *A* as a generative device, it may be held that the first stage, that of analysis, has been dealt with successfully by Indian commentators since Patañjali and by Western scholars of the last two centuries; however, this task is by no means completed. The second stage, that of formalization, has perhaps just begun to receive attention; it depends on analysis, but is not determined by it. The third stage, that of automation, itself depending on formalization, is not determined by formalization; it may not even be effectively realizable. Despite Fowler's laudable effort, the fact remains that this has been hardly touched upon.

J. F. STAAL

UNIVERSITY OF AMSTERDAM

<sup>11</sup> Nāgojibhaṭṭa, *Paribhāṣendusekhara*, ed. F. Kielhorn, II, Translation and Notes, Bombay 1874 (ed.<sup>2</sup> K. V. Abhyankar, Poona 1960), 221 sq.

<sup>12</sup> P. Boudon, "Une application du raisonnement par l'absurde dans l'interprétation de Pāṇini" *JA* 230 (1938) 65-121: 72-8.

<sup>13</sup> L. Renou, *Études védiques et Pāṇinéennes*, II,

Paris 1956, 140 sq.; cf. the same. *La Durghatavṛtti de Śaraṇadeva*, Introduction, Paris 1940, 118-9; *Terminologie grammaticale du Sanskrit*, Paris 1957, s.v. *antaraṅga*.

<sup>14</sup> See e.g. N. Chomsky & G. A. Miller, "Introduction to the formal analysis of natural languages", *Handbook of mathematical psychology*, II, ed. Luce, Bush & Galanter, New York & London, 1963, 313-8.

SYNTACTIC AND SEMANTIC RELATIONS  
IN PĀṆINI

P. KIPARSKY AND J. F. STAAL

I

It is gradually becoming known that Pāṇini's grammar is not confined to the analysis of words (*śabdānuśāsana*) but also provides a theory for the analysis of sentences. Pāṇini's main contribution to syntax is the theory of the *kāraka* relations. These relations comprise such notions as have in the West been traditionally called 'logical subject-of', 'logical object-of', etc., and correspond to the 'underlying' or 'deep' syntactic relations of transformational grammar. In some sentences these underlying relations coincide with the corresponding surface relations, viz., 'surface subject-of', 'surface object-of', etc. But these constitute a special case. In general, the underlying relations are different from the relations defined in the surface structure.

An example may make clear what is meant here. 'Mary' is the surface object in the sentence:

John sees Mary. (1)

Let us assume that the underlying deep structure of (1) is also the deep structure of its passive, i.e.:

Mary is seen by John. (2)

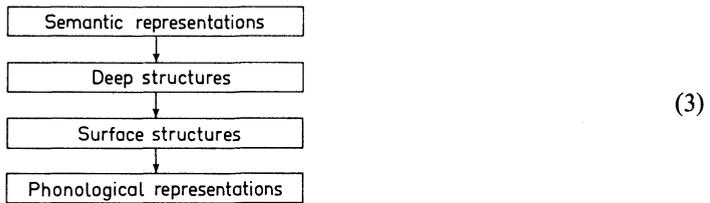
Now 'Mary' is the surface subject in (2). But 'Mary' is at the same time the underlying object in (2); this merely means that 'Mary' is the object in the underlying deep structure of (2).

These examples illustrate that one underlying relation in deep structure (e.g., 'underlying subject-of') may be expressed by various grammatical relations in surface structure ('surface subject-of', 'surface object-of'). They illustrate at the same time that two different underlying relations in deep structure (e.g., 'underlying subject-of', 'underlying object-of') may be expressed by one grammatical relation in surface structure ('surface subject-of' in (1) and (2), respectively). Had there actually been a one-one correspondence between underlying and surface relations, one of these two notions would have been redundant.

This paper will seek to establish that Pāṇini's *kāraka* relations are underlying relations of exactly this kind. This has been argued elsewhere (Staal, 1967, 36-45), but the earlier treatment was given in a particular context, was

incomplete (e.g., Pāṇini’s analysis of the locative absolute and of the causative was omitted), did not show what crucial role is played by *sūtra* 2.3.1, and did not consider the semantic level. Moreover, apart from the literature quoted there, another recent article (Cardona, 1967) has treated the *kāraka* theory afresh and has brought the problems it raises tantalizingly close to what we believe to be their solution.

A major source of misunderstanding in previous interpretations of Pāṇini’s syntactic theory has been their tendency to confuse *kārakas* with semantic relations on the one hand, or with cases on the other hand, and sometimes even with both at once.<sup>1</sup> It is essential that these three kinds of entities be clearly distinguished, since they belong for Pāṇini on three different levels of linguistic description. Pāṇini’s grammar is a system of rules for converting semantic representations of sentences into phonetic representations, via two intermediate levels which may be respectively compared with the levels of deep (underlying) structure and surface structure in a generative grammar.



In the specific domain which is under discussion here, Pāṇini’s procedure was as follows. He interrelated four levels: (1) The level of semantic relations, characterized in terms of concepts such as ‘agent’, ‘goal’, ‘location’; (2) the level of *kāraka* relations, corresponding to deep structure relations such as ‘(underlying) subject’, ‘(underlying) object’, ‘place adverbial’; (3) the level of surface structure, represented in terms of morphological categories, e.g. by nominal cases, derivational affixes, prepositions, or verbal voices; (4) the level of phonology, e.g., case endings which express cases, or verbal affixes which express voices. The derivation of a sentence starts at level (1) from its meaning. A set of rules then specifies the *kāraka* relations. Pāṇini next introduces the cases or other morphological categories which correspond to these *kāraka* relations. Lastly he introduces the actual phonological forms which express these morphological categories, thereby reaching level (4).

For example, the sentence *akṣair divyati* ‘he plays (with) dice’ has as part of its semantic representation the information that the dice stand to the action

<sup>1</sup> For example, Böhthlingk’s identification of the *kārakas* as “der Begriff des Akkusativs”, “der Begriff des Dativs”, and so on, makes them simultaneously morphological and semantic entities. In fact they are neither.

of playing in the instrument relation (*sādhakatama*, literally ‘the most effective means’). This relation can be expressed on the next lower level (2) by several *kāraṅkas* or deep structure relations. The chief of these, given in rule 1.4.42, is termed *karāṇa*. *Karāṇa* can in turn have several realizations on level (3), the chief one being the Instrumental case (2.3.18). On the next level there are again several realizations, one, the suffix *-bhis*, introduced by rule 4.1.2. In our example, however, rule 7.1.9 *ato bhis aī* then obligatorily applies. This rule, which may be written as:

$$bhis \rightarrow aī \quad / \quad aT \text{---} \quad (4)$$

turns *-bhis* to *-aī* in *a*-stems, converting *\*akṣabhis* into *\*akṣaī*. Further phonological rules then turn *\*akṣaī* *divyati* into the ultimate phonetic form *akṣaī* *divyati*. Going back to level (2) in this derivation, we may note that a special rule (1.4.43) at that point provides the option of expressing the same semantic relation by *karman* ‘deep object’ (instead of by *karāṇa* as above) in the case of this particular verb *div-*. By taking this option we would arrive at a quite different range of surface forms, e.g. the Accusative object *akṣān divyati*.

It is evident from this example that there is no one-one correspondence between any of these four levels. But the relation between them is not arbitrary. In the mapping rules that relate each pair of successive levels, Pāṇini throughout makes use of the device of introducing a ‘preferred’ or ‘basic’ replacement, to which subsequent, more detailed rules then enumerate the exceptions and corrections. For example, rule 1.4.42 states that the semantic instrument relation is expressed in the deep structure by *karāṇa*. However, there are a few special cases which the next two rules take care of: with *div-* ‘play’ there is optionally *karman* (1.4.43), as was just mentioned; and with *parikrī-* ‘rent’ there is optionally *saṃpradāna* (1.4.44). *Saṃpradāna* is a *kāraṅka* whose principal manifestation in the surface structure is the Dative case. A second example is provided by the semantic relation of *ādhāra* ‘locus’. The *kāraṅka* which normally expresses this semantic relation is *adhikarāṇa*, typically expressed in the surface structure as a Locative case, e.g. *grāme vasati* ‘he lives in the village’ (1.4.45). But under certain conditions, detailed in 1.4.46–48, the same semantic relation is expressed by *karman*, the direct object relation, e.g. *grāmam upavasati* ‘he inhabits (occupies) the village’.

These examples show that the relation between the semantic and deep structure levels not only fails to be one-one, but is in fact many-many. In some cases the same semantic relation is expressed by different *kāraṅkas*, as in the case of ‘he plays dice’, ‘he plays with dice’, where the instrument relation is expressed alternatively by *karman* and *karāṇa*. In other cases, different semantic relations are expressed by the same *kāraṅka*, as when ‘he plays dice’

expresses the instrument relation, and ‘he inhabits the village’ expresses the locative relation, although both are syntactically instances of *karman* or deep object.

The device of basic and subsidiary mapping rules between levels becomes particularly important in the case of the relation between levels (2) and (3). Here Pāṇini combines it with a convention requiring disjunctive application of the several rules which spell out a *kāraka* morphologically. This convention is established by introducing these rules with the governing sūtra 2.3.1: *anabhihite* ‘if it is not expressed (already)’. This sūtra is applicable to the following ones by virtue of *anuvṛtti* ‘recurrence’<sup>2</sup> and functions as in the following example. At level (2), 1.4.49 provides a characterization of one of the *kāraka* relations, viz., *karman* ‘underlying object’. At level (3), 2.3.2 provides the corresponding case: *karmaṇi dvitīyā* ‘when the *karman* relation is to be expressed, the Accusative case is used’. However, this interpretation is incomplete. The Accusative case is the basic expression for the *karman* relation, but it is not its only expression. Rule 2.3.2 comes under the scope of 2.3.1. Hence 2.3.2 means: ‘when the *karman* relation is to be expressed, and it is not already expressed, the Accusative case is used’.

What happens if *karman* is already expressed, and how could it be already expressed? An example is provided by the passive of the verb, which may also express the *karman* relation. This is stated in 3.1.67, which prescribes that the affix *yaK*<sup>3</sup> is attached to the verbal root if, e.g., *karman* (through *anuvṛtti* from the previous rule) is to be expressed. So the Accusative is the basic expression for *karman*, and the passive is an alternative expression.

Let us look a little more closely at an application of this procedure. The underlying relation *karman* ‘object-of’ holds between the verb and the object in *kumbhaṃ karoti* ‘he makes a pot’. Here the ending *-am* expresses the Accusative case, and this is accounted for as the basic case because it expresses a *karman* which is not already expressed. But *karman* could be alternatively expressed by attaching the suffix *yaK* to the root *kr-* ‘make’, yielding *kriyate* ‘(it) is made’. This accounts for *kumbhaḥ kriyate* ‘the pot is made’. Since *karman* is already expressed by *kriyate*, it is not expressed by attaching an Accusative ending to the nominal stem *kumbha-*. The case ending which is actually attached is that of the Nominative, as follows from a different rule (viz., 2.3.46).

<sup>2</sup> Technical and methodological devices and expressions such as *anuvṛtti* are not described here. The reader may refer to Renou (1940, 1957).

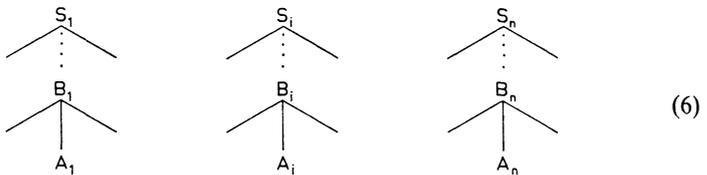
<sup>3</sup> *yaK* is the name for the suffix *ya*, where *K* is an indicatory letter. It is customary to write elements of the object-language in small letters, and elements of the metalanguage of linguistic description in capital letters (as was already done in (4) with respect to *aT*). We shall also use capitalization to distinguish cases (e.g., the Instrumental case) from semantic relations (e.g., the instrumental semantic relation).

Just as several cases, affixes, or voices may be used to express the same underlying relation, several case endings may be used to express the same case; analogously for verbal suffixes and endings. These case endings come in groups and constitute in modern grammars the paradigms of the nominal declensions. Here too Pāṇini introduces a set of preferred or basic endings. They are given in 4.1.2 and constitute the nominal endings attached to consonant stems. Again there are alternatives, e.g., the endings attached to nominal stems ending in *-a*. These are substituted for those of 4.1.2 by later rules (e.g. 7.1.9 and following). There is a parallelism, then, between the expression of a semantic relation by various deep syntactic relations, of a deep syntactic relation by various morphological categories, and of a morphological category by various affixes. In each case, Pāṇini first introduces a basic set and obtains the alternatives by a certain kind of shift or with the help of substitutions. As we shall see, the general form of these techniques is to some extent similar.

The general form of Pāṇini's syntax may be described in the following terms. Pāṇini starts on level (1) with a semantic relation, say  $[A, B]$ :

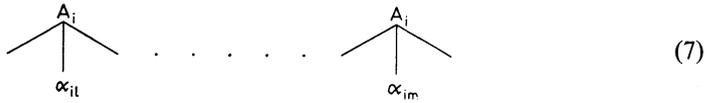


At level (2) such a relation underlies, or is expressed by, various deep structures in which syntactic relations  $[A_1, B_1], \dots, [A_n, B_n]$  take its place:



One of these deep structures is regarded as basic; the others are alternatives.

At level (3) the deep structure relations are replaced by surface structure relations. These are manifested by relationships between noun and verb, most often case relationships. At level (4) such relationships are expressed by specific affixes, and each grammatical relation may be expressed by various affixes, e.g.,  $[A_i, B_i]$  by  $\alpha_{i1}, \dots, \alpha_{im}$ :



One of these affixes is again regarded as basic; the alternatives are obtained by substitution. In the most frequent case,  $[A, B]$  and the  $[A_i, B_i]$  are of the form  $[NP, VP]$  and the  $\alpha_{ij}$  are case terminations, the basic ones being those of the consonant stems.

The general form of the rules which interrelate the four levels may be expressed as:

$$P \rightarrow Q \text{ if } X. \tag{8}$$

Pāṇini introduces the alternative expressions on each following level by rules of this form. Let us assume that there are  $n$  alternatives and that the rules introducing these are written as:

$$\begin{aligned} P &\rightarrow Q_1 \text{ if } X_1 \\ \dots \\ P &\rightarrow Q_n \text{ if } X_n \end{aligned} \tag{9}$$

Now Pāṇini could have introduced the basic alternative  $B$  in two ways by ordering his rules. Either he could have started with (9) and added subsequently:  $P \rightarrow B$ , or else he could have started with  $P \rightarrow B$  and added subsequently (9), changing in (9) ‘ $P$ ’ throughout into ‘ $B$ ’.

Pāṇini adopts neither of these alternatives in a straightforward fashion, i.e., by ordering the rules themselves (as he does in the *Tripāḍī*). But by formulating  $P \rightarrow B$  (e.g., *karman*  $\rightarrow$  *dvitīyā*) in the way he does, i.e., with the addition of ‘if it is not already expressed’ (from the previous rule), he adopts in fact the first alternative and requires that the rules (9) should have applied first. (This is characteristic for all the partial orderings which occur in his grammar outside the *Tripāḍī*.)<sup>4</sup> By this convention he introduces disjunctive ordering.

When introducing the affixes which express case relationships and the like, Pāṇini adopts the second of the two alternatives mentioned. He starts with a context-free rule of the form  $P \rightarrow B$  and adds context-sensitive rules of the form (9), with ‘ $P$ ’ changed into ‘ $B$ ’ throughout. Thus, the basic affix for the Accusative is first introduced by the context-free rule:

$$dvitīyā \rightarrow am \tag{10}$$

<sup>4</sup> E.g., *antarāṅga/bahiraṅga* orderings, orderings on account of the metarule *vipratīśedha param kāryam* (1.4.2), *siddha/asiddha* orderings: cf. Buiskool (1939) and Staal (1966b).

(corresponding to part of 4.1.2). Next the alternatives are introduced by context-sensitive rules:

$$Q \rightarrow R / W \text{---} Z, \quad (11)$$

where 'Q' is the basic affix, i.e., the affix introduced by the preceding context-free rule. An example, given before, is 7.1.9 *ato bhisa ais*, which may be written as:

$$bhis \rightarrow ais / aT \text{---}.^5 \quad (4)$$

Here the Instrumental Plural ending *-bhis*, which 4.1.2 introduced in general though it occurs only in some declensions, is replaced by *-ais* after nominal stems ending in *-a*.

## II

The *kāraka* relations will now be discussed one by one in the order in which they are given by Pāṇini, i.e., *apādāna*, *sampradāna*, *karāṇa*, *adhikarāṇa*, *karman*, *karṭṛ*, and *hetu*. This is roughly in order of increasing complexity and interest.<sup>6</sup> After each *kāraka* relation has been introduced, its relation to semantic concepts on level (1) will be considered and the basic and some of the alternative expressions on level (3) will be discussed. The expressions on level (4) will in general be ignored, but the result of their introduction is illustrated in the examples.

In order to facilitate the recognition by non-Sanskrit readers of the Sanskrit forms introduced at level (4), numerical subscripts will be attached to nominal forms to indicate case terminations. The numbers refer to the cases in Pāṇini's order:

- |                 |             |
|-----------------|-------------|
| 1. Nominative   | 5. Ablative |
| 2. Accusative   | 6. Genitive |
| 3. Instrumental | 7. Locative |
| 4. Dative       |             |

Thus, *kumbhaṃ karoti* will be written as *kumbhaṃ<sub>2</sub> karoti*; *kumbhaḥ kriyate* as *kumbhaḥ<sub>1</sub> kriyate*; etc. No such device will be used in order to mark different kinds of verbal endings, which express *kāraka* relations less frequently.

### 1. *apādāna*

Sūtra 1.4.24 gives a characterization of *apādāna*: the fixed point from which something recedes. A series of special rules gives other semantic conditions

<sup>5</sup> For these expressions in general see Staal (1965a).

<sup>6</sup> There may be historical reasons for this particular order, but this does not concern us in the present context.

for the *apādāna* relation. It holds for the object of verbs of fearing, e.g. *caurebhyo<sub>5</sub> bibheti* ‘he is afraid of thieves’ (1.4.25). Other verbs whose objects stand in the *apādāna* relation include those with the meaning of hiding from, hindering from, learning from, originating from, and the verb *parāji-* ‘to be tired of’ (1.4.26–31). The commentators discuss at length whether some of these special cases might not represent separation in a figurative sense and would then fall under the main characterization of *apādāna* by rule 1.4.24. On the other hand, an ingenious quibble was found concerning the way in which that rule characterizes the *Grundbedeutung*: what about someone who falls off a running horse? There is no moving away from a fixed point, and yet the deep structure relation remains that of *apādāna*. (Cf. *vārttikā* 3 on rule 1.4.24 and the commentary on 1.4.25–8.) What must command our respect in such discussions is the seriousness, precision, and attention to detail which the Indian linguists brought to the study of syntactic-semantic problems – qualities which in Western linguistic traditions have more often been reserved for the study of the phonetic side of language.

Rule 2.3.28 lays down that the basic expression for *apādāna* is the Ablative case, e.g., *grāmād<sub>5</sub> āgacchati* ‘he arrives from the village’. A number of special rules enumerate special uses of the Ablative. This case is for example used with *prati*, when a comparison is made: *abhimanyur<sub>1</sub> arjunataḥ<sub>5</sub> prati* ‘Abhimanyu can be compared to Arjuna’ (2.3.11). With verbs of fearing, the *apādāna* relationship may be expressed by a nominal compound, viz., an Ablative *tatpuruṣa* (cf. Staal, 1966a, 170f, for an analysis of the nominal compounds expressing *kāraka* relations): e.g., *caurabhayam – caurebhyo<sub>5</sub> bhayam* ‘fear of thieves’ (2.1.37). Ablative compounds may also be formed with *-ja*, equivalent to *jātaḥ* ‘born from, produced by’, e.g., *duḥkhaja* ‘born from grief’; not, however, if the source is a species, as in ‘born from a horse’: *aśvāj<sub>5</sub> jātaḥ* as against: *\*aśvaja* (3.2.98).

The precise specification of some of the semantic categories on level (1) which are related to the *apādāna* on level (2) may be illustrated by comparing 1.4.24 with 1.4.25. The first of these rules says that *apādāna* expresses ‘the fixed point from which something recedes’, the second that it expresses ‘the cause of fear’ in connection with verbs meaning ‘to fear’, ‘to protect’. The *Kāśikā*, commenting on these rules, asks why the expression ‘cause of fear’ is used. The answer given is that the *apādāna* is not used with regard to, e.g., ‘forest’ in such sentences as *araṇye<sub>7</sub> bibheti* ‘he is afraid in (not: because of) the forest’, *araṇye<sub>7</sub> trāyate* ‘he protects in (not: from) the forest’, where the forest is not the cause of fear.

## 2. *saṃpradāna*

Sūtra 1.4.32 gives a characterization of *saṃpradāna* ‘indirect object’ in terms

of *karman* 'deep object': whom one has in view in connection with a deep object. The major flaw with this definition is that it does not allow for indirect objects when there is no direct object. Yet they occur, as in Patañjali's example *śrāddhāya*<sub>4</sub> *nigarhate* 'he finds fault with the Śrāddha ceremony' (quoted by Renou, 1957, 325). Patañjali enters into a thorough discussion of whether sūtra 1.4.32 can be saved by interpreting *karman* also in the sense of 'action', its normal (*laukika*) sense outside of technical use in grammar.<sup>7</sup> Aside from the main meaning, there is a long list of special conditions under which the *saṃpradāna* relation also holds in deep structure. We shall mention only a few of them here. There is *saṃpradāna* with verbs of pleasing: *devadattāya*<sub>4</sub> *rocate modakaḥ*<sub>1</sub> 'sweetmeat pleases Devadatta' (1.4.33). Verbs of anger, hate, envy, and annoyance take *saṃpradāna* of the person against whom the emotion is directed: *devadattāya*<sub>4</sub> *krudhyati* 'he is angry with Devadatta' (1.4.37).

According to 2.3.13, the basic expression for *saṃpradāna* is the Dative case, as in *viprāya*<sub>4</sub> *gām*<sub>2</sub> *dadāti* 'he gives a cow to the brahman'. A Dative *tatpurūṣa* compound may be formed in several ways, for example with substances made for the sake of something, as in *yūpadāru* = *yūpāya*<sub>4</sub> *dāru* 'wood for the sacrificial post' (2.1.36). Finally, a *kṛt*-suffix (which in general expresses the dēep subject: see below) may express the relationship of *saṃpradāna*: *dāśaḥ* = *dāśanti tasmai*<sub>4</sub> 'the person to whom they donate'; *goghnaḥ* = *tasmai*<sub>4</sub> *dātuṃ gām*<sub>2</sub> *ghnanti* 'the person for whom (to whom to give) they kill a cow', i.e., 'a guest' (3.4.73).

Rule 1.4.36 is of special interest since it mentions a semantic category, expressed by *saṃpradāna*, which is more regularly expressed by another *kāraka* relation (i.e., *karman*: see below). This rule states that *saṃpradāna* expresses *īpsita* 'the desired (object)' in connection with the verb *sprh-* 'to want', e.g. *puṣpebhyaḥ*<sub>4</sub> *sprhayati* 'he wants flowers'. The *Kāśikā* commentary now asks why the term *īpsita* is used. The answer given is that this is so because *saṃpradāna* is not used with regard to 'forest' in *puṣpebhyo*<sub>4</sub> *vane*<sub>7</sub> *sprhayati* 'he wants flowers in the forest', where the forest is not the desired object.

### 3. *karāṇa*

Sūtra 1.4.42 specifies *karāṇa* as the relation with the most effective means (*sādhakatama*). According to 2.3.18 this is basically expressed by the Instrumental case, as in *paraśunā*<sub>3</sub> *vrkṣaṃ*<sub>2</sub> *chinatti* 'he cuts the tree with an axe'. In special cases the Genitive may be used alternatively: e.g., *ghṛtasya*<sub>6</sub> *yajate* = *ghṛtena*<sub>3</sub> *yajate* 'he sacrifices with clarified butter' (2.3.63); or the

<sup>7</sup> Such ambiguity is sometimes made use of by the grammarians: cf. Renou (1957), 124 s.v. *karman*.

Ablative, as in: *stokān<sub>5</sub> muktaḥ* = *stokena<sub>3</sub> muktaḥ* ‘hardly (with a drop) released’ (2.3.33). In compounds: *paraśucchinnah* = *paraśunā<sub>3</sub> chinnah* ‘cut with an axe’ (2.1.32). Many suffixes express *karāṇa*. A *kṛtya*-suffix (which in general forms the Future Passive Participle) expresses *karāṇa* when attached to *vah-*: *vahya* = *vahati anena<sub>3</sub> yad* ‘that with which one travels’ (3.1.102). The suffix *-in* attached to *yaj* ‘sacrifice’ when compounded with a noun, expresses that the noun occurs as *karāṇa*: *agniṣṭomayājīn* ‘he who has sacrificed with the Agniṣṭoma’ (3.2.85). The suffix *-tra* expresses *karāṇa* in a number of cases: *netram* ‘eye’ (by which one is guided: *nī-*), *śāstram* ‘sword’ (by which one slaughters or is slaughtered: *śas-*), *stotram* ‘laud’ (by which one praises or is praised: *stu-*), etc. (3.2.182). Also the suffix *-ana*: *palāśaśā-tana* ‘a leaf-cutter’ (i.e., an instrument to cut – *śat-* – leaves, *palāśa*) (3.3.117). Pāṇini gives many smaller details of the use of the Instrumental, in many of which the *karāṇa* relation appears to be expressed as well. For further discussion on *karāṇa*, see below, Section 7, on *hetu*.

The *Kāśikā* asks why the ending of the Superlative (*tamaP*) is used in the expression *sādhakatama* ‘most effective means’ on the semantic level. The answer given is that this is necessary since *karāṇa* is not used, e.g., with regard to ‘Ganges’ in *gaṅgāyām<sub>7</sub> ghoṣaḥ* ‘the hamlet on (the banks of) the Ganges’, where the Ganges is not the most effective means, but rather the mediate cause of the hamlet. Similarly, *karāṇa* is not used with regard to ‘well’ in *kūpe<sub>7</sub> gargakulam* ‘the Garga family on (the property enclosing) the well’, where the well is not the most effective means, but rather the mediate cause of the Gargas.

That *sādhakatama* is a technical concept on the semantic level is suggested by the *Kāśikā*’s explicit statements that it is *sādhakatama* which is referred to in 1.4.43 and 1.4.44.

#### 4. *adhikarāṇa*

Sūtra 1.4.45 characterizes *adhikarāṇa* as the locus (*ādhāra*). Its basic expression according to 2.3.36 is the Locative, e.g. *kaṭa<sub>7</sub> āste* ‘he sits on a mat’. The Locative expressing *adhikarāṇa* may be compounded with a verbal noun, as in: *cakrabandhaḥ* = *cakre<sub>7</sub> bandhaḥ* ‘tied on a wheel’ or ‘a stanza arranged in the shape of a wheel’ (2.1.41). When the suffix *-i* is attached to another type of compound, the first member of which expresses the deep object, the *adhikarāṇa* is expressed: e.g., *jaladhi* = *jalam<sub>1</sub> dhīyate* ‘*smin<sub>7</sub>*’ ‘where water is placed’, i.e., ‘ocean’ (3.3.93; note the expression of the deep object with the help of the surface subject *jalam* and the passive *dhīyate*). Other suffixes express *adhikarāṇa* directly: e.g., *-ana* as in *rājadhānī* ‘where the king resides’, i.e., ‘palace’ (3.3.117).

An *adhikarāṇa* of time between two *kāraka* relations represented in suc-

cessive sentences may be expressed by a Locative or an Ablative: *adya bhuktvā dvyāhe*<sub>7</sub> *bhoktā* or *adya bhuktvā dvyahād*<sub>5</sub> *bhoktā* 'having eaten today he will eat two days from now' (2.3.7). A kind of generalized *adhikaraṇa*, i.e., a situation which characterizes another situation, is also expressed by the Locative (the modern grammars call this the *locative absolute*): e.g., *goṣu*<sub>7</sub> *duhyamānāsu*<sub>7</sub> *gato devadatta dugdhāsv*<sub>7</sub> *āgataḥ* 'Devadatta went when the cows were being milked, and returned after they had been milked' (2.3.37). If under these circumstances neglect or disregard is expressed, the Genitive is a possible alternative: *rudati*<sub>7</sub> *pravrajitavān* or *ruditaḥ*<sub>6</sub> *pravrajitavān* 'he wandered away (i.e., became a wandering ascetic) despite the crying of...' (2.3.38).

It is worth noting that the deep structure relation of *adhikaraṇa* in the last two cases obtains between sentences. This is quite consistent with the definition of [A, B] as expressed by (6) (above, p. 87): in this particular case, A is a sentence which is embedded in a sentence B. This reflects the fact that, within a complex sentence, the relationships between embedded sentences may be like the relations between noun-phrases within a simple sentence. The precise nature of this parallelism, which is clearly observable in the surface structure of inflected languages, is captured by the *kāraka* relations. Thus, *adhikaraṇa* is the relationship expressed by the Locative ending both in *kaṭa*<sub>7</sub> *āste* 'he sits on the mat' and in *rudaty*<sub>7</sub> *āste* 'he sits despite the crying of...' (or 'he sits while ... is crying').

In modern linguistics the same purpose is achieved through rules for the nominalization of embedded sentences. Whatever rules apply to noun-phrases apply to the results of such nominalizations. Pāṇini, too, requires nominalization, e.g., in order to derive the participle form *rudati*<sub>7</sub>. In modern linguistics grammatical relations are defined for constituents among which noun-phrases resulting from the nominalization of embedded sentences may occur. In Pāṇini similarly, the *kāraka* relations obtain between the matrix and embedded sentences as well. In *rudati*<sub>7</sub> *pravrajitavān*, there is *adhikaraṇa* of the underlying sentence *rudati* 'he cries' with respect to *pravrajitavān* 'he wandered away'.<sup>8</sup>

That *ādhāra* 'locus' is the semantic concept on level (1), to which *adhikaraṇa* is one of the corresponding *kāraka* relations on level (2), is clearly seen in sūtra 1.4.46. This rule states that the *karman* relation is used instead in connection with the verbal roots *śi-* 'to lie', *sthā-* 'to stand', *ās-* 'to sit' when preceded by the preverb *adhi-*. The *Kāśikā* says explicitly that it is the *ādhāra* which is then expressed by *karman*, e.g., in *grāmam*<sub>2</sub> *adhiṣete* 'he is lying in

<sup>8</sup> In later Sanskrit, the embedding of nominalized sentences in order to express logical relations was developed into a semi-artificial device: see Staal (1965b).

the village', *grāmam*<sub>2</sub> *adhitiṣṭhati* 'he is standing in the village', *parvatam*<sub>2</sub> *adhyāste* 'he is sitting on the mountain'. Commenting on 1.4.47 and 1.4.48, where other cases are enumerated, the *Kāśikā* refers to *ādhāra* in the same manner.

### 5. *karman*

Sūtra 1.4.49 defines *karman* as that which is primarily desired (*īpsitatama*) by the deep subject. Rule 2.3.2, quoted above, lays down that the basic expression for *karman* is the Accusative case, e.g.: *kumbhān*<sub>2</sub> *karoti* 'he makes pots'. The *karman* relation may also be expressed by other cases. To start with a very specific example: in the Veda, for the object of the verbal root *hu-* 'to make an oblation', the Instrumental case is a possible alternative: *yavāgūm*<sub>2</sub>/*yavāgvā*<sub>3</sub> *agnihotraṃ*<sub>2</sub> *juhōti* 'he makes an oblation of/with rice-gruel to Agni' (2.3.3). The *karman* of verbs of going is expressed by the Accusative or the Dative: *grāmam*<sub>2</sub> *gacchati* or *grāmāya*<sub>4</sub> *gacchati* 'he goes to the village' (2.3.12). This rule mentions two exceptions: if the going does not involve real movement, i.e., if it is used metaphorically, or if the object is *adhvan* 'path', only the Accusative occurs: *mohaṃ*<sub>2</sub> *gacchati* 'he falls into error' (not: *\*mohāya*<sub>4</sub> *gacchati*); *adhvānam*<sub>2</sub> *gacchati* 'he goes along the path' (not: *\*adhvane*<sub>4</sub> *gacchati*).

A use of the Dative for expressing *karman* which involves greater depth is described in 2.3.14. The Dative expresses *karman* if there is an alternative where a verbal noun with its logical object occurs: e.g., *edhebhyo*<sub>4</sub> *vrajati* 'he is going for firewood' with its paraphrase: *edhān*<sub>2</sub> *āhartum* *vrajati* 'he is going to fetch firewood', in which the Infinitive *āhartum* occurs. The next rule states that the Dative of a *nomen actionis* is equivalent to an Infinitive or similar verbal noun: *pākāya*<sub>4</sub> *vrajati* = *paktum* *vrajati* 'he is going to cook' (2.3.15); both express *karman*. In these cases the generalized notion of *kāraka* operates. In both *edhebhyo*<sub>4</sub> *vrajati* and *pākāya*<sub>4</sub> *vrajati* the Dative expresses the deep object. The two parallel sentences are complex sentences: the Infinitive *paktum* is, presumably, transformationally derived from *pacati* 'he cooks', and *edhān*<sub>2</sub> *āhartum*, similarly, from *edhān*<sub>2</sub> *āharati* 'he fetches firewood'. In the last sentence *edhān*<sub>2</sub> is the deep object. In both complex sentences we have the same deep structure relationship with the embedded sentence: in *paktum* *vrajati* there is *karman* of the underlying sentence *pacati* with respect to *vrajati*, and in *edhān*<sub>2</sub> *āhartum* *vrajati* there is *karman* of the underlying sentence *edhān*<sub>2</sub> *āharati* with respect to *vrajati*. Thus, the Dative of a lexical item *pāka* derived from the verbal root *pac-* 'cook', and the Infinitive *paktum* derived transformationally from the sentence *pacati* 'he cooks', express the same deep structure relationship. In the firewood case no lexical derivation is involved, yet the deep relationship remains the same. These examples

illustrate how the concept of *kāraka* is used to state generalizations which are difficult to capture in other ways.

Within the nominal system of cases there are other more particular possibilities to express *karman*. The object of the verb *saṃjñā-* ‘recognize’ is expressed not only by the Accusative, but also by the Instrumental: *mātaraṃ<sub>2</sub> saṃjñānīte* or *mātrā<sub>3</sub> saṃjñānīte* ‘he recognizes his mother’ (2.3.22). The Genitive is used in many ways to express the object. Examples include: *mātuḥ<sub>6</sub> smarati* ‘he remembers his mother’ (accounted for by 2.3.52), *alaṃkārasya<sub>6</sub> upaskurute kāmakaḥ* ‘a lover imparts a new quality to an ornament’ (2.3.53), *caurasya<sub>6</sub> rujati rogaḥ* ‘illness afflicts the thief’ (2.3.54), etc.

The so-called ‘objective Genitive’ is treated along with the ‘subjective Genitive’ in 2.3.65, which states that the Genitive may express the deep object, *karman*, as well as the deep subject, *karṭṛ* (see next section): e.g., *apāṃ<sub>6</sub> sraṣṭā* ‘creator of the waters’ and *bhavata<sub>6</sub> āsikā* ‘yours is the sitting’, i.e., ‘it is your turn to sit’. In both cases the governing noun (*āsikā*, *sraṣṭā*) is derived from the verbal root (*ās-*, *srj-*, respectively) by attaching a *krṭ-* suffix. The same deep structure relations are therefore expressed in the examples given here and in the underlying sentences, where the verbal root occurs in a finite verbal form. Confining attention in the present context to the object, *karman* is accordingly expressed in *apāṃ<sub>6</sub> sraṣṭā* and in the underlying sentences, which uses the basic case, i.e., the Accusative: *apaḥ<sub>2</sub> srjati* ‘he creates the waters’. – A *vārttikā* under 2.3.22 notes that this Genitive is also used in the special case quoted above, where the object in the underlying sentence is expressed by the Instrumental: e.g., *mātuḥ<sub>6</sub> saṃjñānīte* ‘recognizer of his mother’ from *mātaraṃ<sub>2</sub>/mātrā<sub>3</sub> saṃjñānīte*.

Such analyses show in a simple case the real connection between Pāṇini’s introduction of basic or preferential expressions and the modern notions of underlying and derived, deep and surface structures. Pāṇini nowhere says that *apāṃ<sub>6</sub> sraṣṭā* is ‘derived from’ *apaḥ<sub>2</sub> srjati*. But he clearly states that the same underlying relation, i.e., *karman*, is expressed in both sentences, and he clearly assigns priority to the second sentence by listing the Accusative first among expressions for *karman*, with *anuvṛtti* of 2.3.1.

Sūtra 2.3.66 lays down that the Genitive expresses only the deep object whenever there is scope (in accordance with the previous rule) for both the deep subject and the deep object to be expressed. For example: *adbhutaḥ samudrasya<sub>6</sub> bandho<sub>1</sub> vānaraiḥ<sub>3</sub>* ‘wonderful how the ocean is spanned by monkeys’ (not \**adbhutaḥ samudrasya<sub>6</sub> bandho<sub>1</sub> vānarām<sub>6</sub>*). Here the object *samudrasya<sub>6</sub>* is expressed by the Genitive; the subject *vānaraiḥ<sub>3</sub>* is expressed by the Instrumental (see next section), not by the Genitive. In other words, the underlying sentence is *vānarāḥ<sub>1</sub> samudraṃ<sub>2</sub> badhnanti* ‘monkeys span the ocean’, where the object is expressed by the (basic) Accusative *samudram*

and the subject by the (basic) Nominative *vānarāḥ*. Another example is *āścaryo gavāṃ<sub>6</sub> doho<sub>1</sub> 'gopālakena<sub>3</sub> 'the milking of cows by someone who is not a cowherd is strange' (not: \*āścaryo gavāṃ<sub>6</sub> doho<sub>1</sub> 'gopālakasya<sub>6</sub>)*.

We have already seen that the forms of the middle voice (Ātmanepada) of the verb may express the object (1.3.13), as in: *kumbhaḥ<sub>1</sub> kriyate* 'the pot is made'. The equivalent underlying sentence, accordingly, is *kumbhaṃ<sub>1</sub> karoti* '(he) makes the pot'. This includes instances where the object of the underlying sentence is expressed by other cases than the Accusative; e.g., to *gacchati grāmaṃ<sub>2</sub> devadattaḥ<sub>1</sub>* or *gacchati grāmāya<sub>4</sub> devadattaḥ<sub>1</sub>* 'Devadatta goes to the village' there corresponds *gamyate grāmo<sub>1</sub> devadattena<sub>3</sub>* 'the village is gone to (reached) by Devadatta'. The actual form of the passive is explained by 3.1.67, which introduces, as we have seen, the affix *yaK*.

That the object may be expressed by a compound seems fairly obvious: *kumbhakāraḥ* 'pot-maker', corresponding to *kumbhaṃ<sub>2</sub> karoti* 'he makes a pot' and *kumbhānāṃ<sub>6</sub> kartā<sub>1</sub>* 'maker of pots' (cf. Rocher, 1964a, 52). Also: *grāmagataḥ* 'one who has gone to a village' (2.1.24). The form *-kāra* derived from the verbal root *kr-* 'make', which does not occur by itself, is accounted for by 3.2.1. Several suffixes or endings, lastly, express the object: *-ta* in: *kṛtaḥ<sub>1</sub> kaṭaḥ<sub>1</sub>* 'the mat is made', and *-tavya* in: *kartavyaḥ<sub>1</sub> kaṭaḥ<sub>1</sub>* 'the mat should be made' (3.4.70).

That not only *īpsitatama*, but also *īpsita* 'desired (object)' is a semantic category follows, e.g., from 1.4.27, where it is declared that the *apādāna* relation is used to express what is *īpsita* in connection with verbs meaning 'to ward off'. For example, *yavebhyo<sub>5</sub> gāṃ<sub>2</sub> vārayati* 'he wards the cow off from the barley-corns', *yavebhyo<sub>5</sub> gāṃ<sub>2</sub> nivarttayati* 'he drives the cow away from the barley-corns'. The *Kāśikā* accounts for the use of *īpsita* by quoting *yavebhyo<sub>5</sub> gāṃ<sub>2</sub> vārayati kṣetre<sub>7</sub>* 'he wards the cow off from the barley-corns in the field', where *apādāna* is not used for 'field' since the field is not the desired object.

## 6. *karṭr*

Sūtra 1.4.54 gives a characterization of *karṭr* as what is independent. Its basic expression according to 2.3.18 is the Instrumental case, as in: *devadattena<sub>3</sub> kṛtaḥ<sub>1</sub>* 'it is done by Devadatta'. The *karṭr* is also expressed by the Active voice of the verb: *pacati* 'he cooks' (1.3.78; 3.1.68, which accounts for the affix *-a* by which the verbal root *pac-* is turned into the present stem *paca-*: see Shefts, 1961, for a detailed treatment of this topic). That the subject is expressed by the Nominative along with the verbal endings, as in *pacati devadattaḥ<sub>1</sub>* 'Devadatta cooks', is not explicitly stated by Pāṇini, but must be inferred from the rule which defines the Nominative as the unmarked case, i.e., as that which designates nothing but the gender and the number

of the nominal stem notion (2.3.46, as interpreted in Thieme, 1956; Thieme's views on this sūtra are undoubtedly correct, though he also is of the opinion that for Pāṇini there exists no grammatical category corresponding to 'subject').

A special instance of *karṭr* occurs with verbs that express reciprocity. They have the endings of the middle voice (1.3.14). For example, *vyatibhavate śrīh<sub>1</sub> padmānām<sub>6</sub>* 'the beauty of lotuses contends for excellence', i.e. 'lotuses vie with each other in beauty'; *vyatipacante brāhmaṇāh<sub>1</sub>* 'the brahmins cook for each other'. The logical subjects are *śrīh<sub>1</sub>* 'beauty' and *brāhmaṇāh<sub>1</sub>*, respectively.

Apart from the Instrumental, the *karṭr* relation may be expressed by the (objective) Genitive, as we have seen (*bhavata<sub>6</sub> āsikā*). In connection with the future passive participle, the Genitive is a possible alternative to the Instrumental, e.g., *bhavatā<sub>3</sub> kaṭaḥ<sub>1</sub> kartavyaḥ<sub>1</sub>* or *bhavataḥ<sub>6</sub> kaṭaḥ<sub>1</sub> kartavyaḥ<sub>1</sub>* 'a mat should be made by you', i.e., 'you should make a mat' (2.3.71). The deep subject is expressed generally (i.e., unless they are specified otherwise) by *kṛt* suffixes, e.g., *odanasya<sub>6</sub> pācakaḥ* 'cooker of rice', *nandanah* 'person who rejoices' (3.4.67). It is only to be expected, lastly, that the subject may also be expressed by a nominal compound, e.g., *caurahataḥ = caureṇa<sub>3</sub> hataḥ* 'killed by a thief' (2.1.32).

### 7. *hetu*

Sūtra 1.4.55 defines *hetu* as that which prompts the *karṭr* (for criticism of this interpretation, which follows the Indian commentators, see Rocher, 1964b). This is basically expressed by the causative, e.g., *kārayati devadattaḥ<sub>1</sub>* 'Devadatta causes (or allows) (someone else) to make'. The sentence with the causative is equivalent to a sentence where causality is expressed by a separate verb which has for its grammatical object the person performing the activity: *kārayati = kurvantaḥ<sub>2</sub> prayūṅkte* 'he causes someone else making...' (3.1.26). This appears to imply that in sentences such as *rāmaḥ<sub>2</sub> kārayati devadattaḥ<sub>1</sub>* 'Devadatta causes Rāma to make' = *rāmaḥ<sub>2</sub> kurvantaḥ<sub>2</sub> prayūṅkte devadattaḥ<sub>1</sub>* 'Devadatta causes Rāma making...', the surface subject *devadattaḥ* expresses the *hetu* and the surface object *rāmam* expresses the deep subject of the corresponding simple sentence *karoti rāmaḥ* 'Rāma makes'.

These specifications apply only to the active causative. Endings of the middle voice are attached to the causative if the underlying object of the corresponding sentence in which no causative occurs is expressed as the surface subject: e.g., in *kareṇur<sub>1</sub> ārohayate niṣādinam<sub>2</sub>* 'the elephant allows itself to be mounted by the elephant-driver', *kareṇuh<sub>1</sub>* 'elephant' is the surface subject which expresses the underlying object expressed as the surface

object in the corresponding simple sentence: *ārohati kareṇum<sub>2</sub> niṣādi<sub>1</sub>* ‘the elephant-driver mounts the elephant’ (1.3.67).

The *kartr* which undergoes the prompting of the *hetu* is expressed by the Instrumental case, e.g., Yajñadatta in: *pācayaty odanam<sub>2</sub> devadatto<sub>1</sub> yajñadattena<sub>3</sub>* ‘Devadatta makes Yajñadatta cook the rice’ or ‘Devadatta causes the rice to be cooked by Yajñadatta’ (*Mahābhāṣya* ad 3.1.26 quoted by Renou, 1957, 228). According to some grammarians, the Accusative is optional: *pācayaty odanam<sub>2</sub> devadatto<sub>1</sub> yajñadattam<sub>2</sub>* (Renou, ad 1.4.52). The Accusative is obligatory after verbs of going, knowing, absorbing and several others (amended by *vārttikās*), as well as intransitives, e.g.: *yāti vipro<sub>1</sub> grāmam<sub>2</sub>* ‘the brahman goes to the village’ → *yāpayati vipram<sub>2</sub> grāmam<sub>2</sub>* ‘he makes the brahman go to the village’; *bodhati śiṣyo<sub>1</sub> vyākaraṇam* ‘the pupil learns grammar’ → *bodhayati śiṣyam<sub>2</sub> vyākaraṇam<sub>2</sub>* ‘he teaches grammar to the pupil’; *āste putraḥ<sub>1</sub>* ‘the son sits down’ → *āsayati putram<sub>2</sub>* ‘he makes his son sit down’ (1.4.52).

The notion of *kāraka* as applied to complex sentences is relevant here in that the *hetu* relation may be represented by an embedded sentence (the following illustrations are taken from Rocher, 1964b, 38–9). The *hetu* may in general be expressed by an Instrumental, as in: *vidyayā<sub>3</sub> yaśaḥ<sub>1</sub>* ‘fame due to learning’ (2.3.23). It may therefore be expressed by the Instrumental of a noun-phrase which is the result of a nominalization of an embedded sentence, e.g. *puruṣa-dakṣiṇa-gamanam* ‘the man’s going to the right’, derived from (cf. Staal, 1965) the sentence: *dakṣiṇena<sub>3</sub> gacchati puruṣaḥ<sub>1</sub>* ‘the man goes to the right’. The Instrumental may be used to turn this into a *hetu*, e.g.: *puruṣadakṣiṇagamanena<sub>3</sub> śakaṭam<sub>1</sub> na paryābhaviṣyati* ‘through the man’s going to the right the cart will not turn over’. Alternatively this *hetu* may be expressed by a Nominative of the nominal compound together with a causative of the verb: *puruṣadakṣiṇagamanam<sub>1</sub> śakaṭam<sub>2</sub> na paryābhāvayiṣyati* ‘the man’s going to the right will cause that the cart will not turn over’. Lastly, this *hetu* may be expressed by the potential mood of the verb (3.3.156) in what could be called a ‘causative’ conditional subordinate or embedded sentence: *puruṣo<sub>1</sub> dakṣiṇena<sub>3</sub>* (or: *dakṣiṇam<sub>2</sub>*) *ced yāyān na śakaṭam<sub>1</sub> paryābhaviṣyati* (or: *paryābhavet*) ‘if the man were to go to the right, the cart would not turn over’. This exemplifies a relationship between causatives and ‘if’-clauses and clearly shows the underlying sentence *dakṣiṇena<sub>3</sub> gacchati* (or *yāti*) *puruṣaḥ<sub>1</sub>* ‘the man goes to the right’, which is *hetu* with respect to *śakaṭam<sub>1</sub> na paryābhaviṣyati* ‘the cart will not turn over’ in all three sentences.

Middle endings are attached to the causative of the verbs *bhī-* ‘fear’ and *smi-* ‘be astonished’, provided fear and surprise, respectively, are caused by the *hetu* and not by something else: e.g., *muṇḍo bhīṣayate* ‘the bald man frightens’ (i.e., ‘the bald man is frightening’), *jaṭilo<sub>1</sub> vismāpayate* ‘the hairy

ascetic astonishes'. But the active endings are attached if fear and surprise are caused by something else, e.g., *kāṣṭhikayā<sub>3</sub> bhāyayati* 'he frightens with a stick', *rūpeṇa<sub>3</sub> vismāyayati* 'he astonishes by his form' (1.3.68). This shows that *kāṣṭhikayā<sub>3</sub> bhāyayati* contrasts with *kāṣṭhikā<sub>1</sub> bhīṣayate* 'the stick frightens' not only in case (i.e., Instrumental and Nominative, respectively), but also in verbal voice (i.e., active and middle, respectively): jointly they express the difference in logical relation. In fact, the forms themselves are different. The causative of *bhī-*, in the latter, middle instance, must have the augment *sUK*, resulting in *bhīṣayate*, not *\*bhāyayate*. In the former, active instance, on the other hand, no form *\*bhīṣayati* corresponds to *bhāyayati* (7.3.40). Similarly, with respect to *smi-*, the middle case form has the augment *pUK*, resulting in *vismāpayate*, not *\*vismāyayate*. In the active instance, on the other hand, there is no form *\*vismāpayati* corresponding to *vismāyayati* (6.1.57; 7.3.36).

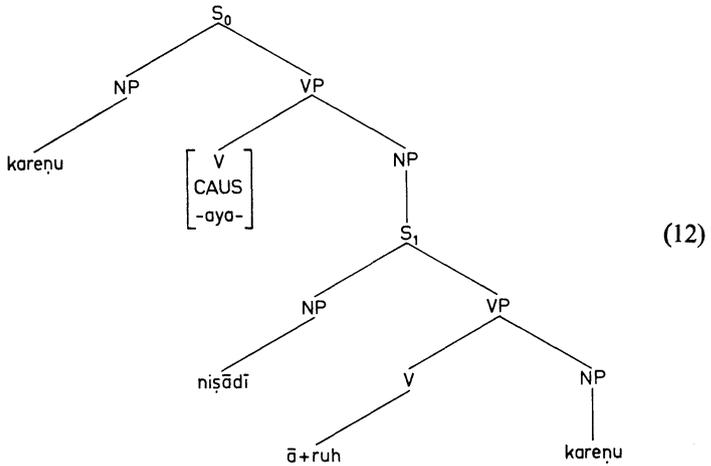
Thus we have according to the rules: *manuṣyavāk<sub>1</sub> vismāpayate* 'the human voice astonishes' and *siṃho<sub>1</sub> vismāpayate* 'the lion astonishes', but *siṃho<sub>1</sub> manuṣyavācā<sub>3</sub> vismāyayati* 'the lion astonishes by his human voice' or *siṃho<sub>1</sub> manuṣyavācā<sub>3</sub> vismāyayan* 'the lion is astonishing by his human voice'. But what is really astonishing when a lion speaks with a human voice? The lion, because it speaks with a human voice, or the human voice, because it springs forth from a lion? Bhaṭṭojī Dīkṣitā discusses in this connection in his *Siddhānta Kaumudī* (2596) an interesting passage from Kalidāsa's *Raghuvamśa* (2.33). The original of Bhaṭṭojī's puzzle with the citation from Kalidāsa deserves to be quoted in full: "But then how do you explain the following passage: 'The lion, having seized the cow, addressed him [i.e., king Dilipa] – that adherent of nobility, banner of the lineage of Manu, of leonine prowess – with a human voice (*manuṣyavācā<sub>3</sub>*), astonishing (*vismāpayan<sub>1</sub>*) him who was already astonished at his own condition (*vismitam<sub>2</sub> ātmavṛttau<sub>7</sub>*) [i.e., paralyzed and unable to shoot the lion]?" For the astonishment is here due to the *karaṇa*, i.e., *manuṣyavācā<sub>3</sub>*. Otherwise the middle ending should have been used. – True, hence a traditional reading is *vismāyayan<sub>1</sub>*. – Or else the passage has to be interpreted thus. The human voice, being the *karṭṛ* which undergoes the prompting [of the *hetu*], astonishes (*vismāpayate*). By means of that the lion is said to be astonishing (*vismāpayann<sub>1</sub> iti*). This is a double causative and the active ending of the participle is used."<sup>9</sup>

Pāṇini's analysis of the causative differs in an especially instructive way from the treatment required by transformational grammar. This would derive

<sup>9</sup> *katham tarhi vismāpayan vismitam ātmavṛttāv iti | manuṣyavāceti karaṇād eva hi tatra smayaḥ | anyathā śānaj api syāt | sattvam | vismāyayann ity eva pāṭha iti sāmpradāyikāḥ | yad vā | manuṣyavāk prayojyakarṭri vismāpayate tayā siṃho vismāpayann iti nyantān nau śateti vyākhyeyam:* ed. and transl. Vasu (1962?), II, 516.

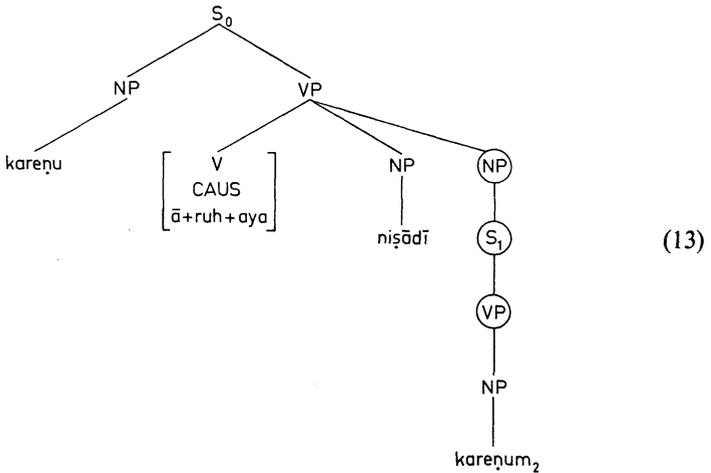
causative sentences from simple sentences embedded as objects of a verb meaning ‘cause, allow’ (Lakoff, 1965; Kuroda, 1965; Chomsky, 1965). We have seen that Pāṇini came close to such an analysis by statements of the form “the causative gets the middle endings if that which is the direct object in the simple form is the agent in the corresponding causative” (1.3.67). The crucial difference between relating causative constructions to simple sentences by such statements of correspondence, and transformationally deriving causative constructions from deep structures in which these simple sentences occur as embedded sentences, is that the transformational derivation makes available intermediate stages of derivation which permit the explanation of formal features of the causative construction. Note that Pāṇini generally passes through intermediate stages in his phonological derivations, but there does not appear to be a single example in his grammar where a syntactic rule converting deep into surface structure results in an intermediate stage which must obligatorily be modified by a further such rule.

Consider again the sentence *kareṇur<sub>1</sub> ārohayate niṣādīnam<sub>2</sub>* ‘the elephant allows itself to be mounted by the elephant-driver’. The deep structure of this sentence would be the following:



This deep structure corresponds most directly to a surface structure of the form ‘The elephant allows that the driver mounts it’. In Sanskrit a number of transformations apply to this deep structure to give it a very different surface form. Following Chomsky (1965) we assume that transformations apply in cyclic order: they first apply to the most deeply embedded sentence (marked S<sub>1</sub> in (12)) and then reapply in the same order to the next deepest sentence (S<sub>0</sub>). In an example with more layers of embedding, the rules would

continue to apply cyclically in the same way until the topmost S node in the tree was reached. In the first cycle, the object of  $S_1$ , *kareṇu*, is marked with the Accusative case. The passive transformation is optionally applicable on this cycle to  $S_1$ . We shall first consider the case where the passive is not applied and return later to the case where the passive is applied. With Accusative case marking being the only transformation relevant to our purposes that has applied to  $S_1$  in this derivation, we move to the next cycle and apply the rules to  $S_0$ . In this cycle, the underlying subject NP of  $S_1$  is raised into  $S_0$ , where it is attached to the VP and becomes its surface object. A similar raising applies in English to give *The elephant allows the driver to mount it*. In Sanskrit, unlike in English, the verb of  $S_1$  is also raised into  $S_0$  and is amalgamated with the causative element into a single causative verb. These two operations completely obliterate the internal structure of the embedded simple sentence. Its subject and verb have been removed, and what is left is the following structure:



The circled nodes in (13) are erased by general tree-pruning conventions (Ross, 1967). What is left is the surface structure of the causative sentence in Sanskrit. Case-marking, applying now to  $S_0$ , marks *niṣādī* accusative, since it is at this point in the derivation the object of the verb, and the subject *kareṇu* is marked with the nominative case. Were it not for the fact that we have two identical noun phrases in the sentence, the derivation would stop here. Substituting *vrkṣam* for *kareṇum* in our example, we would have derived the causative sentence *kareṇur ārohayati niṣādīnaṃ vrkṣam* 'the elephant makes the driver climb the tree'. But the occurrence of two identical noun phrases makes the sentence susceptible to the reflexive transformation. The

reflexive is formed by replacing the second of the identical noun phrases (which may be a direct or indirect object, or a possessor) by a reflexive pronoun; in Sanskrit the verb can then be optionally converted into the middle voice, and if this option is taken, there is a further option of dropping the reflexive pronoun. Thus, 'Devadatta makes his mat' could be expressed in the following three ways:

- (1) *devadattaḥ svaṃ kaṭam karoti* (reflexivization),
- (2) *devadattaḥ svaṃ kaṭam kurute* (reflexivization, middle),
- (3) *devadattaḥ kaṭam kurute* (reflexivization, middle, reflexive pronoun dropping).

In the same way, *kareṇum* in our example is reflexivized, and the reflexive pronoun can be erased if the verb is made middle. From the intermediate stage *kareṇur ārohayati niṣādinam kareṇum* we thus derive our sentence *kareṇur ārohayate niṣādinam*.

The point of this example is that the middle voice which Pāṇini prescribes for the causative verb in certain cases by a highly complicated rule (1.3.67) turns out simply to be a case of the general reflexive use of the middle voice. The reflexive (realized optionally as a middle voice on the verb) appears whenever there is identity of noun phrases within a sentence at a certain point in the derivation. The critical fact is that this situation can be the result of a transformation such as the raising we have described. The correct application of the reflexive rule depends on the form of the sentence at a certain intermediate stage, after the raising rule has applied. A system such as Pāṇini's, in which sentences are not derived transformationally, is therefore unable to treat reflexivization as a single process. Pāṇini in fact splits up the reflexive middle into at least three different rules. In the case of direct objects it is apparently intended to be handled by 1.3.13, according to which the middle endings express the *karman*; the indirect object case is handled by rules 1.3.72–7, which specify the middle voice 'when the agent benefits from the action'; and the causative case that we have just analyzed is given as a completely unrelated process in 1.3.67. What makes this worth mentioning is that at least in the causative case, Pāṇini is *in principle* prevented by the form of his grammar from combining a process with others that are clearly related to it.

The transformational derivation of causatives from (and not merely their equivalence with) simple sentences functioning as objects of a verb of causation also enables us to explain the case forms in which the noun phrases of the causative construction can appear. We saw above that there are two types of causative sentences: one in which both subject and object of the underlying simple sentence are in the Accusative, and one in which the underlying object is in the Accusative, but the underlying subject is in the

Instrumental. We have already shown how the double Accusative type of causative results from case marking of the raised subject of the embedded sentence in the top sentence. The second type of causative is simply the case where the option of *applying the passive inside the embedded sentence* is taken. The subject thereby gets marked with the Instrumental case, and the underlying object becomes the subject. Raising now applies to this surface subject of the passivized embedded sentence, making it the object of the matrix sentence, where it becomes marked with the Accusative case by the usual rule. Otherwise the derivation proceeds exactly as in the double Accusative type.

But this is not all. Pāṇini and the commentators note that certain verbs do not allow the *kartṛ* to appear in the Instrumental case, and thus only allow the double Accusative construction. These are verbs of going and knowing, certain verbs of absorbing, and verbs having as their object a noise or literary work (e.g. verbs of listening or reciting). As it stands in Pāṇini, this is a remarkably specific but inexplicable observation. In terms of our transformational derivation of causatives, however, this restriction translates into the following form: the passive cannot apply inside the embedded sentence in the causative construction if the verb in the embedded sentence is one of going, knowing, etc. Thus, the sentence *gacchati vipro<sub>1</sub> grāmam<sub>2</sub>* ‘the brahman goes to the village’ can be passivized into *grāmo<sub>1</sub> gamyate vipreṇa<sub>3</sub>*, when it is a main clause, but not when it is the sentential object of the verb of causation. But note that the English causative has a very similar restriction. Both the active *I had John carry the piano downstairs* and the passive *I had the piano carried downstairs by John* are grammatical sentences. But with verbs of learning, going, and listening the passive cannot apply in the causative:

- I had John learn the Morse code.
- \*I had the Morse code learned by John.
- I had John approach the village.
- \*I had the village approached by John.
- I had John listen to the concert.
- \*I had the concert listened to by John.

in spite of the fact that the corresponding simple sentences are passivizable:

- The Morse code was learned by John.
- The village was approached by John.
- The concert was listened to by John.

But we can go beyond merely noting this correspondence between English and Sanskrit. The class of verbs which are not eligible to passivization under

these circumstances is close to, if not identical with, a *semantic class*: those denoting actions by which the object is not affected, in an evident sense. The class can be identified by the *do to-test*:

- What I did to the piano was bring it down.
- \*What I did to the Morse code was learn it.
- \*What I did to the village was approach it.
- \*What I did to the concert was listen to it.

E. Klima (personal communication) has pointed out that passivization is restricted to affected objects in the case of prepositional phrases even in main clauses in English, as in his examples

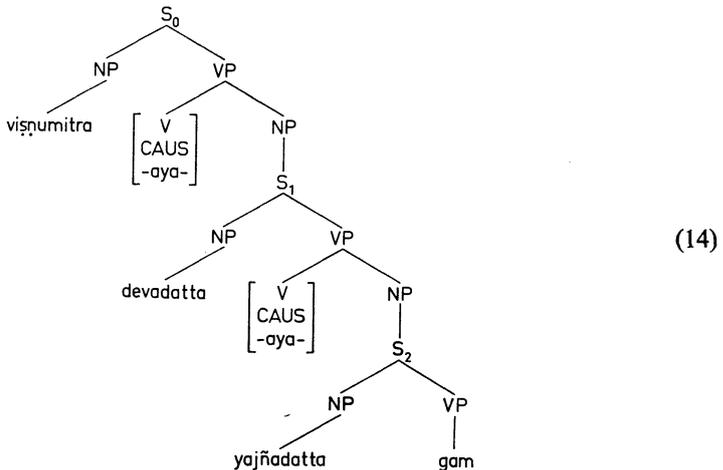
- The bed was slept in by John.
- \*England was slept in by John.

corresponding to

- What John did to the bed was sleep in it.
- \*What John did to England was sleep in it.

This constitutes further support for the plausibility of our explanation.

The passive is of course also applicable on the second cycle to the top sentence, and these processes can be iterated. Consider the more complicated deep structure:



If  $S_1$  were not embedded in another sentence, it would come out as *devadatto<sub>1</sub> yajñadatta<sub>2</sub> gamayati* ‘Devadatta makes Yajñadatta go’. This sentence could be passivized into *yajñadatto<sub>1</sub> devadattena<sub>3</sub> gamyate*. Embedding this

passivized sentence into  $S_0$  we get, correctly, *viṣṇumitro<sub>1</sub> yajñadattaṃ<sub>2</sub> devadattena<sub>3</sub> gamayati* 'Viṣṇumitra makes Devadatta make Yajñadatta go'. Theoretically the passive could apply again to give *yajñadatto<sub>1</sub> viṣṇumitreṇa<sub>3</sub> devadattena<sub>3</sub> gamyate*, a sentence which should be ambiguous and which should also mean 'Devadatta makes Viṣṇumitra make Yajñadatta go'. Such elaborate examples do not appear to be attested, however.

In these complex recursive syntactic processes Pāṇini's syntactic theory has finally shown its limitations. We have discussed the causative at some length in order to show that this class of constructions, which unlike most of the other phenomena treated by Pāṇini's *kāraka* theory transcends the boundaries of the simple sentence, can be analyzed in the framework of transformational grammar far better than within a theory of the general form represented by Pāṇini's grammar. It is a comment on Pāṇini's greatness that the Sanskrit facts which provide the evidence for this deeper analysis come from the very observations which are so carefully recorded in his grammar.

### III

Comparing Pāṇini's description of these grammatical phenomena with ours, we note two essential differences. First, the kind of rule ordering used by Pāṇini is more limited. In one set of cases, the ordering groups together rules which are partly alike in form. The purpose here is to enable the rules to be made as short as possible by factoring out their common part. Rules so abbreviated must be expanded to their full form by the principles of *anuvṛtti* before they can be applied. The other type of ordering encountered in Pāṇini's syntactic rules is the disjunctive ordering imposed according to sūtra 2.3.1 (*anabhihite*) on rules which specify how a certain term at level  $n$  can be realized variously at level  $n + 1$ . This type of ordering serves to express notions like 'otherwise', 'in all other cases', 'alternatively', and so on. Each of these two types of ordering is designed to reflect certain *formal* relationships within a set of rules. But what does not seem to occur in Pāṇini's syntax is ordering which reflects *functional* relationships between grammatical processes. But this is the main purpose of ordering rules in a transformational grammar. For example, the causative and the reflexive are formally quite different grammatical processes, but they are functionally related in that the causative transformation creates structures to which the reflexive rule is applicable and must in fact apply. This functional relation is expressed by ordering the causative rule before the reflexive rule. By doing so we can derive all reflexive pronouns from noun phrases identical with another noun phrase within the same simplex sentence, regardless of whether they were in the same simplex sentence already in the deep structure or came to stand in the

same sentence only through the operation of the causative rule. If the rules are not ordered, two reflexive rules are necessary, which we in fact find in Pāṇini.

The second main difference between the two descriptions is in the use made of constituent structure. In Pāṇini's analysis of the causative, passive, and reflexive constructions, constituent structure plays no significant role. This becomes clear when we compare this treatment with the analysis we have proposed. This analysis draws prominently on there being an embedded sentence in the deep structure of causatives, which can undergo transformational processes characteristic of sentences (notably passivization), and on this sentence being dissolved in the matrix sentence by the causative transformation, with its constituents belonging henceforth to the matrix sentence and being treated as parts of the matrix sentence by subsequently applying transformations (notably the reflexive). For Pāṇini, on the other hand, the constituents of a sentence are all on a par, with no further hierarchical structure.

From the viewpoint of a characterization of Pāṇini's syntactic theory, however, the essential question is not how Pāṇini treated these particular syntactic processes, but what descriptive devices he could in general use. We must, in other words, determine whether the fact that Pāṇini does not use functional ordering and constituent structure in the example at hand is an accidental feature of this example or a consequence of the syntactic theory implicit in the *Aṣṭādhyāyī*. This question can be definitively answered only on the basis of more material than the present study deals with. Still, Pāṇini's treatment of the *kāraḥ* is extensive enough, and covers a sufficiently substantial amount of Sanskrit grammar, for some tentative general conclusions to be drawn.

It seems that the general form of Pāṇini's syntactic rules and levels is in fact rather different from those of a generative grammar. In the theory of Chomsky (1965), deep structures and surface structures have the same formal properties. Both are represented by trees with labeled nodes and ordered branches. The set of deep structure categories (Sentence, Noun Phrase, Verb Phrase, etc.) is a subset (perhaps a proper subset) of the set of surface structure categories. For Pāṇini, on the other hand, each category belongs to one and only one level. That is, the levels are represented by disjoint sets of categories, and there are no categories which do not belong to one of the levels. Moreover, the syntactic rules (that is, the rules relating levels (1) and (2), and the rules relating levels (2) and (3)) appear to be constrained in the following way: if

A → B on the condition that C

(15)

is a syntactic rule, where A is a category belonging to level  $n$ , then

- (i) B must belong to level  $n+1$ , and
- (ii) C is a condition involving categories of levels  $n$  or  $n+1$ .

Thus, each syntactic rule maps some part of the structure at level  $n$  directly into the corresponding structure at level  $n+1$ , and never into any intermediate structure to which further syntactic rules must still apply before the next level is reached. The condition C may involve either of the two levels which the rule relates. For example, the rules relating semantic representations (level (1)) to *kāraḥas* (level (2)) may be limited by conditions having to do with level (1) or level (2). Thus, rule 1.4.25 states that the cause of fear (level (1)) is *apādāna* (level (2)) with verbs of fearing (level (1)). Rule 1.4.49 states that the thing most desired (level (1)) by the kartr (level (2)) is *karman* (level (2)). The condition C may either be internal to the sentence, as in these cases, or it may involve a reference to a related sentence, as in the causative example cited earlier.

If this is the right interpretation, then Pāṇini's syntactic rules, with respect to certain of their formal properties, are closer to *realization rules* of the kind encountered in the 'neo-Firthian' syntax of Halliday and the stratificational grammar of Lamb than they are to transformations. In other ways, however, Pāṇini's rules show some transformational properties, as for example the necessity, in the application of some of the rules, of referring to grammatical features of sentences related to the one under consideration. In any case, it would seem that Pāṇini's syntax differs sharply from his phonology in several important respects. Neither functional ordering nor constituent structure are utilized in the syntax. In the phonology, on the other hand, Pāṇini uses extensive functional ordering, and a notion of constituent structure within the word is implicit in the distinction of primary and secondary suffixes and in the role played by the *pada*.

#### IV

Pāṇini's grammar is an explicit set of rules for deriving the phonetic form of sentences from their semantic form via two intermediate stages which bear significant similarities to the deep and surface structure levels of generative grammar. The semantic form of sentences is characterized mainly in terms of a set of semantic relations between the meanings of the verb and its various adjuncts in the sentence. To a large extent these relations remain sketchy, and where Pāṇini goes into detail it appears to be more for illustration than for exhaustiveness. It does not appear that Pāṇini intended to generate semantic representations. To the extent that the semantic level is developed and its categories specified, it is taken for granted, and no attempt

is made to characterize the set of possible semantic representations by any rules. For these reasons, it is hardly possible to attribute to Pāṇini a fully developed theory of semantics in the sense in which he clearly can be said to have a theory of phonology.<sup>10</sup>

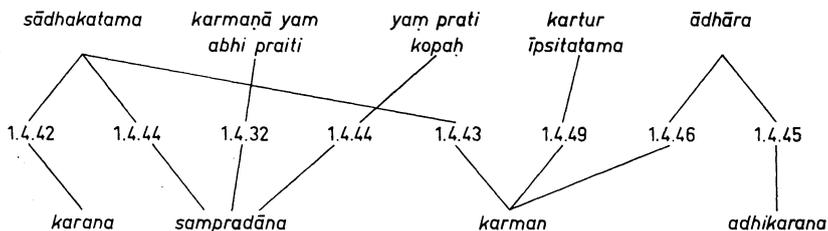
Nothing has been said here so far about the role of the lexicon in Pāṇini's linguistic theory. Associated with the grammar there is a lexicon, the *Dhātupāṭha*, which has among its functions the grouping of roots into classes on the basis of the applicability of various grammatical rules. In any case it is an integral part of the grammatical description furnished by Pāṇini (which has nothing to do with the fact that part of the *Dhātupāṭha*, as we know it, may antedate Pāṇini). Some formal peculiarities of the *Dhātupāṭha* suggest that one might think of it as having a function in the mapping of the semantic level into the level of deep structure. Each root listed in the lexicon is defined by an abstract noun in the locative case, e.g. *i gatau* (II, 36). The root *i* means 'go' and *gatau* is the locative of *gati* 'going'. The Locative is the case in which the environment before which an operation takes place is specified in the grammar (1.1.66). But the Locative, more generally, indicates the circumstances under which something takes place (cf. Rocher in *JAOS* 87 (1967), 584) and may thus specify an environment regardless of order. This might indicate that the lexical entries of the *Dhātupāṭha* should be viewed as a set of lexical insertion rules intended to apply in the semantic environment specified by the gloss given in the Locative case. The glosses would thus also belong to the metalanguage for semantic description. This interpretation would explain the fact that synonymous roots tend to get the same gloss, e.g. *i gatau* (II, 36), *vraj gatau* (I, 272), *dram gatau* (I, 494). The practice is carried through even where it leads, as often happens, to glossing a word with its own derivative, e.g. *gam gatau* (I, 1031). This would be utterly pointless unless the glosses were regarded as metalinguistic terms and the lexicon was considered as formally belonging with the grammatical system. We may in fact plausibly regard it as part of the mechanism for relating levels (1) and (2), i.e., what we have been terming semantics and deep structure.<sup>10a</sup>

<sup>10</sup> It is at least possible to assume that Pāṇini would have left questions about possible semantic representations to *logic*. It is true that we know little about the development of logic during Pāṇini's time. But there are interesting similarities between the semantic terms he refers to and technical terms in the later systems of logic. We have seen that Pāṇini's semantic categories include *īpsitatama*, *īpsita* and *sādhakatama*, but not *sādhaka*. In Indian logic *sādhaka* is a frequently used synonym of *hetu*, which denotes the reason or antecedent. Similarly, *sādhya* denotes the thing inferred or consequent. The technical term for 'locus' (a very fundamental concept in Indian logic) is generally *adhikaraṇa*, and sometimes *ādhāra*. Such connections could be further pursued, but go beyond the limits of the present article.

<sup>10a</sup> If it is true that the glosses were added later to the text of the *Dhātupāṭha* (which seems likely: B. Liebich, *Kṣīrataraṅgīnī*, Breslau 1930, 244) and were not implicitly un-

There is no justification for supposing that the semantic level, let alone the level of *kāraḥas*, has any kind of ontological significance, as Rocher (1964a, b) has come close to suggesting. Direct reference to the outside world is outside the scope of linguistics. Its place is taken by reference which the speaker intends or supposes to obtain. If this would not already follow from the status of the science itself, it could be clearly established from linguistic usage, e.g. from the use of anaphoric pronouns.<sup>11</sup> Similarly, a noun which occurs in the Instrumental case may express *karāḥa* or *hetu*. But this does not imply that this noun refers to what is really an instrument or cause in the outside world; it merely signifies that the speaker supposes or intends it to be such. In the passage quoted from the *Rāghuvamśa*, Kalidāsa wishes it to be understood that king Dilipa was astonished when he heard the lion speak with a human voice. In reality things may be different.<sup>12</sup>

That the semantic level is distinct from the deep structure or *kāraḥa* level in Pāṇini's grammar has been one of the points we have tried to establish in the above discussion. We illustrate the point again by diagramming a few of the relationships that obtain between these two levels.



(16)

It must be admitted that the distinction between these two levels is often unclear even in the Indian commentators on Pāṇini. The commentary of the *Kāśikā* on 3.2.1, however, recognizes the distinction and its importance very clearly. It enumerates three semantic relations which the direct object relation corresponds to: *nirvartya* 'product' (e.g. *kumbhaḥ<sub>2</sub> karoti* 'he makes a pot'), *vikārya* 'thing affected' (e.g. *kāṇḍān<sub>2</sub> lunāti* 'he cuts stalks'), *prāpya* 'thing reached' (e.g. *vedam<sub>2</sub> adhyeti* 'he studies the Veda'). There now follows the

derstood at the time of Pāṇini, the above argument may be valid only with regard to the situation after Bhimasena.

<sup>11</sup> E.g., if Lindsay is mayor of New York, a speaker not knowing this will say 'I met Lindsay and later phoned the mayor of New York'. On the other hand, he will say 'I met Lindsay and later phoned him' if he supposes both to be the same person, even if this is not in fact the case.

<sup>12</sup> Further evidence in this respect is supplied by Patañjali in a passage which briefly touches upon the relation between *kāraḥa* and the ontological categories: see Staal (1967), 44.

significant remark that *each one* of these different types of *karman* can form a certain type of compound with the suffix *aṅ*, e.g. *kumbhakāraḥ* 'one who makes pots', *kāṇḍalāvaḥ* 'one who cuts stalks', *vedādhyāyaḥ* 'one who studies the Veda'. The implication is that the *kāraka* level plays a functional role in the system and could not be eliminated without complicating the description. If semantic representations were mapped directly into surface structures, without the mediating *kārakas*, the list of surface structures to which *karman* can correspond, namely the compounds with *aṅ*, the Accusative (as the object case), the Objective Genitive, and so on, would have to be enumerated anew for each semantic relation which can underlie a *karman* relation. It is in such cases that the *kāraka* level enables generalizations to be made, and they are presumably the motivation for introducing this level. It is not clear, however, that such motivation can be given for every one of the *kārakas* of Pāṇini's grammar.

The distinction between *kārakas* and morphological categories has been somewhat better appreciated. It was put concisely in a *vārttikā* of Kātyāyana under 2.3.1 which enumerates the kinds of expressions that may represent *kāraka* relations: *tiṅ* (verbal terminations), *kṛt* (a class of primary suffixes), *taddhita* (a class of secondary suffixes) and *samāsa* (nominal compounds). Examples of such forms have occurred throughout this study (for further discussion see Staal, 1967, 41–2). These represent as it were the 'surface' term of the *kāraka* relationships.

It has been said that Pāṇini's method was taxonomic, and his subject largely confined to the analysis of words. But even the cursory analysis given here of the *kāraka* theory suggests not only that such statements are untrue, but also that Pāṇini's treatment of syntax and semantics presents analogies to various aspects of several modern linguistic theories, without being directly identifiable with any of them. The importance of his treatment becomes the more apparent once it is realized that, unlike *kāraka* relationships, which determine the structure and 'construction' of sentences, problems of arrangement and constituent order in the finally derived strings, which play so prominent a role in English syntax, have no deep significance in inflected languages and occupy little space in a syntax of Sanskrit. The *kāraka* relations therefore perform much of the work that needs to be done in Sanskrit syntax.

#### APPENDIX

In order not to burden the exposition unduly, the sūtras referred to in the course of this paper have not been quoted in the original. This loss is here repaired, most references through *anuvṛtti* to preceding sūtras have been supplied between parentheses, and translations have been added.

- 1.1.66 *tasminn iti nirdiṣṭe pūrvasya* ‘when an element is referred to in the Locative, (the operation takes place) with regard to the preceding’
- 1.3.13 *bhāvakarmanoh (ātmanepadam: 12)* ‘(the middle endings are used) when the verb expresses a state [i.e., the impersonal] or an object [i.e., the passive]’
- 1.3.14 *kartari karmavyatihāre (ātmanepadam: 12)* ‘(the middle endings are used) when the verb expresses an agent [i.e., the active] if the activity is reciprocal’
- 1.3.67 *ner añau yat karma nau cet sa kartānādhyāne (ātmanepadam: 12)* ‘(the middle endings are used) when the verb has the affix *NI* [i.e., the causative], if that which is the deep object when the verb is without *NI*, is the subject when it is provided with *NI*, excepting verbs of remembering’
- 1.3.68 *bhīsmiyor hetubhaye (ātmanepadam: 12; ner: 67)* ‘(the middle endings are used when the verb has the affix *NI*) for *bhi* “fear” and *smi* “be astonished” provided the *hetu* is the cause of fear’
- 1.3.78 *śeṣāt kartari parasmaipadam* ‘the active endings are used after all remaining verbal roots when a subject is expressed’
- 1.4.2 *vipratīṣedhe paraṃ kāryam* ‘in case of contradiction (between two rules) the latter has to be applied’
- 1.4.23 *kārake* ‘(the following rules apply) when a *kāraka* is expressed’

N.B.: This rule is valid through 1.4.55, but has not been repeated in the following translations.

- 1.4.24 *dhruvam apāye’pādānam (kārake: 23)* ‘*apādāna* is the fixed point from which something recedes’
- 1.4.25 *bhītrārthānām bhayahetuḥ (kārake: 23; apādāna: 24)* ‘the cause of fear (is *apādāna*) with verbs meaning “fear” and “protect”’
- 1.4.27 *vāraṅārthānām īpsitaḥ (kārake: 23; apādāna: 24)* ‘the desired object (is *apādāna*) with verbs meaning “ward off”’
- 1.4.32 *karmanā yam abhi praiti sa saṃpradānam (kārake: 23)* ‘whom one has in mind in connection with an object is *saṃpradāna*’
- 1.4.33 *rucyarthānām priyamāṇaḥ (kārake: 23; saṃpradāna: 32)* ‘the person pleased (is *saṃpradāna*) with verbs meaning “please”’
- 1.4.36 *spr̥her īpsitaḥ (kārake: 23; saṃpradāna: 32)* ‘the desired object (is *saṃpradāna*) with the verb *spr̥h* “want”’
- 1.4.37 *krudhadruherśyāsūyārthānām yaṃ prati kopaḥ (kārake: 23; saṃpradāna: 32)* ‘the person against whom anger is directed (is *saṃpradāna*) with verbs meaning “be angry”, “hate”, “be jealous”, “be annoyed with”’

- 1.4.42 *sādhakatamaṃ karaṇam* (kārake: 23) ‘*karaṇa* is the most effective means’
- 1.4.43 *dīvaḥ karma ca* (kārake: 23; *sādhakatama*: 42) ‘also the deep object with the verb *div* “play” (is the most effective means)’
- 1.4.44 *parikrayaṇe saṃpradānam anyatarasyām* (kārake: 23; *sādhakatama*: 42) ‘(the most effective means) may be *saṃpradāna* in connection with renting’
- 1.4.45 *ādhāro’ dhikaraṇam* (kārake: 23) ‘*adhikaraṇa* is the locus’
- 1.4.46 *adhiśīṅsthāsām karma* (kārake: 23; *ādhāra*: 45) ‘(the locus is) *karman* with the verbs *śīṅ* “lie”, *sthā* “stand”, *ās* “sit”, when preceded by *adhi*’
- 1.4.47 *abhinivīśaś ca* (kārake: 23; *ādhāra*: 45; *karman*: 46) ‘and (the locus is *karman*) with the verb *viś* “enter” when preceded by *abhi ni*’
- 1.4.48 *upānvadhyānvasaḥ* (kārake: 23; *ādhāra*: 45; *karman*: 46) ‘(the locus is *karman*) with the verb *vas* “inhabit” when preceded by *upa, anu, adhi, āṅ*’
- 1.4.49 *kartur ipsitatamaṃ karma* (kārake: 23) ‘*karman* is the thing most desired by the deep subject’
- 1.4.52 *gatibuddhipratyavasānārthaśabdakarmākarmakāṇām aṅikartā sa ṇau* (kārake: 23; *karman*: 49) ‘when verbs of going, knowing, absorbing, verbs having as their object a sound or intransitive verbs have the affix *ṆI* [i.e., the causative affix], the subject of the verb when without the affix *ṆI*, is *karman*’
- 1.4.54 *svatantraḥ kartā* (kārake: 23) ‘*karṭṛ* is the independent’
- 1.4.55 *tatprayojako hetuś ca* (kārake: 23; *karṭṛ*: 54) ‘that which prompts the *karṭṛ* is also *hetu*’
- 2.1.24 *dvitiyā śrītātītapatitagaṭātīyastaprāptāpannaiḥ* (*samāsa*: 3; *saha supā*: 4; *tatpuruṣa*: 22) ‘a noun with an Accusative ending (forms a *tatpuruṣa* compound) with the nouns *śrīta* “undergoing”, *atīta* “passed”, *patīta* “fallen”, *gata* “gone”, *atyasta* “gone beyond”, *prāpta* “reached”, *āpanna* “arrived at”’
- 2.1.32 *karṭṛkaraṇe kṛtā bahulam* (*samāsa*: 3; *saha supā*: 4; *tatpuruṣa*: 22) ‘the deep subject and the instrumental often (form a *tatpuruṣa* compound) with a noun ending in a *KṚT* suffix’
- 2.1.36 *caturthī tadarthārthabalihitasukharakṣitaiḥ* (*samāsa*: 3; *saha supā*: 4; *tatpuruṣa*: 22) ‘a noun with a Dative ending (forms a *tatpuruṣa* compound) with a noun expressing something for its sake, with *bali* “oblation”, *hita* “good for”, *sukha* “agreeable” and *rakṣita* “guarded”’
- 2.1.37 *pañcamī bhayena* (*samāsa*: 3; *saha supā*: 4; *tatpuruṣa*: 22) ‘a noun with an Ablative ending (forms a *tatpuruṣa* compound) with the noun *bhaya* “fear”’

- 2.1.41 *siddhaśuṣkapakvabandhaiś ca* (*samāsa*: 3; *saha supā*: 4; *tatpuruṣa*: 22; *saptamī*: 40) ‘(a noun with a Locative ending forms a *tatpuruṣa* compound) also with *siddha* “established”, *śuṣka* “dry”, *pakva* “cooked”, *bandha* “tied”’
- 2.2.13 *adhikaraṇavācinā ca* (*samāsa*: 2.1.3; *saha supā*: 2.1.4; *tatpuruṣa*: 2.1.22; *ṣaṣṭhī*: 2.2.8; *na*: 2.2.10; *ktena*: 2.2.12) ‘and (a noun with a Genitive ending does not form a *tatpuruṣa* compound) with (a participle ending in *Kta*) denoting the locus’
- 2.3.1 *anabhihite* ‘(the following rules apply) when it [i.e., the *kāraka*] is not (already) expressed’

N.B.: This rule is valid through 2.3.71, but has not been repeated in the following translations.

- 2.3.2 *karmaṇi dvitīyā* (*anabhihite*: 1) ‘the Accusative [i.e., the endings of the Accusative] is used when a *karman* is expressed’
- 2.3.3 *ṛtīyā ca hoś chandasi* (*anabhihite*: 1; *karmaṇi dvitīyā*: 2) ‘the Instrumental is also used (for the *karman*) of *hu* “make an oblation” in the Veda’
- 2.3.7 *saptamīpañcamyau kārakamadhye* (*anabhihite*: 1) ‘the Locative or the Ablative are used when there is an interval between two *kāraḥ*’
- 2.3.11 *pratīnidhipratidāne ca yasmāt* (*anabhihite*: 1; *karmapravacanīyayukte*: 8; *pañcamī*: 10) ‘and (the Ablative is used together with a preposition) for that for which something is substituted or exchanged’
- 2.3.12 *gatyarthakarmaṇi dvitīyācaturthyau ceṣṭāyām anadhvani* (*anabhihite*: 1) ‘the Accusative or the Dative are used for the deep object of verbs of going, provided there is real movement and excepting *adhvan* “path”’
- 2.3.13 *caturthī sampradāne* (*anabhihite*: 1) ‘the Dative is used when a *sampradāna* is expressed’
- 2.3.14 *kriyārthopapadasya ca karmaṇi sthāninaḥ* (*anabhihite*: 1; *caturthī*: 13) ‘(the Dative is used) also for the object of an underlying verb accompanied by another word with verbal connotation [i.e., the infinitive]’<sup>13</sup>
- 2.3.15 *tumarthāc ca bhāvavacanāt* (*anabhihite*: 1; *caturthī*: 13) ‘(the Dative is used) also for a word expressing a state and synonymous with an infinitive’
- 2.3.18 *kartṛkaraṇayos ṛtīyā* (*anabhihite*: 1) ‘the Instrumental is used when *kartṛ* or *karaṇa* are expressed’

<sup>13</sup> Literally, ‘the endings of the Dative are used after a word ...’

- 2.3.22 *saṃjñō'nyatarasyām karmaṇi* (*anabhihite*: 1; *karaṇa*: 18) '(the Instrumental is used) optionally for the deep object of *saṃjñā* "recognize"'
- 2.3.23 *hetau* (*anabhihite*: 1; *karaṇa*: 18) '(the Instrumental is used) for a cause'
- 2.3.28 *apādāne pañcamī* (*anabhihite*: 1) 'the Ablative is used when an *apādāna* is expressed'
- 2.3.33 *karaṇe ca stokālpakṛcchrakatipayasyāsattvavacanasya* (*anabhihite*: 1; *pañcamī*: 28; *ṛṭiyā*: 32) '(the Instrumental) as well as (the Ablative are used) when a *karaṇa* is expressed with *stoka* "drop", *alpa* "a little bit", *kṛcchra* "difficult", *katipaya* "some", unless a substance is designated'
- 2.3.36 *saptamī adhikaraṇe ca* (*anabhihite*: 1; *dūrāntikārthebhyah*: 35) 'the Locative is used (for words meaning "far" and "near") also when a locus is expressed'
- 2.3.37 *yasya ca bhāvena bhāvalakṣaṇam* (*anabhihite*: 1; *saptamī*: 36) '(the Locative is used) also for that state which characterizes a(nother) state'
- 2.3.38 *ṣaṣṭhī cānādare* (*anabhihite*: 1; *yasya bhāvena bhāvalakṣaṇam*: 37) 'the Genitive is used also (for a state which characterizes another state) when disregard is expressed'
- 2.3.46 *prātipadikārthalingaparimāṇavacanamātre prathamā* (*anabhihite*: 1) 'the Nominative is used when nothing but the gender and the number of the nominal stem notion are expressed'
- 2.3.52 *adhigarthadayeśāṃ karmaṇi* (*anabhihite*: 1; *ṣaṣṭhī*: 50) '(the Genitive is used) for the deep object of verbs meaning remembering, and of *day* "allot" and *īś* "own"'
- 2.3.53 *kṛñāḥ pratiyatne* (*anabhihite*: 1; *ṣaṣṭhī*: 50; *karmaṇi*: 52) '(the Genitive is used for the deep object) of *kṛñ* "do" when it imparts a new quality'
- 2.3.54 *rujārthānām bhāvavacanānām ajvareḥ* (*anabhihite*: 1; *ṣaṣṭhī*: 50; *karmaṇi*: 52) '(the Genitive is used for the deep object) of verbs meaning *ruj* "afflict" when expressing a state, excepting *jvar* "suffer"'
- 2.3.63 *yajeś ca karaṇe* (*anabhihite*: 1; *ṣaṣṭhī*: 50; *bahulaṃ chandasi*: 62) '(the Genitive is often used in the Veda) for a *karaṇa* in connection with *yaj* "sacrifice"'
- 2.3.65 *kartṛkarmaṇoḥ kṛti* (*anabhihite*: 1; *ṣaṣṭhī*: 50) '(the Genitive is used) for the deep object and the deep subject of nouns ending in a *KṚT* suffix'
- 2.3.66 *ubhayaprāptau karmaṇi* (*anabhihite*: 1; *ṣaṣṭhī*: 50) '(the Genitive is used) for the deep object when there is scope for both (the deep subject and object) to be realized'

- 2.3.71 *kṛtyānāṃ kartari vā* (*anabhihite*: 1; *ṛtīyā*: 18; *ṣaṣṭhi*: 50) ‘(the Genitive or the Instrumental) may be used for the deep subject of nouns ending in a *KṚTYA* suffix [i.e., the future passive participle]’
- 3.1.26 *hetumati ca* (*ṇic*: 25) ‘(the affix *ṆIC* is attached to a verbal root) also when (the activity is) caused by someone else’
- 3.1.67 *sārvadhātuke yak* (*bhāvakarmanoh*: 66) ‘the affix *yaK* is attached before a *sārvadhātuka* suffix when the verb expresses a state [i.e., the impersonal] or an object [i.e., the passive]’
- 3.1.68 *kartari śap* (*sārvādhātuke*: 67) ‘the affix *ŚaP* is attached (before a *sārvādhātuka* suffix) when the verb expresses a subject’
- 3.1.102 *vahyaṃ karaṇam* ‘the noun *vahyam* expresses a *karaṇa*’
- 3.2.1 *karmaṇy aṇ* ‘the suffix *aṆ* is attached when there is a deep object’
- 3.2.85 *karāṇe yajaḥ* (*ṇini*: 78; *bhūte*: 84) ‘(the suffix *ṆinI* is attached) to *yaj* “sacrifice” with past meaning when the accompanying noun expresses a *karaṇa*’
- 3.2.98 *pañcamyām ajātau* (*janer ḍaḥ*: 97) ‘(the suffix *Ḍa* is attached to *jan* “be born”) in the meaning of the ablative provided it does not denote a species’
- 3.2.182 *dāmnīśasayuyujastutudasisicamihapatadaśanahaḥ karāṇe* (*ṣṭraṇ*: 181) ‘(the suffix *ṢṭraṆ* is attached) to *dāP* “cut”, *nī* “guide”, *śas* “slaughter”, *yu* “join”, *yuj* “join”, *stu* “praise”, *tud* “strike”, *si* “bind”, *sic* “sprinkle”, *mih* “urinate”, *pat* “fall”, *daś* “bite”, *nah* “tie”, when the *karaṇa* is expressed’
- 3.3.93 *karmaṇy adhikarāṇe ca* (*ghoḥ kiḥ*: 92) ‘(the suffix *Ki* is attached to *dā* “give (etc.)” and *dhā* “place (etc.)”) also if the locus is expressed in combination with a deep object’
- 3.3.117 *karaṇādhikaraṇayoś ca* (*lyuṭ*: 113) ‘(the suffix *LYUṬ* is attached) also when the *karaṇa* or the *adhikaraṇa* are expressed’
- 3.3.156 *hetuhetumator liṅ* ‘verbal forms ending in *LIN* [i.e., the potential mood] are used to express the *hetu* and what follows from it’
- 3.4.67 *kartari kṛt* ‘a *KṚT* suffix is used to express the deep subject’
- 3.4.70 *tayor eva kṛtyaktakhalarthāḥ* (*karmaṇi bhāve cākarmakebhyaḥ*: 69) ‘*KṚTYA* suffixes, the suffix *Kta* and those meaning *KHAL* are used to express both [i.e., the deep object and the impersonal with intransitive verbs]’
- 3.4.73 *dāśagoghnau saṃpradāne* ‘the nouns *dāśa* and *goghna* express *saṃpradāna*’
- 4.1.2 *svaujasamauṭchaṣṭābhyāmbhisṇebhyāmbhyasṇasibhyāmbhyasṇaso-sāmnyossup* (*nyāpprātipadikāt*: 1) ‘(after feminine stems ending in *Ṇī* or *āP*, or nominal stems) *sU*, *au*, *Jas*, *am*, *auṬ*, *Śas*, *Ṭā*, *bhyām*, *bhis*, *Ṇe*, *bhyām*, *bhyas*, *ṆasI*, *bhyām*, *bhyas*, *Ṇas*, *os*, *ām*, *Ṇi*, *os*, *suP*’

- 6.1.57 *nityaṃ smayateḥ* (ādecaḥ: 45; ṇau: 54; hetubhaye: 56) ‘(āT is substituted for e, o, ai, au in causatives when the fear – [in this case: the surprise] – is caused by the *hetu*) obligatorily in the case of *smi* “be astonished”’
- 7.1.9 *ato bhisa ais* (aṅgasya: 6.4.1) ‘after (nominal stems ending in) aT, *ais* is substituted in the place of *bhis*’
- 7.3.36 *artihrivliriknūyikṣmāyyātāṃ pug ṇau* ‘the infix pUK is inserted before *ṆI* [i.e., the causative] after *r* “go”, *hrī* “be ashamed”, *vli* “crush”, *rī* “flow”, *knūy* “be humid”, *kṣmāy* “tremble”, and after roots ending in *ā*’
- 7.3.40 *bhiyo hetubhaye ṣuk* (ṇau: 36) ‘the infix ṣUK is inserted (before *ṆI*) after *bhī* “fear” when fear is caused by the *hetu*’

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*Part Three*

Reviews



D. H. H. Ingalls, *Materials for the study of Navya-Nyāya Logic* (= *Harvard Oriental Series*, 40), Cambridge, Mass., 1951.

Though this book was published eight years ago and has been reviewed several times,<sup>1</sup> its importance seems to justify a reassessment of its significance, which takes into account some further investigations it has given rise to and which seeks to indicate in which directions further developments may be expected.

Although *ex nihilo nihil fit* it may be said that Professor Ingalls' book is a pioneering work both in respect of its object and in respect of its method. Its object is Navya-Nyāya, the modern school of Indian logic which developed in Mithilā and Bengal from Gaṅgeśa's *Tattvacintāmaṇi* (XIIIth century) onwards and to which many logicians belonged, of whom Raghunātha Śīromaṇi (ca. 1475-ca. 1550) may especially be mentioned. Its method is inspired by the techniques of contemporary symbolic logic, with the help of which the author has been able to give a clear account of several of the extremely complicated logical structures with which the Navya-Naiyāyikas manipulate. In both these respects his only predecessor was Sailesvar Sen, whose thesis *A study on Mathurānātha's-Tattva-Cintāmaṇi-Rahasya* (Wageningen, 1924), written when the author was working with Faddegon in Amsterdam, has unfortunately never been given the attention it deserved. Apart from this work, from which Ingalls acknowledges he has derived most help, the same field had never been studied with the same method. Earlier translations of Navya-Nyāya texts exist, but some of these are hardly less obscure than the originals. The methods of modern logic seem to have been referred to only by S. Schayer in the field of Buddhist logic (from 1932 onwards), but Ingalls' bibliography does not mention any of Schayer's articles.

The author has edited, translated and interpreted selections of texts and has devoted more than fifty concise pages to "an examination of some theories and techniques of Navya-Nyāya logic", which will prove extremely useful not only to logicians who wish to inform themselves about Indian logic of the modern school, but also to Sanskrit scholars who want to study another text from the large field of Navya-Nyāya literature. This section constitutes the second part of the book. It is preceded by "Biographical Notes" dealing with Gaṅgeśopādhyāya, Jayadeva Paṅśadhara (ca. 1425-ca. 1500), Raghunātha Śīromaṇi and Mathurānātha Tarkavāgiśa (ca. 1600-ca. 1675), where considerable attention is paid to - sometimes amusing - local traditions which are utilised with great prudence. The discussion of theories and techniques is followed by editions, translations and interpretations of three texts. The first is the one paragraph from Gaṅgeśa, known as Vyāpti-pañcaka, which enumerates the five provisional definitions of *vyāpti* and the general reason for their invalidity (they are all rejected and later replaced by the *siddhānta-lakṣaṇa*, the final definition, not dealt with in Ingalls' work). The second text is the commentary of Mathurānātha upon these

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<sup>1</sup> I. M. Bocheński in: *The Journal of Symbolic Logic*, 17 (1952), 117-119; J. Brough in: *Journal of the Royal Asiatic Society*, 1953-54, 87-88; J. Filliozat in: *Journal Asiatique* 240 (1952), 409-410; J. Gonda in: *Tijdschrift voor Philosophie*, 13 (1951), 727-8; J. Jordens in: *Revue Philosophique de Louvain*, 50 (1952), 143-4; K. H. Potter in: *Philosophy East and West*, 4 (1954-55), 271-3.

definitions, called the Vyāpti-pañcaka-rahasya. The last is an extract from Raghunātha's Dīdhiti of the section commenting on these same definitions.

The texts are based upon modern Indian editions. In Navya-Nyāya it will take time before the need will be felt for editions based upon manuscripts. First the attempt will have to be made to read and understand the bulk of literature which Indian Naiyāyikas have published (one text sometimes in several editions) during the last century. Moreover, the material is generally not old and the texts with their numerous commentaries and subcommentaries seem to be rarely corrupt. For the Vyāpti-pañcaka-rahasya Ingalls has utilised the modern subcommentary of Śivadatta Miśra. For the Dīdhiti use is made of the classical commentaries of Gadādhara and Jagadīśa (XVIIth century).

The "examination of some theories and techniques" is mainly but not exclusively based upon the material occurring in the texts which are given later. Sometimes the reader is referred to more detailed examples provided by these texts, sometimes the explanations of the latter refer back to the general "examination" for fuller discussions. In this central part Ingalls introduces methods and uses results of modern logic. His investigations are always interesting and often brilliant: for instance the discovery of the modern definition of the disjunction ( $p \vee q \equiv \sim(\sim p \sim q)$ , "a disjunction is equivalent to the negation of the conjunction of the negations") in an apparently obscure and at first sight insignificant Sanskrit expression (*anyatara*, "alternation", is: *bhedadvayāvachinna-pratīyogitāka-bhedavat*, "that which possesses a mutual absence to which the counterpositive, is limited by two mutual absences") (63). Nowhere, on the other hand, does the author seem to go too far in this direction by reading modern ideas into the text. See for instance the scrutiny with which the problem of the possible occurrence of De Morgan's law is considered: the text seems to justify fully the author's careful statements, that Gaṅgeśa, though he applied the principle involved, was probably unconscious of it, whereas Mathurānātha seems to have been aware of it (66). It may be asked whether perhaps the above quoted definition for *anyatara* does not show that its author (Tārānātha Tarka Vācaspati) was aware at least of one of the two laws of De Morgan.

Ingalls' highly interesting applications of the methods of modern logic give rise to questions regarding the role played by the language of the Sanskrit expressions: do the expressions of formal logic represent and correspond to the *logical structures* of Navya-Nyāya, or also to the *linguistic expressions* of these structures? How can the two be distinguished (at first sight the former seems accessible only through the latter)? Does the tool, the symbolism of modern logic itself, reflect the structure of modern language(s)? An example may illustrate these problems, for which the definition of disjunction quoted above may be referred to again. Johannes Versor, commenting upon Petrus Hispanus' *Summulae Logicales*, expressed the two laws of De Morgan as: *copulativa et disiunctiva de partibus contradicentibus contradicunt*, "conjunctions and disjunctions with contradictory members are contradictions".<sup>2</sup> By utilising the term "contradictories" the Latin expression denotes not only what modern logic expresses by: "a and non-a", but also: "non-a and a". Hence it achieves much greater concision and corresponds to four expressions in symbolic logic:  $((p \wedge q) \equiv \sim(\sim p \vee \sim q))$ ;  $(p \vee q) \equiv \sim(\sim p \wedge \sim q)$ ;  $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$ ; and  $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$ .<sup>3</sup> The general structure of each of these four expressions however, disregarding the negations, is similar to the Latin expression: *copulativum de ...* corresponds to: ... $\wedge$ ... and *disiunctivum de ...* corresponds to: ... $\vee$ ...

<sup>2</sup> J. Łukasiewicz, "Zur Geschichte der Aussagenlogik", *Erkenntnis*, 5 (1935), 111-131, note 40. See, however, P. Boehner, "Bemerkungen zur Geschichte der De Morgansche Gesetze in der Scholastik", *Archiv für Philosophie*, 4 (1951), 113-146.

<sup>3</sup> This remarkable fact is not mentioned by Łukasiewicz, who says that this idea "hier etwas zu knapp ausgedrückt ist".

In the Sanskrit expression *anyatara* corresponds to: ...v... and *bheda* corresponds to:  $\sim$ . Professor Brough suggested further that, in some sense, *pratiyogi* corresponds to the brackets in  $\sim(\sim p \wedge \sim q)$  and *avacchinna* to (the nature of) the contents of the brackets or to the relation between the brackets and their contents. In these respects the Sanskrit is more explicit than the formal expression. If we say that we realise nevertheless that the Sanskrit expression corresponds to:  $(pvq) \equiv \sim(\sim p \wedge \sim q)$ , we consider a logical structure which is not expressed in the same way in language as in the formula.

Sometimes Ingalls establishes a strict correspondence between Sanskrit expressions and formulae. For instance, *sādhyābhāvavadavṛttitva* (scl. *hetoh*), “non-occurrent, (of h) to the locus of absence of s” (the first definition) is represented by:  $\sim o_1 \perp 1-s_1 \bar{P} h$ , a formula which is inspired by modern logic but introduces new symbols. It is probable that in this direction further applications of methods of symbolic logic may be expected. It is at the same time clear, that Ingalls’ work provides precious material for anyone who wishes to study the functioning of language in the expression of logical structures<sup>4</sup> or who wants to compare the relation between thought and expression in Indian and Western thought.

A few more topics may be mentioned out of the many theories and techniques discussed. – Some of the logical terms seem to be related to grammatical terms and usages. For instance (apart from *vyāpti* itself<sup>5</sup>), *gaurava* and *lāghava* (47) are relatively common in grammar. There is some reminiscence of the terms “strong” and “weak” as applied to systems of axioms in modern logic. More important is the essential identity of “a pot” with “a blue pot”, which has puzzled Potter who in his review has suggested that it is due to some confusion on the part of the Naiyāyikas. Without excluding this as a possibility, it might be suggested that this fact may be related to a general feature of Sanskrit (a linguistic structure, which is neither true nor false), exemplified by the status of the *karmadhāraya* compound. This compound is a *tatpuruṣa* and as such it denotes its second member.<sup>6</sup> In other words, “a blue pot” has in Sanskrit always denoted “a pot”.<sup>7</sup>

Difficult is the subject of double negation. Professor Brough observed in his review that the difficulty of the argument on  $-\dot{\neg}x$  (71) seems to lie “in a confusion on the part of the Sanskrit writer between (c of -a) and (c of  $\dot{\neg}a$ )”. Actually the ambiguity may merely lie in the representation. Where Ingalls states: “c of  $-\dot{\neg}x \equiv c$  of  $\dot{\neg}x$ ”, the validity of the further argument depends on the assumption that both *pratiyogi*’s are of the same type in this line but of different types in the next line. If it be assumed that all the c’s represent only *atyantābhāviya-pratiyogi*,  $c_{at}$ , the argument establishes merely:  $-\dot{\neg}x \equiv c_{at}$  of  $\dot{\neg}x$ . This is what the Sanskrit text says: ... *ghaṭa-bhedāyantābhāva-rūpasya ... ghaṭa-bheda-pratiyogitvāt* (102). Consequently, an x occurs on the first line of the next page (72), where the text has merely *pratiyogi*.

It seems difficult to accept the first argument of the same section, adduced in order to establish  $-\dot{\neg}x \equiv x_1$ , for it seems to be based upon the assumption that potness occurs in all things that are  $-\dot{\neg}$ pots. – From a modern point of view, moreover, in this situation where levels of abstraction are involved the need for a kind of theory of types is felt.

<sup>4</sup> No use is made of Ingalls’ work in: P. Hartmann, *Nominale Ausdrucksformen im wissenschaftlichen Sanskrit* (Heidelberg, 1955).

<sup>5</sup> Cf. L. Renou, *Terminologie grammaticale du Sanskrit* (Paris, 1957), 301–2.

<sup>6</sup> See the present reviewer’s “Correlations between language and logic in Indian thought”, *Bulletin of the School of Oriental and African Studies*, 23 (1960), 109–22.

<sup>7</sup> An attempt to differentiate seems to have been made in the Nyāyavārtika: cf. M. Biarreau, “La définition dans la pensée indienne”, *Journal Asiatique*, 245 (1957), 371–384: 376. For Aristotle cf. *Metaphysica* Δ 29, 1024 b 29–32 and Z 6, 1031 a 19–24.

A formal representation of the difficulties met with in these passages may be suggested here.<sup>8</sup> Use is made of the notation of restricted-variables, expressed by  $\alpha x R(x, y)$  and denoting the idea "x such that R (x, y)", and of familiar symbols of the logic of relations. In addition, A (x, y) denotes: "relation (*sambandha*) of x to y", and B (x, y) denotes: "x possesses y (*x y-vat*)".

*Definition 1. (anyonyābhāva)*  $\dot{-}a =_d \alpha x (x \neq a)$ .

*Definition 2. (atyantābhāva)*  $-a =_d \alpha y \sim A(y, \alpha x B(x, a))$ .

*Statement:*  $--a = a$ .

*Explanation:* According to definition 2,

$--a = \alpha w \sim A(w, \alpha z B(z, \alpha y \sim A(y, \alpha x B(x, a))))$  (1).

It is likely that:  $\sim A(y, \alpha x B(x, a))$  (2)

implies:  $\sim A(a, \alpha x B(x, y))$  (3).

Substitution of  $\alpha y$  (2) for  $y$  in (3) yields:

$\sim A(a, \alpha z B(z, \alpha y \sim A(y, \alpha x B(x, a))))$  (4).

Comparison of (1) and (4) yields:  $--a = a$ .

*Statement:*  $-\dot{-}a = a_1$ .

*Explanation:* According to definitions 1 and 2,

$-\dot{-}a = \alpha z \sim A(z, \alpha x B(x, \alpha y (y \neq a)))$  (5).

It is likely that:  $A(a_1, \alpha x B(x, a))$

implies:  $\sim A(a_1, \alpha x B(x, \alpha y (y \neq a)))$  (6).

Comparison of (5) and (6) yields:  $-\dot{-}a = a_1$ .

*Statement:*  $-\dot{-}a = a$ .

*Explanation:* Analogous reasoning from  $A(a, \alpha x B(x, a))$ .

Ingalls' comparison with intuitionistic logic and the reference to Heyting (68 note 135) are very interesting. It may be asked more generally whether it is justified to say that the realism of Nyāya possesses constructivistic tendencies. The stress laid upon unnegatable (*kevalānvayi*) terms and the elimination of all the negations of the Vyāpti-pañcaka in the final definition is reminiscent of the negationless mathematics and logic of G. F. C. Griss.

Potter has in his review<sup>9</sup> objected against the translation of *guṇa* by "quality" and has suggested "trope". If "quality" is unsatisfactory, it might be suggested to use "quale": just as *nīla*, "blue", inheres in one substance only, whereas *nīlatva*, "blueness", inheres in many, "qualia" might be restricted to one entity, whereas "quality" inheres in many.<sup>10</sup>

It is a characteristic of modern Indian logic that quantification is expressed without quantors but by means of abstracts and concepts such as *avacchinna*. Is it perhaps to some extent justified to look upon the extremely frequent suffixes *-ādi*, "etc." (cf. e.g. 97) as expressions of generalisation?

In Navya-Nyāya Sanskrit repeated abstraction is expressed by using more than one abstracting suffix: *-tvaiva*, *-tvaṭā*, etc. (45). It may be worth noting that this occurred earlier only in the Rgveda,<sup>11</sup> from where it might have been adopted by the Naiyāyikas (sometimes Brahmins).

<sup>8</sup> I am most grateful to Professor Brough for the discussions I have had with him on this and on other points, though naturally I remain responsible for the particular view expressed here.

<sup>9</sup> Also in: "Are the Vaiśeṣika 'Guṇas' qualities?", *Philosophy East and West*, 4 (1954-55), 259-264; and in: *The Padārthatattvanirūpanam of Raghunātha Śīromaṇi* (Cambridge, Mass., 1957), 13.

<sup>10</sup> The term "quale" has been used in contemporary philosophy by N. Goodman in: *The structure of appearance* (Cambridge, Mass., 1951) (also quoted by Potter, *Padārthatattvanirūpanam*, 18).

<sup>11</sup> Renou (*Etudes védiques et Pāṇiniennes*, I, Paris, 1955, 54) says that these double suffixes "n'ont pas eu de suite dans l'usage ultérieur".

The grammatical rule that “a possessive suffix should not be appended to a *karma-dhāraya* if a *bahuvrīhi* can give the same meaning” (90–91) goes back to Patañjali’s *Mahābhāṣya*.<sup>12</sup> – The following article, to which Dr. Friedman kindly drew my attention, might be added to the bibliography: T. S. Bhattacharya, “The five provisional definitions of *vyāpti*”, *The Journal of the Ganganatha Jha Research Institute*, 3 (1945), 67–88, 169–188, 315–348. – The view held by Bocheński in his review, that the Indians did not discriminate between connotation and denotation, cannot be maintained.<sup>13</sup>

A short reference may be made to a discussion on Ingalls’ book which took place between the author and S. Bhattacharya.<sup>14</sup> Bhattacharya rightly objects against the translation of *pakṣa*, *sādhya* and *hetu* by minor, major and middle term respectively, and stresses the realism of Nyāya, which does not deal with statements, but with objects and “knowledges” – a point repeatedly made by Ingalls and stressed already in his Introduction. Without mentioning here some other topics of the discussion, it may be worth quoting Ingalls, according to whom the essential difference between the two standpoints lies “in the degree to which we believe translation to be possible. It appears that for him the degree is low; for me relatively high.” In particular Bhattacharya does not seem to believe that Nyāya can be translated in terms of symbolic logic. The present reviewer feels, that expressions and arguments of Navya-Nyāya can be represented by expressions of modern logic even to a higher degree than was done in Ingalls’ work. The test for such representations should be, that not only can the Sanskrit be actually converted into formulae according to explicit rules, but also vice versa (disregarding minor points, such as synonymous expressions). This seems to be possible because “the various technical terms, the formation of compounds, the morphological means of expression (e.g. suffixes and case-endings) and the syntactical means of expression (e.g. appositional clauses) in the technical Sanskrit of Navya-Nyāya are analogous to the terms, the formulas and the rules of modern Western logic”.<sup>15</sup>

In conclusion the various contexts may be mentioned in which the book under review is significant and has given (or may be expected to give) rise to further research.

(1) The study of Indian logic. This line of research has been followed by K. H. Potter in his *Padārthatattvanirūpaṇam* (Cambridge, Mass., 1957). This basic text, about which Ingalls had given the first information, is less close to formal logic, but contains much material which is of interest to modern philosophers (a statement on page 67: *saty api cānumānikādīsukhādiniścaya sukhādikam anubhāvāmīti pratyayābhāvād iti*, “for we do not say ‘I am experiencing pleasure, etc.’ in cases where we ascertain pleasure, etc. by inference, etc.” is reminiscent of Wittgenstein).

(2) The study of Indian philosophy (especially the three main schools of the Vedānta since the XIVth century: e.g. the *Advaitasiddhi* of Madhusūdana Sarasvatī with its commentaries or the *Tarkatāṇḍava* of Vyāsātīrtha). It should be particularly interesting to see how Indian thinkers applied Navya-Nyāya in philosophy, as the West has as yet

<sup>12</sup> L. Renou, *Le Durghataṅgī de Śaraṇadeva*, I 1 (Paris, 1940), 81–82 note, and cf. P. C. Chakravarti, *The linguistic speculations of the Hindus* (Calcutta, 1933), 457 (quoting the same expression with *matup* in stead of *matu*).

<sup>13</sup> See: J. Brough, “Theories of general linguistics in the Sanskrit grammarians”, *Transactions of the Philological Society*, 1951, 27–46, and cf. M. B. Emeneau, “India and linguistics”, *Journal of the American Oriental Society*, 75 (1955), 151. Cf. also Potter, *Padārthatattvanirūpaṇam*, 63–64.

<sup>14</sup> S. Bhattacharya, “Daniel H. H. Ingalls on Indian logic”, *Philosophy East and West*, 5 (1955–56), 155–162 and: D. H. H. Ingalls, “A reply to Bhattacharya”, *ibid.*, 163–166.

<sup>15</sup> See the present reviewer’s “Means of formalisation in Indian and Western logic”, *Proceedings of the XIIth International Congress of Philosophy* (Venice, 1958).

hardly directly applied the techniques of modern logic to purely philosophical problems in a significant way.

(3) The study of logic. It is known that the history of logic has not been a continuous evolution, but a succession of partly independent periods. Hence the same laws were re-discovered several times (e.g. Frege's definition of number discovered by Ingalls in Mathurānātha (76–77)). It is therefore important that Bocheński, to whom we owe the first comprehensive survey of formal logic which takes Indian material into account,<sup>16</sup> has largely drawn from Ingalls' work, which, in his words, "erschloss den Historikern der Logik eine neue geistige Welt".<sup>17</sup> The experience of the historians of logic justifies the expectation that Indian logic may also possess doctrines which modern logic has not yet evolved.

In the light of the above Professor Ingalls' work may contribute to research into the problem of the universality of logical principles, as well as throw light upon the relation between language and thought.

London

J. F. Staal

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<sup>16</sup> I. M. Bocheński, *Formale Logik* (Freiburg/München, 1956). Twenty years earlier, the same author had given a survey of Indian logic in: "Notiones historiae logicae formalis", *Angelicum*, 13 (1936), 109–123. In 1948 he gave expression to "le sentiment que les grands travaux de Schtcherbatsky et des autres indianistes devraient être refaits par des logiciens" ("L'état et les besoins de l'histoire de la logique formelle", *Proceedings of the Xth International Congress of Philosophy*, Amsterdam, 1949, 1062–1064).

<sup>17</sup> *Formale Logik*, 485.

*Manikāna. A Navya-nyāya manual.* Edited with English translation and notes by E. R. SREEKRISHNA SARMA. Foreword by H. H. RAMAVARMA OF COCHIN. Pp. xliii + 149. Adyar: THE ADYAR LIBRARY AND RESEARCH CENTRE, 1960.

With this publication Dr. S. K. Sarma has rendered an invaluable service to all scholars interested in Indian logic. The author, whose thesis *Die Theorien der alten indischen Philosophen über Wort und Bedeutung, ihre Wechselbeziehung, sowie über syntaktische Verbindung* (Marburg/Lahn: Philipps-Universität, 1954) has unfortunately never been published, is known from an article on Indian syntactical theory ("Syntactical meaning—two theories," *The Adyar Library Bulletin* 23 (1959) 41-62), a paper on Thoreau ("A short study of the Oriental influence upon Henry David Thoreau with special reference to his

*Walden*," *Jahrbuch für Amerikastudien* 1 (1956) 76-92) and a translation of Thoreau's *Walden* into Malayalam. With the present work he joins the small group of distinguished scholars—Y. V. Athalye, D. H. H. Ingalls, Swami Madhavananda, S. Kuppaswami Sastri, Saileswar Sen and S. C. Vidyabhusana—who have made Navya-nyāya accessible beyond the specialised circle of paṇḍits to the English reading public. Dr. Sarma knows most of the work of his predecessors and has utilised their results. But while his predecessors worked on texts which were at least published, though not necessarily understood until then, Dr. Sarma has done pioneer work by publishing a new and important text of which only one manuscript has been found. The result is an impressive addition to the already valuable series of Adyar publications. The *Manikāna* is a relatively short manual (*prakāraṇa*) of modern logic, the only one known so far, and is therefore an indispensable instrument for the study of this school.

The manuscript belongs to the Adyar Library and though the script is Devanāgarī, this may indicate that Navya-nyāya was not confined to the North East, but flourished also in the South of India. This would corroborate Fillozat's suggestion (*Journal Asiatique* 240 (1952) 409-10) that the "Chintāmani" mentioned by de Nobili in the beginning of the seventeenth century as a classic of Indian philosophy studied at Madurai, was the same as the famous *Tattvacintāmaṇi* of Gaṅgeśa. At present Navya-nyāya certainly flourishes in the South; it is a special branch of examination in the *pāthasālās* in Mysore and in Cochin (*Report of the Sanskrit Commission 1956-1957*, Government of India, Delhi, 1958, 44). The Maharaja of Cochin, himself a distinguished logician and the author of a commentary on the *Siddhānta-muktāvali* (*Subodhini*, Tripunithura, 1956), fostered a school of Navya-nyāya in Cochin. It is therefore not only fortunate, but also appropriate that the Maharaja has contributed a foreword to Dr. Sarma's work.

The *Manikāṇa* consists of four chapters dealing with the four *pramāṇas*. Dr. Sarma gives a complete edition and translation of the text, a lucid introduction and an important section consisting of notes which elucidate obscure passages and explain implicit references. These notes give life to a text which is otherwise very difficult to understand, for the *Manikāṇa* uses concise expressions and presupposes a good amount of prior information on the part of the student. This also implies that, without the notes, the English is often no more intelligible than the Sanskrit. In a technical work of this kind the translation performs in the first place one function: it enables the reader to understand the original. The technical Sanskrit used here can only be made intelligible to a reader who is willing to study the logical concepts and relationships carefully, just as an English translation of Euclid's *Elements* can only be intelligible to a person who is familiar with the elements of geometry. Dr. Sarma's introduction and notes are therefore indispensable.

As the name indicates, the *Manikāṇa* is an abridgement of the *Tattvacintāmaṇi*. In subject matter it resembles works such as the *Siddhānta-muktāvali* and the *Nyāyasiddhāntamañjarī*. The author is not known, nor is the date. It would not be easy to determine the date or at least the relative place of this work within the Navya-nyāya

tradition, and the editor has not attempted this, mainly on account of the fact that the book is a *prakaraṇa*. For, if certain doctrines or further elaborations of doctrines are missing, this could either mean that they had been deleted or that they had not yet been developed at the time the *Manikāṇa* was written. For example the section on non-existence (*abhāvaprakaraṇa*) does not deal with the theory of double and multiple negation which was developed in the *Nyāyasiddhāntamañjarī* (c. 1550 A. D.) and in Mathurānātha (c. 1650 A. D.). On the other hand the text adopts the doctrine that *saṃkara* is an impediment to being a *jāti*, a doctrine maintained in the *Siddhānta-muktāvali* (seventeenth century), but discarded at the time of the *Dinakariya*, a commentary on this text (see: D. H. H. Ingalls, *Materials for the study of Navya-nyāya logic*, Cambridge, Mass., 1951, 42 note 49). The *Manikāṇa*, then, could well have been written in the seventeenth or eighteenth century. Its conciseness, a certain textbook-like clarity, and the fact that it is a *prakaraṇa* may point to a later date within this period.

The text provides some evidence for the assumption that the author of the *Manikāṇa* was in some respects a follower of the great logician Raghunātha (beginning of the sixteenth century), whose revolutionary doctrines have not been adopted by the conservative majority of Navya-nyāya logicians. Mathurānātha, for example, seems to share Raghunātha's opinion on some points, but "in deference to those who do not, he refines his statements so that they will hold true under the traditional theory" (Ingalls, 69). The author of the *Manikāṇa* seems to refer to double negation only once, and on this occasion he unreservedly accepts Raghunātha's view: *ākāśābhāvasyātyantābhāvo na pratiyogisvarūpaḥ* "the absolute non-existence of the non-existence of ether is not the same as the counter-correlate (i. e., ether)" (40-1). This corresponds to the denial of the law of double negation:

$$\sim (\sim \sim x = x).$$

In another passage, Raghunātha is referred to (as Śiromaṇi) and his views are adopted (61). Since Raghunātha's logic is of central importance and has not been much studied (despite K. H. Potter's *The Padārthātattvanirūpaṇam of Raghunātha Śiromaṇi*, Cambridge, Mass., 1957, which deals more with the Vaiśeṣika categories than with the more specifically Navya-naiyāyika topics such as

*anumāna*), the *Mañikāṇa* is also in this respect a valuable source of information.

Dr. Sarma's explanation (119) of the reason which Raghunātha adduces for rejecting the law of double negation seems preferable to Ingalls' explanation (*op. cit.* 68), though both are basically similar. According to Ingalls, "all absences have as their nature absence<sub>1</sub> (*abhāvatva*) . . . and no amount of legerdemain can turn these absences into entities whose nature is presence<sub>1</sub> (*bhāvatva*)." According to Sarma, "as the pot exists on the floor through the relation of contact (*samyoga-sambandha*), the nonexistence of the nonexistence of the pot must also be held to be so related. This is absurd, since nonexistence abides in its substrate through attributiveness (*viśeṣanātā*) . . ." This seems to be more in keeping with the usual arguments adduced in Navya-nyāya.

Sarma's Introduction is at the same time an introduction to Navya-nyāya in general and can be profitably studied by logicians who have no access to the Sanskrit originals. It is at points reminiscent of the more detailed expositions of Ingalls in the second section of his book (entitled: "An examination of some theories and techniques of Navya-nyāya logic"), to which it refers occasionally. Dr. Sarma has divided this introduction into four parts, "Methodology," "Metaphysical Concepts," "Some technical terms," and "Some premises." These parts, and especially the last, contain information not previously covered by Ingalls. Among the premises one meets with the rule *guṇe guṇo na* "no quality exists in (another) quality" (xxxix). This rule embodies a principle which is not dissimilar to the premise upon which the first order predicate calculus in modern logic is based.

The author's translations of technical terms are generally excellent, though there are a few inconsistencies. It is somewhat unfortunate, for example, when *ativyāpti*, *avyāpti* and *asambhava* are rendered by respectively "overapplicability," "non-inclusiveness," and "nonapplicability" (xviii). In this translation one Sanskrit term (*vyāpti*) is translated by two different English terms, while one English term ("applicability") renders two different Sanskrit terms. Another instance is provided by the term *vyāpāra*, which is translated by "operation" (a usual rendering) but also by "(intermediary) function."

The translation of the text is careful and literal. This results unavoidably in a number of passages

where the English is very hard to understand without referring to the Sanskrit. In a few passages the translation could have been more simple. An example occurs on page 5, where *tadvati tatprakā-rako'nubhavaḥ pramā / tathāhi—rajate rajatamiti jñānaṃ rajatatvavati rajatatvaprakāraḥ* is translated as: "the experience which has reference to a substantive possessed of that (attribute) which is an adjunct to it (*i. e.*, experience) is valid cognition. This may be explained as follows: The cognition '(This is) silver' with regard to silver is one which has reference to a substantive possessed of silverness, which is an adjunct (to the cognition)." This passage could be rendered as follows: "the experience that the chief qualifier occurs in what possesses it, is called valid knowledge. For example, the knowledge of silver with regard to silver is the knowledge that the chief qualifier silverness occurs in what possesses silverness (*i. e.*, silver)."

It might be objected that both English translations of the above expression are cumbersome and hard to understand. In these and similar cases the question therefore arises whether a translation into a more purely technical terminology could not do greater justice to the original, than a translation into a necessarily artificial kind of English. The present reviewer has for this reason made use of a formal terminology and language, constructed along similar lines as the symbolic terminology and language of modern mathematical logic. The above expression, for example, can be translated into this terminology and the result gives an exact representation of the logical relationships expressed in the original. For this purpose we shall make use (cf. *Bulletin of the School of Oriental and African Studies* 23 (1960) 109-22) of the expressions of elementary mathematical logic and in addition of the expression  $\alpha xF(x)$  for a *restricted-variable* denoting the idea "x such that F(x)." Special use will be made of two binary relations, A and B, defined as follows: A(x, y) denotes: "there is occurrence (*vṛttitva*) of x in y"; and B(x, y) denotes: "x is the locus (*adhikāraṇa*) of y," or: "x possesses y (x y-*vati*)."

With the help of this formal language the above passage can be expressed as follows. If p denotes the *prakāra* "chief qualifier," the expression denoting: "x such that x possesses p" becomes:  $\alpha xB(x, p)$ . The definition of knowledge as the experience that the chief qualifier occurs in what possesses it, can be written as:

$$A[p, \alpha xB(x, p)].$$

If *rajata* "silver" be donated by  $x$ , and *rajatatva* "silverness" by  $\hat{r}$ , the example can be formalised as follows:

$$A[\hat{r}, \alpha xB(x, \hat{r})],$$

denoting "silverness occurs in what possesses silverness." This expression can be further simplified by means of the formula:

$$\alpha xB(x, \hat{r}) = r,$$

expressing the rule: *tattvatat tad eva* "that which possesses thatness is (just) that" (*BSOAS* 114). Thus we arrive at:

$$A(\hat{r}, r),$$

which expresses that silverness occurs in silver.

Many doctrines of the *Manikāna* can be translated and analysed along these lines. The most important one is the definition of *vyāpti*. This definition is similar to the definition of the *Bhāṣā-pariccheda* (analysed in *BSOAS* 119), but contains one extra clause. In order to show where the difference lies the present definition may be analysed here. This definition is formulated as follows: *pratiyogyanadhikaraṇahetvadhikaraṇaniṣṭhābhāvapratyogitānavacchedakasādhyatāvachhedakaviśiṣṭasādhyasamānādhikaraṇahetutvaṃ vyāptih* (30). This is translated by Dr. Sarma as: "pervasion is to be that probans which has a common substrate with the probandum, as specified by the determinant of being the probandum, which (determinant) is not the determinant of the counter-correlateness of the nonexistence resting (read (?): residing) in the substrate of the probans, which (substrate) is not a substrate of the counter-correlate (of the non-existence)." This definition is further explained and exemplified with well-known cases on p. 110, note 9.

The main clause is: *sādhyasamānādhikaraṇahetutvam*. If  $h$  and  $s$  denote respectively *hetu* and *sādhya*, this clause can be expressed as:

$$\alpha xB(x, s) = \alpha xB(x, h). \quad (1)$$

The preceding part of the expression (*hetvadhikaraṇa . . . sādhyatā*) can be formalised step by step in the following manner:

*hetvadhikaraṇa* (e.g., *parvata*):  $\alpha xB(x, h)$ .  
*hetvadhikaraṇaniṣṭhābhāva* (e.g., *jalābhāva*):

$$\sim A[y, \alpha xB(x, h)].$$

*hetvadhikaraṇaniṣṭhābhāvapratyogitā* (e.g., *jalatva*):

$$\alpha \hat{y} \sim A[y, \alpha xB(x, h)].$$

*hetvadhikaraṇaniṣṭhābhāvapratyogitānavacchedakasādhyatā* (e.g., *vahnitvam*):

$$\hat{s} \neq \alpha \hat{y} \sim A[y, \alpha xB(x, h)]. \quad (2)$$

This expression constitutes the second clause of the definition. The clauses expressed in (1) and (2) correspond to the two clauses constituting the *Bhāṣā-pariccheda* definition, as (2) is equivalent to the expression arrived at there, i.e.:

$$A[s, \alpha xB(x, h)].$$

The remaining clause: *pratiyogyanadhikaraṇa(hetutvam)* is an addition of the *Manikāna*. Substituting the value for the *pratyogī* given above, this can be expressed as follows:

$$\alpha zB\{z, \alpha y \sim A[y, \alpha xB(x, h)]\} \neq \alpha xB(x, h). \quad (3)$$

Thus (1), (2), and (3) constitute together the definition of *vyāpti* according to the *Manikāna*.

This definition is subjected to further refinements in the usual fashion. Many refinements consist in further conditions enjoined upon the *sambandha* "relation," through which the entities concerned occur in their locus. The relation through which  $x$  occurs in  $y$  can be expressed by a subscript attached to  $A$ . For example, when  $h$  denotes *samavāya* "inherence,"  $A_h(x, y)$  will denote that  $x$  occurs in  $y$  through the relation of inherence.

The *Manikāna* imposes one condition of this type, which does not occur in the *Siddhāntamuktāvalī* in connection with the definition of *vyāpti*, but which is given in a different context, i.e., when dealing with *anvaya-vyatireka* (see the present reviewer's "Contraposition in Indian logic," to be published in *Proceedings of the International Congress for Logic, Methodology and Philosophy of Science*, Stanford 1960). This condition requires that the relation determining the counter-correlateness (*pratyogitāvachhedaka-sambandha*) be the same as the relation determining the probandum-ness (*sādhyatāvachhedaka-sambandha*) (see p. 111, note 13, reading "probandum-ness" for "subjectness"). This condition can be expressed as follows:

$$\alpha z \sim A_z[y, \alpha xB(x, h)] = \alpha z A_z[s, \alpha xB(x, h)].$$

Dr. Sarma's publication provides a basic text in Navya-nyāya, gives a wealth of information and can be recommended as an excellent text book on the subject. As a manual of Navya-nyāya the *Maṅikāṇa* seems to be more useful than the more special texts translated previously by Sen, Ingalls and Madhavananda. It is to be hoped that Dr.

Sarma will continue to work in this field and will perhaps one day provide us with the much needed edition and translation of the *Tattvacintāmaṇi* itself.

J. F. STAAL

SCHOOL OF ORIENTAL AND AFRICAN STUDIES,  
UNIVERSITY OF LONDON

**Grammatical method in Pāṇini:** His treatment of Sanskrit present stems. By BETTY SHEFTS. (American oriental series, Essay 1.) Pp. x, 45. New Haven: American Oriental Society, 1961.

Reviewed by J. F. STAAL, *University of Amsterdam*

Since Bloomfield described Pāṇini's Sanskrit grammar (about 350 B.C.) as 'one of the greatest monuments of human intelligence', it has become customary for linguists to honor Pāṇini as a remote ancestor. That such homage could easily become lip service is due to the particularly difficult and inaccessible character of the works of the Indian grammarians. Linguists may not have enough Sanskrit to follow a description of Sanskrit in terms of a technical metalanguage, constructed from Sanskrit and treated as Sanskrit (e.g. metalinguistic elements are declined like nouns of the object language). On the other hand, Sanskritists who have not made a special study of Sanskrit grammatical tradition may react to the rules of Pāṇini in the same way as English speakers might react to the formulae in a textbook on algebra written in English. Still, difficulty and inaccessibility are no excuses for neglect; and if the Sanskrit grammarians have something important to offer a modern linguist, the latter must be provided with expositions which he can understand, provided he is willing to go to some trouble.

Miss Shefts's dissertation provides an exposition of this kind which is worth the trouble required to read it. It consists of an introduction, a text and translation of 18 rules (*sūtras*) of Pāṇini, a list of definitions of technical terms of the metalanguage, and a chapter with extracts, edited and translated, from two commentaries, the *Mahābhāṣya* (ca. 150 B.C.) and the *Kāśikā* (seventh century A.D.). The former extract covers part and the latter covers all of the earlier translated rules of Pāṇini. This is followed by a short bibliography. All this material is contained in 45 pages, but the treatment combines with brevity the lucidity which was so much sought after by the Indian grammarians themselves. The author was not only taught by a series of excellent Western scholars, but—like other Western Sanskritists who have contributed to the field—she was trained by an Indian pandit as well.

Since Miss Shefts illustrates Pāṇini's method by means of his treatment of present stems, a few words will be devoted to introducing this topic. In the discussion reference is made not only to Pāṇini's grammar, but also to the *Dhātupāṭha*, a list of verbal roots also attributed to Pāṇini.

The Sanskrit grammarians set themselves the task of describing as concisely as possible verbal forms like *tudati*, *tudyante*, *atudāma*, *tutuduḥ*, *tutoda*, *totsyati*,

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*tunna, todīta, todayati, ataatsīt*. They considered it convenient and simple (in this they were followed by modern grammarians) to postulate not only a root *tud-* in terms of which these forms can be described by adding other elements, but also a present stem *tuda-*, derived from the root, in terms of which some of the above forms can be described (e.g. the present indicative *tudati* 'he strikes'). This method of description constitutes an analysis based purely upon considerations of simplicity. The verbal forms are thereby divided into two classes, the first directly described in terms of the root, the second described in terms of the present stem, itself derived from the root. Simplicity and uniformity are achieved because the elements affixed to the root *tud-* can also be affixed to other roots; and similarly the elements affixed to the stem *tuda-* (e.g. *-ti*) can also be affixed to other stems. The *-a(-)* used to form the present stem *tuda-* and the other present-stem-forming morphemes can be called suffixes (from the point of view of the root or stem) or infixes (from the point of view of the final form); the Indian grammarians call them *vikaraṇa*. The various kinds of *vikaraṇa* differ from each other in form and function. Pāṇini made use of metalinguistic forms to indicate particular functions whenever necessary. An example of present-stem-forming morphemes which differ not in form but in function:

ROOT	PRESENT STEM	PRESENT INDICATIVE
<i>tud-</i>	<i>tudá-</i>	<i>tudáti</i>
<i>budh-</i>	<i>bódha-</i>	<i>bódhati</i>

The first present-stem-forming *-a-* is accented and affixed to the root; the second present-stem-forming *-a-*, also affixed to the root, causes the vowel of the root to be accented and to be replaced by another (this is called *guṇa*; in the present instance, *u* is replaced by *o*). The formally identical *-a-* morphemes of the object language are functionally different and can be formally differentiated in the metalanguage by adding certain indicatory elements. Subscripts could be used:

ROOT	VIKARAṆA	PRESENT STEM
<i>tud-</i>	<i>a<sub>1</sub></i>	<i>tudá-</i>
<i>budh-</i>	<i>a<sub>23</sub></i>	<i>bódha-</i>

Here 1 indicates that the vowel to which it is attached is accented; 2 that the foregoing vowel is accented; 3 that the foregoing vowel undergoes *guṇa*.

Pāṇini uses metalinguistic sounds (these will be set in roman type): instead of *a<sub>1</sub>* and *a<sub>23</sub>* he writes *śa* and *śap* respectively. The metalinguistic elements *ś* and *p* indicate particular features accompanying the sounds of the object language to which the metalinguistic elements are attached. Sounds marked by these indicatory elements belong to classes of which the members share particular features. Such classes are denoted by the technical term *it*; Pāṇini therefore speaks of *pit*, *śit*, etc. The *its* belong to the metalanguage or, as Pāṇini says, occur 'in grammatical instruction' (*upadeśe*) only. Particular rules state that these indicatory elements should disappear from the finally derived forms of the object language, so that we arrive at *tudati* and never at *\*tudśati*.

Methods of description by means of *its* are succinctly explained by Miss Shefts

on the first page of her book and under some of the definitions in the second chapter. Another device which Pāṇini uses is called *anuvṛtti* 'recurrence'—in the author's words, 'the "following" of a term used in the *sūtra* in other, later *sūtras*, where it then has to be supplied (I have enclosed such terms in square brackets)'. By *anuvṛtti* superfluous repetitions are avoided. For instance, to describe written English Pāṇini could have given a rule that 'the plural of nouns is formed by adding -s'. A following rule need not be phrased, 'but the plural of *man* is *men*', merely 'but of *man*, *men*'. Here there is *anuvṛtti* of the term 'plural', which can therefore be omitted in the later rule. The formulation of rules will therefore depend on their order. Hence order is important in achieving the desired economy of expression; in the instance above, the order is determined by the principle that the exception should follow the general rule. Order can also be determined by other factors; for example, rules which say that in certain circumstances the forms A and B occur in free variation should be placed after rules that describe the circumstances in which A and B occur.

Present-stem formation is also connected with problems of order, for Pāṇini distinguishes ten *vikaraṇas*, or ten different ways in which stems can be derived from roots: e.g. in addition to *tud* + *śap* → *tuda* we have *div* + *śyan* → *dīvyā*, *su* + *śnu* → *suno*. Among these *vikaraṇas* there are two zero morphemes, accompanied by different features:

ROOT	VIKARAṆA	PRESENT STEM
<i>ad-</i>	0	<i>atti</i>
<i>hu-</i>	0	<i>juhoti</i>

Pāṇini again expresses the different functions by different metalinguistic forms. In the case of written English he might have referred to the plurals of *man* and *sheep* as follows:

$$man + 0_1 \rightarrow men$$

$$sheep + 0_2 \rightarrow sheep$$

Here the subscript 1 denotes that the preceding vowel *a* is replaced by *e*, while the subscript 2 denotes that the preceding vowel remains unchanged. Since Pāṇini generally calls such zero morphemes *LU*, and the above distinction can be indicated by using *kit* and *śit* respectively, he arrives at

ROOT	VIKARAṆA	PRESENT STEM
<i>ad-</i>	<i>luk</i>	<i>atti</i>
<i>hu-</i>	<i>ślu</i>	<i>juhoti</i>

The author devotes an excellent section (12) to these and other zero morphemes. While the zero *vikaraṇas* appear in the second and third place in the *Dhātupāṭha* (see below), they are placed among the other zero morphemes in the grammar. As these other zero morphemes are often of the type *luk*, the rule for *ad-* (which requires *luk*) follows these other cases and precedes the rule for *hu-* (which requires *ślu*; 3).

In both the grammar and the *Dhātupāṭha* the first *vikaraṇa* is *śap*. Since according to the Indian view zero morphemes imply that something is being replaced, the first *vikaraṇa* can most easily perform this function. Consequently the two zero morphemes occupy the second and third place, and the list of *vikaraṇas* begins: *śap*, *luk*, *ślu*. After this seven more *vikaraṇas* follow and the question arises of explaining both their order and the priority of *śap*. For this we should bear in mind that rules which describe the occurrence of two forms A and B in free variation with each other follow the rules which describe the occurrence of A and B.

We have in Sanskrit free variation of the *vikaraṇas* *śap* and *śyan*. Hence one meets with *bhramati* as well as *bhrāmyati*, both derived from a root *bhram-*. Pāṇini teaches that there are three possible combinations of two *vikaraṇas* which occur in free variation: *śap* and *śyan*, *śap* and *śnu*, *śnā* and *śnu*. Since there are two rival forms which alternate with *śap*, the economy criterion suggests the *anuvṛtti* of *śap*. So two possibilities arise:

I. <i>śap</i>	or	II. <i>śap</i>
<i>śyan</i>		<i>śnu</i>
<i>śnu</i>		<i>śyan</i>

These would enable Pāṇini to insert the following alternations respectively:

I. <i>śap</i>	or:	II. <i>śap</i>
<i>śyan</i>		<i>śnu</i>
<i>śap</i> or <i>śyan</i>		<i>śap</i> or <i>śnu</i>
<i>śnu</i>		<i>śyan</i>
<i>śap</i> or <i>śnu</i>		<i>śap</i> or <i>śyan</i>

Since there is, lastly, the alternation *śnu/śnā*, one could expect Pāṇini to adopt I, where *śnu* appears at the end, so that *śnā* can follow immediately:

<i>śap</i>
<i>śyan</i>
<i>śap</i> or <i>śyan</i>
<i>śnu</i>
<i>śap</i> or <i>śnu</i>
<i>śnā</i>
<i>śnu</i> or <i>śnā</i>

However, Pāṇini actually adopts I, but places *śnā* much further down. At this point apparently an explanation based upon considerations of simplicity can go no further. Miss Shefts offers a very plausible explanation for the order adopted by Pāṇini.

According to her, the *vikaraṇas* are ordered by length (in terms of *mātrā* and twice in a special way) and this is applied first to *vikaraṇas* which alternate with *śap* and then to *vikaraṇas* which do not alternate with *śap*. This principle explains the order adopted both in the grammar and in the *Dhātupāṭha*; the only difference is that the grammar treats the two zero morphemes separately.

If the priority of *śap* is correctly explained by systematic considerations (the fact that only *śap* occurs in free variation with two other *vikaraṇas*; correctly stated by Miss Shefts, 2-3), there is no need for other reasons of a different nature. The reasons listed under §6 (3) appear superfluous.

After this thorough discussion of order, partly based upon considerations of *anuur̥thi*, it is surprising that in the translation of the rules (4-5) some cases of *anuur̥thi* (always enclosed in square brackets and disregarding *viparināma*) have been omitted: in 3.1.75 [*śnuḥ*: 3.1.73] and in 3.1.76 [*śap*: 3.1.68; *śnuḥ*: 3.1.73].

The chapter on definitions offers, in addition to its lucid discussion of zero morphemes (s.v. *lopa*), some interesting suggestions about Pāṇini's difficult uses of the terms *ārdhadhātuka* and *sārvadhātuka*. Miss Shefts derives these terms from the convincingly postulated pre-Pāṇinean terms *\*ardhadhātu* 'half root' and *\*sarvadhātu* 'whole root', presumably used to denote what were later called 'root' and 'stem' respectively.

The rest of the book is devoted to the discussions of the *Mahābhāṣya* and the *Kāśikā*. Readers who look for simple illustrations of Pāṇini's rules and wish to get accustomed to the style of the Sanskrit grammarians, should read the *Kāśikā* first. The author of this text was pragmatic in outlook and eager to provide practical solutions. A linguist who looks for more sophisticated discussions of the actual and possible implications of Pāṇini's rules should turn to the *Mahābhāṣya*.

Miss Shefts's dissertation raises some very interesting problems on the relation between language and metalanguage—terms which the author avoids, though they are fully justified in view of such expressions in the texts as *upadeśa*, *saṃjñā*, and *paribhāṣā*. Similar problems may arise whenever a book on grammar with identical object language and metalanguage is translated into another language. These problems could presumably disappear in a correct translation, where elements of the object language (e.g. Sanskrit) would be preserved while elements of the metalanguage (e.g. technical Sanskrit) would be replaced. The large number of technical terms introduced by Pāṇini, however, prevents this simple solution. Typographical attempts to express the distinction, e.g. by writing the object language in italics and the metalanguage in roman type, are difficult to maintain throughout. Miss Shefts has tried to do this with the *its*, as we saw before. But then the *its* should be called *its*, and not (as she calls them) *its*. Similarly the suffix of a declined it belongs to the metalanguage and should be in roman. The author writes *śapaḥ* 'in the place of *śap*', where one would expect *śapaḥ*. Is this because *-aḥ* is a case ending of the object language as well? But surely *ś* is also a sound of the object language.

Another difficulty appears when *ślu* (author's italics) is compared with *śap*. Since *ś* is always it, this sound should certainly appear in roman. But *lu* appears nowhere in the object language and is a typical element of the metalanguage. The only correct expression is therefore *ślu*. When elsewhere, as is often the case, roots of the object language are referred to by the root + an indicatory sound, e.g. *adi* denoting *ad-*, the author correctly writes *adi*. But why not generalize this to all metalinguistic talk about roots, e.g. by writing *divādibhyaḥ* 'after *div-* etc.' instead of *divādibhyaḥ*? No ambiguity arises even when sandhi appears, e.g. *svādibhyaḥ* 'after *su-* etc.'

More difficult to deal with are interesting cases like *asteḥ* 'in the place of *asti*'. Here a third-person verbal conjugation (*asti* 'he is') is treated and declined as a noun ending in *-i* (genitive *-eh*). This practice can be compared to a hypothetical Latin grammar of Latin, expressing a rule about *amāmus* 'we love', e.g. 'in *amāmus ...*', by *amāmō ...*, or a rule about *sum* 'I am', e.g. 'of *sum ...*', by *sī ...* When the Sanskrit pronoun *aḥam* 'I' is declined like a substantive (as was actually done by some later Indian philosophers)—e.g. *aḥamaḥ* 'of the ego'—this could still be expressed by *aḥamaḥ* whenever a metalinguistic statement about the form *aḥam* is meant. But in cases such as *asteḥ* we are at a loss how to express the relationship by typographical means.

I suggest that to avoid most difficulties of this kind we return to philological tradition and always print the original text in italics. In India the distinction between language and metalanguage was never expressed by typographical means. What is read by modern scholars was originally spoken or recited; how far special metalinguistic nasalizations and accents were pronounced is a separate problem. In the translation of a text which contains object-language and metalanguage material, elements of the object language should remain in italics and elements of the metalanguage, generally translated into another language, should appear in roman. Untranslated or untranslatable metalinguistic elements can generally be left in roman as well. By such conventions most difficulties may be avoided, e.g. *divādibhyaḥ śyan* 'after *div-* etc., *śyan*'; *aster bhuh* 'in the place of *asti*, *bhū-*'. In practice these conventions would leave the typography of most of Miss Shefts's translations unaltered. The texts would again have to appear in the traditional italics. I have treated the matter in some detail because these conventions may prove practical in future work on the Indian grammarians.

Since in a book of this kind the distinction between roman and italics can be crucial, typographical mistakes should be avoided with special care. Unfortunately they are not absent in this book, though on the whole it is admirably produced. *śivasūtras* for *śivasūtras* (7) is an innocent mistake, but *dhaṣ* for *dhaṣ* (7) can be misleading. The same applies to *bhāvati* for *bhāvati* and *tip* for *tip* (11, twice). Another kind of confusion may be caused by a printing mistake in the text of one of the *sūtras* (5): 3.1.81 *śna* for *śnā*.

These are only minor criticisms, which will not detract from the value of this welcome contribution. Let us hope that Miss Shefts will continue to give expositions of other methods of the Indian grammarians, or further studies on the order of Pāṇini's 3996 rules. The earlier studies of Buiskool, Faddegon, and Boudon, not mentioned in the bibliography, deal partly with similar problems and could be further developed. The number of scholars who are well equipped in this field, aside from pandits (who often have little knowledge of English), is nowadays very small; much remains to be done if the brilliant contributions of the past are to be continued. Now that a translation of the beginning of the *Kāśikā* is also appearing, a complete translation of the *Mahābhāṣya* remains an important desideratum. Despite Kielhorn's excellent edition of 1878–84, generations of scholars have shrunk from this task. Miss Shefts is in a position to disregard in this respect the example of her predecessors.

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*Die Logik im Mahābhāṣya.* By HARTMUT SCHARFE. (Deutsche Akademie der Wissenschaften zu Berlin, Institut für Orientforschung, Veröffentlichung Nr. 50). Pp. 173. Berlin: AKADEMIE-VERLAG, 1961. Price: DM 34.50.

Patañjali's *Mahābhāṣya* is not one of the first technical books in India, for the technical manuals of the ritualists are older. But it certainly is one of the oldest commentaries in which we find a detailed reflection of technical discussions. Patañ-

jali obviously takes great pleasure in proofs, refutations and all kinds of arguments. Add to this the fact that as a linguist he is used to express himself carefully and exactly, and the conditions are created for a book of which the logical analysis should be rewarding. Moreover we are in the fortunate situation, relatively rare with regard to Indian literature, of possessing a first rate edition of the text in the three volumes published by Kielhorn between 1892 and 1909. In addition the subcommentaries of Kaiyaṭa and Nāgēśabhaṭṭa are available in good Nirṇaya-Sāgara editions and in

the edition of Gurukula Jhajjara (Rohataka). Finally there is a word index (*Mahābhāṣyaśabdakośa*) published by S. Pathak and S. Chitrao (Poona, 1927).

The work of Scharfe consists in numerous passages from the *Mahābhāṣya* dealing with logical topics, which are translated, analysed and grouped under various headings. The translations are reliable and the author is obviously at home in the *Mahābhāṣya*. Still there is a tendency to find fault with Patañjali which pervades the work and which pictures Patañjali, after all, as a rather primitive thinker. This seems hardly justifiable since there is every reason to consider Patañjali a brilliant and scholarly commentator, independent in his judgment, equipped with great factual information as well as respect for facts and obviously steeped in the grammatical doctrines of his predecessors. As a scholar and thinker on the one hand, and as a forerunner of innumerable themes of Indian scientific thought on the other hand, Patañjali could be compared in the West with no less a thinker than Aristotle. That both have made mistakes does not change the fact that both are rightly regarded as persons who set the standard of thought for centuries.

It is not surprising therefore that Scharfe is sometimes wrong in his hasty criticisms. Another fact also comes in his way: for while Patañjali was an astute though untrained logician, Scharfe repeatedly makes logical errors and appears not very much at home in the modern logical writings to which he refers. Without detracting from the value of his work, which is exceedingly useful as a collection of passages which are of logical interest, some examples will show the shortcomings and may at the same time exhibit something of the logical level of Patañjali's investigations, as well as some later influence of his ideas and formulations.

In the section on *Zeichen und Bezeichnetes* an interesting use of the distinction between *use* and *mention* is met with (see pages 23-4). Pāṇini refers in *sūtra* 4.1.83 to the word *divyati* "he plays" which occurs in *sūtra* 4.4.2. The context of 4.1.83 requires that the word *divyati* appears in the ablative case, which is *divyateh*. But instead of this Pāṇini uses *divyatah*, the ablative of the present participle *divyat* "playing." That is, Pāṇini does not speak about 'he plays' (in quotes) but speaks about the playing one or the player (without quotes). Though this is

logically incorrect according to Scharfe, Patañjali is quite right in maintaining that it is completely natural. It is as if we refer to that rule about the player, though we are perfectly well aware that it is a rule about the grammatical form 'he plays.' According to Scharfe Patañjali "hat hier das Wesen der Zitierung (Quotation) nicht erkannt" (p. 23). This seems extremely unlikely, since Pāṇini and Patañjali not only clearly saw the function of quotation and the fundamental distinction between use and mention, but may well have been the first scholars to do so. This is nowhere stated by Scharfe, though he refers in another context (p. 99) to the relevant *sūtra* (1.1.68), but was brilliantly demonstrated by J. Brough ("Theories of General Linguistics in the Sanskrit Grammarians," *Transactions of the Philological Society* (1951) 27-46). Strange as it may seem, Brough's article is quoted by Scharfe on pages 25 and 26 in a way which suggests that it supports his own thesis. Actually it clearly establishes the opposite. In the Bibliography Brough's paper (which is also quoted elsewhere) is not mentioned. A possible explanation of this state of affairs is that the author misunderstood Brough's argument because he himself ignored the precise meaning of the distinction between use and mention. For the same reason he was not in a position to appreciate that Patañjali did understand this same distinction perfectly well.

A similar confusion appears to be at the background of the argument on p. 25, where it is claimed that the distinction between language and metalanguage is not likely to arise anywhere unless these two are embodied in different natural languages. This is not only falsified by the situation in modern logic, where the distinction is made *within*, e.g., English, but also by the fact that Patañjali himself operates with metatheorems (*paribhāṣā*) which necessarily belong to a metalanguage.<sup>1</sup> In his discussion of *paribhāṣā* (p. 50 sq.) (where it is rightly maintained that the later grammarians enlarged the scope of the *vipratīśedha-paribhāṣā*) Scharfe does not mention this. These passages fail to show that Patañjali, following Pāṇini, made a contribution which is of the greatest importance in the history of logic and which was in the West discovered only many centuries later.

<sup>1</sup> Cf. J. F. Staal, "The Theory of Definition in Indian Logic," this JOURNAL 81 (1961) 122-6.

With respect to definitions, to which another section of Scharfe's work is devoted, it might have been mentioned that the grammatical terms *lakṣaṇa* and *lakṣya* denoting respectively the rules of grammar and the object of grammar, have to be made to coincide in such a way that *lakṣaṇa* is based upon *lakṣya*. This develops later into the logical theory of definition where *lakṣaṇa* and *lakṣya* refer to respectively the *definiens* and the *definiendum*.<sup>2</sup> This trend of thought was not confined to the Sanskrit grammarians. The Tamil *tadbhava* forms of Sanskrit *lakṣaṇa* and *lakṣya* are respectively *ilakkaṇam* and *ilakkiyam*; the first term is used by the Tamil grammarians to denote grammar in general.

In the chapter on *Aussagen, ihre Verbindung und ihre Verneinung* the elucidation of Patañjali's use of *eva* "only" as an expression for *niyama* "restriction" is instructive. Actually this use serves the purpose of quantification as the example shows: *ajādi gunavacanād eva* does not only mean: "the suffixes beginning with a vowel (-iṣṭha and -īyas) are only affixed to adjectives" but also: "it holds for all forms, that if the suffixes beginning with a vowel (i. e., -iṣṭha or -īyas) are affixed to a form, that form is an adjective." More explicit use of the quantifying function of *eva* was made by the Buddhist logician Dharmakīrti.<sup>3</sup> Another expression, namely *apūrvō vidhīh* (p. 41) leads in Mīmāṃsā to the celebrated doctrine of *apūrvā* as was observed by Renou.<sup>4</sup>

Elsewhere in the chapter on *Aussagen* (p. 42) there is a confusion between the *interpretation* of a conjunction and the *truth values of the matrix* of a conjunction, which to unravel would take us perhaps too far. The result of this confusion is that a case of *saṃnīyoga* occurring in 3.1.11: *kartuḥ kyañ salopaś ca* is expressed by:<sup>5</sup>

$$X \ \& \ (Y \vee \bar{Y})^5$$

The underlying idea is that X is an *unconditional* rule whilst Y is *conditional*. However in logical terminology X & (Y ∨  $\bar{Y}$ ) is merely equivalent

to X, since Y ∨  $\bar{Y}$  always has the truth value 't.' What Patañjali actually says can only be expressed by:

$$X \ \& \ (Y \rightarrow Z)$$

where X denotes *kyañ* " -ya- is affixed,"

Y denotes *sa* " -s occurs" and

Z denotes *salopaḥ* " -s is dropped,"

so that the entire expression comes to denote: " -ya- is affixed and whenever -s occurs, it is dropped." Patañjali expresses this by saying that X is *pradhānaśiṣṭah* "als Hauptsache gelehrt" (Scharfe) (i. e., unconditional) whilst  $\bar{Y}$  is *anvācayāśiṣṭah* "als sekundäre (eventuelle) Hinzufügung gelehrt" (i. e., conditional).

The passage I 5.14-9 (Kielhorn) appears twice: on pages 38 and 56. The logical content is closely related to the law of double negation and can only be adequately formalized when a symbol is introduced for *pratiśeḍha*-negations.<sup>6</sup> The law of double negation:  $\bar{\bar{X}}$  äq X is equivalent to the law of the excluded middle (*principium tertii exclusi*): X ∨  $\bar{X}$ . The way in which this is expressed is to such an extent reminiscent of the roundabout and verbose manner in which some German philosophers express their struggle with simple logical laws, that it is worth quoting: "Das logisch wichtige dieser beiden Stellen ist, dass sich hier Positives und Negiertes als gleichberechtigte begriffliche Realitäten gegenüberstehen und das Positive und Negative zusammen das Ganze ausmachen: das heisst, ein Drittes gibt es nicht. Eine Ergänzung muss man allerdings dazu noch anbringen: Dieses 'Ganze' ist nicht 'alles,' 'die Welt,' auch nicht sämtliche möglichen Laute (samt denen, die in anderen Sprachen vorkommen), sondern ein begrenztes 'Ganzes,' die Umgangssprache der Zeit und Heimat Patañjalis samt ihrer Hochform. Aber innerhalb dieses relativen 'Ganzes' gilt die Darlegung Patañjalis: Das Negierte ist die Ergänzung des Positiven zum 'Ganze' und umgekehrt" (p. 56). Patañjali would merely consider this an amusing example of *gaurava*.

The law of contraposition is rightly discovered in the passage I 219 (on pp. 93-4).<sup>7</sup> Actually this law is also at the background of Patañjali's dis-

<sup>2</sup> See previous note.

<sup>3</sup> See J. F. Staal, 'Contraposition in Indian logic,' *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress*, Stanford, 1962, 634-49. This paper will be abbreviated as C.

<sup>4</sup> *Journal Asiatique* 233 (1941-42) 126-7.

<sup>5</sup> I use here and in the following Scharfe's terminology which is adopted from: D. Hilbert-W. Ackermann, *Grundzüge der theoretischen Logik*, Berlin-Göttingen-Heidelberg, 1949.

<sup>6</sup> See J. F. Staal, "Negation and the Law of Contradiction in Indian thought: a comparative study," *Bulletin of the School of Oriental and African Studies* 25 (1962) 52-71. This paper will be abbreviated as N.

<sup>7</sup> Cf. C 648.

cussion dealt with on page 60, which could be formalized as follows: the three expressions  $A \rightarrow \bar{B}$  (*yadi santi nāprayuktāḥ*),  $B \rightarrow \bar{A}$  (*aprayuktā na santi*) and  $\overline{A \& B}$  (*santi cāprayuktāśceti vipratīṣiddham*) are equivalent. It is also used in the passages quoted on pages 90 and 119, as Scharfe rightly notes.

In the treatment of *pariyudāsa* and *prasajya-pratiṣedha* (p. 63-4) there is considerable confusion both in the interpretation of the use of these technical terms and in their evaluation in terms of modern logic. Had the author applied the definitions which are mentioned and correctly translated (on p. 64) to the case under discussion (on p. 63), then no confusion in the interpretation could have arisen and the erroneous comparison with mathematical logic could not have been made.

A correct treatment of Patañjali's discussion could be given along the following lines.<sup>8</sup> We shall introduce the following symbols:

- a for 'thing'  
 $A(x, y)$  for 'x denotes y'  
 $B(x)$  for 'x is a particle.'

The interpretation of Pāṇini 1.4.57 *cādayo'sattve* is under discussion. Patañjali rightly declares that:

(1) if the negation is *pariyudāsa* it has to be combined with the next word and the *sūtra* means: "if *ca*, etc., denote not-thing, they are particles." This can be expressed by:

$$A(x, \bar{a}) \rightarrow B(x) \quad (1)$$

(2) if the negation is *prasajya-pratiṣedha* it has to be combined with the verb and the *sūtra* means: "if *ca*, etc., do not denote a thing, they are particles." This can be expressed by:

$$\overline{A(x, a)} \rightarrow B(x) \quad (2)$$

Now consider the noun *vipra* (for which we shall write  $v$ ), which denotes both an action and a thing, i. e.:

$$A(v, \bar{a}) \& A(v, a) \quad (3)$$

(1) if it is *pariyudāsa*, we derive from (3):  $A(v, \bar{a})$  and hence with the help of (1):  $B(v)$ , i. e., *vipra* is a particle.

(2) if it is *prasajya-pratiṣedha*, we do not have  $\overline{A(v, a)}$  so we cannot derive with (2):  $B(v)$ , i. e., *vipra* is not a particle. Hence the negation used is *prasajya-pratiṣedha*.

Scharfe compares this problem with a passage from Hilbert-Ackermann which is quoted in *extenso*. This passage however deals with something quite different, i. e., the negation of predicates of which the quantification is not properly formalized. Hilbert and Ackermann do themselves not pursue this symbolism, for in the following chapter of their work proper symbols for quantified predicates are introduced and it becomes easy to state the problem in the following terms: the negation of  $(x)F(x)$  can be either  $(x)\bar{F}(x)$  or  $(x)F(\bar{x})$ .

Scharfe concludes his treatment of *pariyudāsa* and *prasajya-pratiṣedha* with the words: "eine formale, sofort erkennbare Trennung, die jede Unklarheit beseitigt, bietet aber erst die formale Logik." Unfortunately the opposite is true: the distinction between *pariyudāsa* and *prasajya-pratiṣedha* has no counterpart in modern Western logic because variables are never directly negated and expressions such as we have used in the preceding lines, e. g.,  $A(x, \bar{a})$ , are not well-formed expressions.<sup>9</sup> This does neither mean that modern logicians are wrong, nor that Patañjali is wrong. But it certainly implies that Scharfe is wrong.

The argument of Patañjali in which the expression *arthād āpannam* is used (p. 98) seems quite distinct from the later *arthāpatti* with which Scharfe compares it. The latter is rightly formalized in such a way that it can be expressed by the following meta-theorem:

$$A \rightarrow (B \vee C), A, \bar{B} \vdash C.^{10}$$

Patañjali's argument is formalized by Scharfe in such a way that it can only be expressed by:

$$(A \& B) \rightarrow C, C \rightarrow D \vdash (B \rightarrow C) \rightarrow D.$$

But this is false. Actually Patañjali argues in accordance with the meta-theorem:

$$A \rightarrow B, B \rightarrow C, A \vdash C$$

which consists of merely twice *modus ponens* and is quite distinct from *arthāpatti*, where it is essential that a negation occurs in one of the premisses (for this reason the Mīmāṃsakas hesitated to consider it a valid case of *anumāna*).

On Patañjali's use of examples Scharfe has also some harsh comments to make (p. 83-4). Again it is unreasonable to suppose that Patañjali would lower his usual standard of rigour considerably and believe that *exempla probant*. In Scharfe's

<sup>8</sup> Cf. N 58.

<sup>10</sup> See C 645.

<sup>8</sup> Cf. N 58-61.

example Patañjali merely avoids invoking a general rule because he does not want to prove his assumption. In general Patañjali is far more logical in the use of examples than the early logicians of the Nyāya. Quite apart from this the method in the *Mahābhāṣya* of dealing with grammatical problems by giving an example (*udāharaṇa*) as well as a counter-example (*pratyudāharaṇa*) is a forerunner of the later practice of the logicians to discuss both *sapakṣa* and *vipakṣa* cases.<sup>11</sup> But Scharfe fails to mention the interesting case, i.e., *pratyudāharaṇa*. This is unfortunate for it remains true that, though any number of examples cannot prove one single proposition, one single counter-example is enough to refute an entire theory.

One final remark may be made. Western scholars sometimes tend to discern "magico-religious" backgrounds in everything Indian. So Scharfe. That the teacher of grammar sits with *darbha*-grass facing East (p. 9-10) is a culturally determined phenomenon which can be compared with the fact that some Western Professors put on their gown before they teach and interrupt their work at definite times to drink coffee or tea. All this has nothing to do with the views they pro-

ound. Similarly Scharfe compares the fact that Pāṇini's grammar presupposes knowledge of Sanskrit, for obscure reasons, with Vedic sacrifices where the sacrificer is only successful if he knows why his action is correct. But this fact shows something different and much more important, i.e., that the Indian grammarians, just as modern linguists, attempted to give an adequate as well as concise *description* of the language. They freed themselves to some extent from common assumptions of their era by stressing the descriptive character of their science at the expense of its possible prescriptive value. According to some this was their main achievement.

Scharfe's work is valuable and important because of his systematic search for logical passages in a major work of Indian scientific culture which is not a manual of logic. His results have established beyond doubt that Patañjali occupies a unique place in the history of logic. It is to be hoped that more research will be devoted to the Indian grammarians from the point of view of both logic in general and Nyāya in particular. Such research may well lead to important discoveries, for grammar is the richest and at the same time most Indian product of scientific thought in India, whereas Western logic is only beginning to realize the importance of its linguistic background.

J. F. STAAL

UNIVERSITY OF AMSTERDAM

<sup>11</sup> See C 634-8. For a wider philosophical perspective see also M. Biarreau, 'Le rôle de l'exemple dans l'inférence indienne,' *Journal Asiatique* 245 (1957) 233-40.

**Studies in Indian grammarians I: the method of description reflected in the Śivasūtras.** By GEORGE CARDONA. (Transactions of the American Philosophical Society, 59:1.) Philadelphia: American Philosophical Society, 1969. 48 pp.

Reviewed by J. F. STAAL, *University of California, Berkeley*

The *Śivasūtra*, the list of sounds which precedes the rules of Pāṇini's grammar and which, according to some traditions, was revealed to Pāṇini by Śiva, has been the subject of a number of studies by modern authors: e.g., Sköld 1926, Breloer 1929, Faddegon 1929, Thieme 1935 (beginning of chapter 4), C. K. Raja 1957, Staal 1962, and Misra 1966 (section 2.1)—all listed in the bibliography of the monograph under review. The reason for this interest seems to be twofold: on the one hand, the *Śivasūtra* appears to constitute, at any rate at first sight, a unity which may be studied by itself and which is in this respect unlike any other sequence of rules of comparable length from Pāṇini's grammar; on the other hand, the *Śivasūtra* poses a number of problems, especially concerning the order and classification of its sounds, and illustrates at the same time several Pāṇinian techniques. Investigations of these problems have made it increasingly clear that the *Śivasūtra* cannot be isolated from the rest of Pāṇini's grammar without artificiality, and that the problems it raises can indeed only be solved when their interrelations with many grammatical rules are taken into account. In this study, Cardona has gone over this well-trodden ground once more, carefully weighing the arguments of his predecessors. He has again drawn attention to numerous rules of the grammar which have to be reckoned with, but he has also considerably enlarged the textual basis for comparative studies by making use of the Prātiśākhya literature and some of the later works in the grammatical tradition, both Pāṇinian (in particular Patañjali's *Mahābhāṣya* and Kaiyaṭa's *Pradīpa* thereon) and non-Pāṇinian (in particular the *Kātantra*).

As far as the *Śivasūtra* is concerned, Cardona has not arrived at any startling or unexpected results. But he derives from its study a general conclusion with regard to Pāṇini's methodology which, though not entirely new either, is of considerable interest: namely, that Pāṇini did not aim at abbreviations for abbrevia-

tions' sake, but that he used functional abbreviations in order to state generalizations. In Cardona's words: 'I do not, of course, mean to deny that Pāṇini was interested in composing as brief as possible a set of grammatical statements. It is quite well known how concise the *Aṣṭādhyāyī* is. However, the guiding principle behind the concise formulation of rules in the grammar itself flows from the basic procedure of abstracting the *sāmānya* "general" and *viśeṣa* "particular". For this abstraction leads to considering together rules concerning *utsarga* "general operations" and rules concerning *apavāda* "contravening" of the *utsarga*' (p. 30b). And elsewhere: 'The mania for *mātrālāghava* cannot be ascribed to Pāṇini: it is a property of lesser original Indian grammarians' (41b).

It has long been known that some of the later grammarians used more abbreviations than Pāṇini. Colebrooke, for example, wrote (1803:213) about the *Muḡdhabodha* of Vopadeva: 'but the author has not been content to translate the rules of Pāṇini, and to adopt his technical terms. He has on the contrary invented new terms, and contrived new abbreviations.' Boehtlingk's and Bhattacharya's later editions amply support this statement. Also, the fact that Pāṇini's method itself aimed at such abbreviations as are functional and express generalizations was quite well known to scholars like Faddegon and Buiskool. Buiskool, for example, says at the end of his book on the *Tripādī*: 'It cannot even be the alleged brevity of expression which seemed of prime importance to Pāṇini (1939:155, corresponding to 1934:208), and he formulates as his conclusion that the *Tripādī* is 'a system of thematical groups which are rationally classified and arranged' (1939:156 = 1934:209). Faddegon (1936:36) refers further back to Speyer, author of the *Sanskrit Syntax* of 1886: 'Indeed, the more one studies the *Aṣṭādhyāyī* the more one realizes the depth and correctness of the characterization given by Speyer of this work as "ein wohl durchdachtes und nicht nur künstlerisches, sondern auch kunstvolles Lehrbuch".'

Despite the general validity of these conclusions, it also remains true that the extent of non-functional abbreviations in Pāṇini, especially in connection with the technique of *anuvṛtti*, remains distressingly large, as every student knows. To give one example from the beginning of the grammar: 1.1.3 *iko guṇavṛddhī* is a meta-linguistic statement dealing with the use of the technical terms *guṇa* and *vṛddhī*. The next sūtra, 1.1.4 *na dhātulope ārdhadhātuke*, with *anuvṛtti* of *guṇavṛddhī* from the previous rule, is a rule which treats a special case where *guṇa* and *vṛddhī*, despite other rules, do not take place. Thus this case of *anuvṛtti* is entirely non-functional and ad hoc. A more interesting case, also of fairly widespread occurrence, has recently been discussed by Kiparsky (1968:173).

Cardona's exposition on the whole is clear, though his inclusion of an enormous amount of detailed information, loosely or not at all connected with the argument and partly known (though indeed not widely known), is somewhat tiresome. An exception to his overall clarity is the repeated claim that Pāṇini 'mainly formed *pratyāhāras* not to INCLUDE sounds to be denoted but rather to EXCLUDE certain sounds from a given group' (24b). The total number of sounds, let alone of those in a given group, is not only finite but very small, and it is difficult to see what this claim amounts to. The premises from which this thesis is apparently derived (*pratyāhāra* condensations are not always available where several sounds

have to be referred to, and even when available, they are not used when only two sounds are referred to, *ibid.*) do not make it any clearer. Nor do the examples, e.g. *yañ* (21, 31): this, as Cardona says himself, 'is used to include four sounds: *y, v, m, bh*' (31a); but it is likewise used to exclude at least four others: *t, th, dh, s*. Perhaps this thesis was meant as a substitute for a different principle which is certainly fundamental to Pāṇini's use of *pratyāhāra* condensations, and which I once formulated as follows: 'When Pāṇini uses a *pratyāhāra* to refer to a group of sounds, this group should contain ONLY sounds occurring in the sequence denoted by the *pratyāhāra*, but it need not contain ALL sounds of the sequence' (Staal 1962:6).

Cardona introduces a staggering amount of extra references over and above the already numerous references to the texts. Simple direct references by means of the customary numbers to the rules—which might have been listed separately, for example in an appendix—would have been much more helpful. Thus in the reference to *yañ* quoted above from Cardona's page 31, *yañ* is only explained by the preceding symbol '(35)'. Dutifully turning back the pages, the reader meets with a '(35)' on page 27; but this belongs to a different context. The '(35)' he really needs occurs on page 21. There, under '(a)', the reader finds 7.3.101 *ato dīrgho yañi*, which is formalized not as  $[\check{a} \rightarrow \bar{a}] yañ$  (as one might expect) but as  $[\check{a} \rightarrow \bar{V}] yañ$ . This is explained with a reference to '[C2]', which stands for 1.1.50 as the reader remembers from page 5. Of course, Cardona had sounded an advance warning: 'These conventions will obviously require a considerable amount of cross reference from the reader. However, this is only a bit of memory work which is required of every student of Indian grammar' (6a). But Cardona submits his readers in fact to much that is—fortunately—not at all required of every student of Indian grammar; the need for familiarity with the new conventions and methods of reference he has introduced is only felt by the readers of his book.

Though Cardona's study raises several interesting problems on points of detail and contains not only restatements, but also improvements upon his predecessors, including myself, only two more general topics will be discussed here: the former deals with the form of the rules used by the Indian grammarians, the latter with Cardona's general outlook and methodology.

As to the form of the rules, Cardona has adopted (though not consistently) the expressions introduced by me in 1965, i.e. expressions of the form  $a[b \rightarrow c]d$ , referring to a rule of the form  $b \rightarrow c$  as restricted by a context of the form  $a \_ d$ . I should like first of all to propose the adoption of the following alternative and entirely equivalent expression:

$$(1) b \rightarrow c / a \_ d,$$

thereby conforming to contemporary practice. This may perhaps increase the Sanskrit grammarians' audience among linguists, if ever so slightly. Now, the interpretation of rules of this form is of course not at all obvious. Cardona rightly contrasts in this respect the usage of Pāṇini with that of the Prātiśākhya literature (p. 13, note 32; pp. 32–3). Though his formulation is more correct than it was on an earlier occasion here also referred to (Cardona 1965:230, note 12), it still leaves something to be desired, mainly because Cardona contents himself

with stressing the rather inconsequential difference in case endings attached to what corresponds to *b* and *c*. The main difference (as stated partly in these terms by Cardona) between the Pāṇinīya and the Prātiśākhya rules is that the former teach *ādeśa* 'substitution' and the latter *vikāra* 'modification' or *pariṇāma* 'transformation'. This is to be understood against a somewhat philosophical background, as is well known (for a translation of Kātyāyana's relevant discussion see Paranjpe 1922:35; cf. also Ruegg 1958, 1959:45; Staal 1960:55). Both interpretations are consistent with formula 1, but the 'substitution' interpretation in Kātyāyana's sense requires 1 to be regarded as an abbreviation of:

$$(2) a b d \rightarrow a c d,$$

since the entire expression on the right is regarded as a substitution for the entire expression on the left. In other words, on Kātyāyana's interpretation context-sensitive rules are re-interpreted as context-free rules, of course at a considerable cost.

As is well known, Pāṇini provides unambiguous meta-linguistic statements specifying his method of referring to what corresponds to *b*, *a* and *d* in 1. The *Vājasaneyī-prātiśākhya* has a similar statement relating to *c* in 1, to which Cardona rightly draws attention. *Ṛkprātiśākhya* 1.56, on the other hand, is not a meta-linguistic statement, but refers in passing to the practice adopted in that work. Moreover, the Prātiśākhya practice with respect to what corresponds to *a* and *d* in 1, though not formulated meta-linguistically, is not at all unprincipled. There are exceptional usages, such as *Vājasaneyī-prātiśākhya* 4.124 *svaramadhye* 'between vowels' (which of course is very natural). But on the whole, the left-context *a* in 1 is expressed by the Ablative (as is generally but not exclusively the case in Pāṇini's grammar, and analogously in ordinary, non-technical Sanskrit). In the *Ṛkprātiśākhya* and the *Vājasaneyī-prātiśākhya*, the right-context *d* in 1 is often referred to by means of nominal compounds ending in °*udaya* or °*para*. The commentaries on Pāṇini, especially the *Kāśikā*, followed by modern pandits, also make use of *para*, though uncompounded. What is significant in all this is that, both in Pāṇini's grammar and in the Prātiśākhya literature, rules of the form 1 are formulated quite consistently.

Since Cardona has paid much critical attention to some of my papers (though they constitute by no means his only target), a general comment on differences in outlook and methodology may be justified. Doubtless over-generalizing, it could be argued that Cardona approaches his subject in the first place historically and as a philologist. Others prefer to approach it in the first place systematically and as linguists. The scope for mutual contradiction between these two approaches is fairly narrow. When Cardona suggests, for example, that it might have been my opinion that 'Pāṇini started from scratch in developing the *Śivasūtra* listing' (32, note 80), or suggests that my views might be weakened because Pāṇini adopted classifications of sounds that were not original with him (42), he clearly misses the point. Cardona quotes a remark of mine in full on this final page, apparently to disagree with it: 'The analysis above may give a glimpse of the techniques used in Pāṇini's workshop. It also shows that a phonological analysis of the sounds of a language, expressed in a mere list with subdivisions, can be fully determined by the grammatical structure of the language' (Staal 1962:9-10).

Actually, this is amply supported by Cardona's monograph, which studies the *Śivasūtra* throughout in terms of 'the grammatical structure of the language' as analysed in the rules of the grammarians. Cardona seems to think that the main aim of my 1962 paper was 'to support conclusions bearing on some recent views of phonology' (42a). I had no such axe to grind.

Only on a high level of abstraction could anyone assume that some actual persons are philologists, and others linguists. But even if these approaches are adopted by the same people, it is worthwhile to keep them clearly distinct. It seems obvious that those who are engaged in linguistic, logical and philosophical treatments of Pāṇini would be well advised to take into account what philology and history have to provide. But the opposite holds equally well, and this state of affairs in particular does not imply that there is any methodological priority of philology over linguistics. Cardona seems to assume something like this, for he says at the beginning of his study: 'I do not think we have yet arrived at a sufficiently detailed understanding of Indian grammatical methods to make a comparison with Western methods truly useful. After such an understanding has been attained, it will be both welcome and valuable to make comparisons' (3a). Acceptance of such a hierarchy of priorities is remarkably naïve, and moreover unhistorical, as any survey of the history of Western Pāṇinian scholarship would show. Whitney, for example, aimed at 'a detailed understanding of Indian grammatical methods' and not at 'a comparison with Western methods'. But Whitney's 'philological' understanding was seriously hampered by his underlying linguistic views. Pāṇini was not a philologist but above all a linguist, and he deserves to be treated as such—just as Plato or Śāṅkara deserve to be treated as philosophers. Any analysis that goes beneath the surface is accordingly systematic and comparative, whether or not avowedly so. Let me not belabor this point, but conclude with a quotation from the English philosopher Gilbert Ryle, recently quoted in a collection of papers on Greek philosophy, which applies to linguists to the extent it applies to philosophers:

'There is, of course, always a considerable hazard in attempting to elucidate a doctrine of an earlier philosopher in the light of subsequent and especially of contemporary doctrines. It is always tempting and often easy to read palatable lessons between the lines of some respected but inexplicit Scripture. But the opposite policy of trying to chart the drift of some adolescent theory without reference to the progress of any more adult theories is subject not to the risk but to the certainty of failure. We cannot even state what was a philosopher's puzzle, much less what was the direction or efficacy of his attempt to solve it, unless subsequent reflections have thrown a clearer light upon the matter than he was able to do. Whether a commentator has found such a light or only a will-of-the-wisp is always debatable and often very well worth debating' (Ryle 1939: 324-5).

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Bimal Krishna Matilal, *The Navya-nyāya Doctrine of Negation. The Semantics and Ontology of Negative Statements in Navya-nyāya Philosophy* (= *Harvard Oriental Series*, vol. 46). Cambridge, Mass., 1968. xiii, 208 pp.

The deeper we penetrate into Indian civilization the more it becomes apparent that we require special skills. This of course is not surprising. No one would attribute to a Slavist the professional competence to expound the Periodic Table of Mendeleev. Yet there are people who expect Indologists and Sanskritists to discourse authoritatively on Indian astronomy, Indian medicine, Indian grammar and Indian what-not. And this despite the fact that such subjects survive only because of the Indian paṇḍits who continue to cultivate highly specialized traditions.

Fortunately, expectations are beginning to change and several of these subjects are beginning to come into their own. Navya-nyāya or Modern Logic was first dealt with in a logical spirit in *A Study on Mathurānātha's Tattva-cintāmaṇi-rahasya* (Wageningen 1924) by Saileswar Sen, who had studied under Pandit Sitanath Siddhanta-bhusana at Calcutta and under Barend Faddegon at Amsterdam. The next landmark was *Materials for the Study of Navya-nyāya Logic* (Cambridge, Mass. 1951; reviewed in these pages: 1960b, 68-73) by Daniel H. H. Ingalls, who had studied under Pandit Sri Kalipada Tarkacharya at Calcutta. The book under review falls into this tradition (though Saileswar Sen is not mentioned even once). Professor Matilal studied Navya-nyāya first at Calcutta under several paṇḍits, and then under Ingalls at Harvard, where he wrote the dissertation upon which this book is based. Four comparable books on Navya-nyāya since Ingalls' work have come to the notice of the present reviewer, and these and Matilal's work all seem to be independent of each other: Potter (1957), Sreekrishna Sarma (1960; cf. review Staal 1962b), Mohanty (1966; cf. review Matilal 1968) and Goekoop (1967). For Barlingay (1965) cf. this reviewer's review (1968).

Matilal's book consists of three parts and an appendix. Part I deals with the basic concepts of Navya-nyāya, and corresponds to Ingalls' chapter "An examination of some theories and techniques of Navya-nyāya logic", which it sometimes refines and extends. Part II gives a translation of the *Abhāvavāda* (Discourse on Absence) of Gaṅgeśa's *Tattvacintāmaṇi*, together with a sketch of the background, an introduction and explanatory notes. Part III gives a translation of Raghunātha's *Nañ-vāda* (Discourse on Negative Particles), together with an introduction and explanatory notes. The appendix provides the Sanskrit texts translated in parts II and III, reproduced photographically from the Bibliotheca Indica edition of the *Tattvacintāmaṇi*. A bibliography, a Sanskrit and an English index conclude the book, which is well-produced in accordance with the unfailingly high standards of the *Harvard Oriental Series*.

The exposition of basic concepts is excellent and contains much material that is not elsewhere available in a Western language. There are very few flaws, but as it is the reviewer's thankless job to concentrate on some of these, the reader is advised to keep in mind this overall excellence. Matilal's exposition starts with an analysis which appears to be epistemological (because its object is *jñāna* "cognition") but which on closer inspection turns out to be semantic. According to the Navya-naiyāyikas, cognitions are "qualificative" and are of the form "x is qualified by y", where x is the qualificand (*viśēya*) and y the qualifier (*viśeṣana*). Matilal is careful to distinguish this distinction from the Western grammatical distinction between subject and predicate (to which we shall return), and relate it to the distinction between substance and quality in earlier Nyāya. In the analysis he proposes for this relation, he makes use of a binary predicate Q(x,y). Accordingly, *nilo ghaṭaḥ* "a pot is blue", which the Indian logicians analyze as "a pot is qualified by blue-color", is written as Q(Pot,Blue-color) (I have supplied and shall continue to supply the comma for clarity). This is a useful representation (which in fact is closely related to the expression B(x,y) introduced by this reviewer in 1960a), but Matilal's applications of it are marred by an unfortunate confusion between terms and propositions. What is interesting about this confusion is that it probably reflects a similar or identical confusion sometimes made by the Naiyāyikas themselves.

Thus Matilal says (p. 15) that "a pot" is represented by Q(Pot,Pot-ness), and "a pot is blue" by Q(Pot,Blue-color). Furthermore, combining these two, "a pot is blue" may according to Matilal also be represented by:

$$Q(Q(\text{Pot},\text{Pot-ness}), Q(\text{Blue-color},\text{Blue-ness})) \quad (1)$$

But it is clear that Q(x,y) must either stand for the *proposition* "x is qualified by y" (thus conforming to the usage of modern logicians), or for the *term* "x which is qualified by y". On the first assumption, Q(Pot,Pot-ness) can only mean "a pot is qualified by potness" or "a pot is a pot", and (1) has no meaning at all since the arguments of Q, which must be terms, are themselves Q's, i.e., propositions. On the second assumption, Q(Pot,Blue-color) can only mean "a blue pot" and (1) something like "a blue pot which is qualified by blueness".

The best way out of this tangle seems to be to adopt the first assumption and use restricted-variables (as was done by this reviewer in 1960a) to stand for terms. Accordingly, Q(x,y) stand for "x is qualified by y" and  $\alpha xQ(x,y)$  for "x which is qualified by y" (literally, "x such that x is qualified by y"). Hence *nilatva-viśiṣṭa-ṇila-rūpa-vān ghaṭatva-viśiṣṭaḥ ghaṭaḥ* will not be represented by (1) but by:

$$Q(\alpha xQ(x,\text{potness}), \alpha xQ(x,\text{blueness})) \quad (2)$$

This is well-formed, unambiguous and stands in very close correspondence to the Sanskrit original.

The very interesting discussion of the Navya-nyāya metalanguage which Matilal presents on pages 29-30 makes, as it stands, no sense either and must also be stated in different terms. According to Matilal, the analysis of *rakta-puṣpavati latā* "the creeper possesses a red flower" may be represented by:

$$Q(Q(a,b),Q(Q(c,d),Q(e,f))) \quad (3)$$

where a, b, c, d, e and f represent, respectively, creeper, creeper-ness, flower, flower-ness, red-color and red-ness (similarly in Matilal 1966). But (3) again is meaningless since none but the innermost Q's have terms as their arguments. A possibly appropriate logical expression should be of the form:

$$Q[\alpha zQ(z,b), \alpha y\{Q(y,d) \wedge Q(y, \alpha xQ(x,f))\}] \quad (4)$$

Notice that the individuals a, c and e are missing from (4), just as "pot" and "blue"

were missing from (2). But this precisely illustrates the point of the Navya-naiyāyika attempt at an analysis of individuals in terms of universals and of the relation Q.

A simple fact about the Sanskrit language appears to underlie the confusion between terms and propositions: optional (and in fact widespread) absence of the copula. Accordingly, *nilo ghaṭaḥ* can either function as a term (meaning “a blue pot”) or as a proposition (meaning “a pot is blue”) (I disregard here the considerable problems arising from the absence of definite or indefinite articles in such expressions). These expressions are disambiguated in larger contexts: for example, *khaṇḍate nilo ghaṭaḥ* can only mean “a (or: the) blue pot breaks”. This is a fairly superficial fact about Sanskrit syntax, however, and I do not mean to imply that the distinction between terms and propositions does not obtain in Sanskrit. On the contrary, it is clearly fundamental to Sanskrit syntax and semantics, and was also clearly recognized as such by Indian logicians (and of course by the grammarians). But this superficial ambiguity nevertheless seems to have generated some confusion.

Matilal looks at these semantic analyses by the Navya-naiyāyikas largely from a logical and philosophical point of view. This in fact constitutes one of the great merits of his work. Still, a linguistic analysis should also be rewarding, as the following few remarks may show. In transformational grammar, noun phrases such as “the blue pot” are generally derived by a transformation from expressions containing embedded sentences (actually corresponding to restrictive variables in logic), viz., from expressions such as:

$$\text{the pot } \left[ \begin{array}{c} \text{the pot is blue} \\ \text{S} \end{array} \right] \quad (5)$$

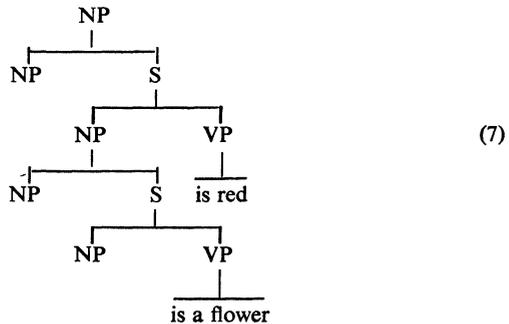
In Sanskrit this transformation could be formulated in a particularly simple manner, yielding derivations such as:

$$\text{ghaṭaḥ } \left[ \begin{array}{c} \text{nilo ghaṭaḥ} \\ \text{S} \end{array} \right] \Rightarrow \text{nilo ghaṭaḥ} \quad (6)$$

(disregarding word-order). This is a typically linguistic transformation, for both sides of (6) correspond to the same restricted variable. The Naiyāyika analysis of individuals in terms of universals appears to parallel a similar process, involving transformations by which nouns are derived from relative clauses where they occur in predicate position (see Bach 1968; cf. Staal, forthcoming). Thereby “red” is derived from “red x”, itself derived with the previous transformation from  $x \left[ \begin{array}{c} x \text{ is red} \\ \text{S} \end{array} \right]$ . Similarly, “a flower” is

derived from “a flower x”, itself derived from  $x \left[ \begin{array}{c} x \text{ is a flower} \\ \text{S} \end{array} \right]$ . Combining these

processes, the underlying structure of the noun phrase “a red flower” might be of the form:



Various alternative underlying structures have been suggested by contemporary linguists. But structures such as (7) may at any rate suffice to show that linguistic

analyses (themselves in part corresponding to the uses of restricted variables in logic) have much in common with the Navya-naiyāyika analysis of expressions by recursive applications of a binary relation. Matilal's remark that the distinction between qualificand and qualifier is different from the subject-predicate distinction in grammar can now be seen to be appropriate only with respect to the *surface* distinction between subject and predicate. However, the corresponding distinction in transformational grammar, i.e., the NP-VP distinction, obtains not only at the surface level but also at deeper levels. The *underlying* NP-VP relation, which may be itself semantic or very close to a semantic relation, is therefore quite similar to the relation Q manipulated by the Navya-naiyāyikas.

These considerations relate to the general analysis of the conceptual apparatus of Navya-nyāya, and though Matilal is fully justified in devoting much attention to them, they are not specific to the central topic of the book, i.e., negation. The Naiyāyikas regarded negation not as a psychological feature of negative judgments, but as a component of cognitions of the kind described before. Though this was a good move (as Matilal rightly stresses), it also opened the door for further confusion between terms and propositions. Matilal describes the Nyāya argument in terms which lead to results that can only be described as meaningless (p. 94): in order to negate propositions of the form "a (is) P", the latter are first reduced to existential statements of the form "there is an a (which is) P", then to assertions from which the existential operator has been dropped, i.e., "someone asserts a (which is) P" and finally to complex terms from which the assertion part has been dropped, i.e., "a (which is) P". Negations of propositions therefore pertain ultimately to these complex terms, and since terms apparently are not negated (this link is missing in Matilal's associative chain), the negation must attach to the predicate. Hence the negation of "a (is) P" is "a (which is) not-P".

On this analysis, the negation of a proposition is a term. This result is reached by arguing in tune with what can only be regarded as a meaningless string, viz.:

$$(Ex)P(x) \rightarrow \alpha xP(x) \quad (8)$$

and arriving at an equally ill-formed jumble, viz.:

$$\neg P(a) \rightarrow \alpha x(\neg P)(x) \quad (9)$$

I think the situation is not quite that bad. It seems that the Navya-naiyāyikas were merely trying to state that a proposition of the form (Pa) is negated by negating its predicate, while at the same time maintaining the distinction between a proposition and its propositional content or *śābdabodha*, a notion which Matilal rightly relates to Frege's notion of content (p. 92-93). Let us consider an actual example, provided by Matilal in his translation and notes on the *Nañ-vāda* (p. 150-151). The proposition:

$$\text{caitraḥ pacati} \text{ "Caitra is cooking"} \quad (10)$$

has for its negation the proposition:

$$\text{caitro na pacati} \text{ "Caitra does not cook"}. \quad (11)$$

The *śābdabodha* of (10) is:

$$\text{pākānukūlakṛti-mān caitraḥ} \\ \text{"Caitra possessing an effort conducive to cooking"} \quad (12)$$

and that of (11):

$$\text{pākānukūlakṛty-abhāva-vān caitraḥ} \\ \text{"Caitra possessing an absence of such an effort as is conducive to cooking"}. \quad (13)$$

Matilal's description of the Nyāya argument suggests that the *śābdabodha* (13) is the

negation of the proposition (10). But this is quite unnecessary, for (12) and (13), as we have seen, are ambiguous and may also be construed as propositions, i.e.:

“Caitra possesses an effort conducive to cooking” (12’)

“Caitra possesses an absence of such an effort as is conducive to cooking” (13’)

But now it is obvious that (13’) must be regarded as the analysis of the negation of (10). All undesirable consequences vanish obligingly.

The notion of *abhāva* “negation”, which is the cornerstone of the Nyāya analysis, is introduced by the *śābdabodha*. In linguistic terms, a *śābdabodha* is arrived at by nominalization; *abhāva* may accordingly be regarded as a nominalization of the negative element *na*. The logical analysis itself may be formalized along the following lines.

Let ‘E’ express the *śābdabodha* of E, which is either a proposition or a predicate; conversely, let the proposition  $\vdash S$  express the assertion that the *śābdabodha* S of a proposition obtains. Accordingly, P(a) and  $\vdash$  ‘P(a)’ are equivalent expressions. Let  $\Omega$  stand for the negation which occurs in a *śābdabodha* and which corresponds to the negative element in the corresponding negative proposition. Lastly, let the binary predicate B(x,y) stand for “x possesses y”.

Then, if (10) and (11) are represented by (Pa) and  $\neg$  P(a) respectively, their respective *śābdabodha*’s (12) and (13) are ‘B(a, ‘P’)' and ‘B(a, ‘ $\Omega$  P’)'’. Consequently, (12’) and (13’) have for their respective representations  $\vdash$  ‘B(a, ‘P’)' and  $\vdash$  ‘B(a, ‘ $\Omega$  P’)'’. The Naiyāyikas, then, analyze  $\neg$  P(a) as  $\vdash$  ‘B(a, ‘ $\Omega$  P’)'’. This is at worst a harmless exercise.

There is at least one other kind of negation which is analyzed along similar lines: mutual absence or the negation of identity. Matilal shows that this negation, which is of the form  $x \neq y$  and which in the early Nyāya was sharply distinguished from the negation of a relation, which is of the form  $\neg A(x,y)$ , are analyzed in similar terms in Navya-nyāya. This is of course a step forward, since identity is also a relation and can very well be written in the form  $A(x,y)$ .

In the previous discussions we have only met with negative propositions and, by implication, with negative predicates. But Indian scholars also evolved a very interesting theory of negative terms (which has nothing to do with the confusion dealt with before). This theory is actually due to the grammarians and to the ritualist philosophers of the Mīmāṃsā, not to the Naiyāyikas. But it was discussed at length in the Nyāya and Navya-nyāya, and Matilal reveals interesting comments on it from the hand of Raghunātha (pp. 156-161). The distinction itself is generally illustrated by the ambiguity of *yajatiṣu ye yajāmahe iti karoti nānuyājeṣu* which can mean either:

“in sacrifices one utters *ye yajāmahe*, but one does not utter it in *anuyāja* sacrifices” (14)

or:

“in sacrifices which are not *anuyāja* sacrifices one utters *ye yajāmahe*” (15)

The former contains what is called a *prasajya-pratiṣedha* (verbally bound negative), the latter what is called a *pariyudāsa* (nominally bound negative) (p. 156).

A formalization was proposed by the present reviewer in 1962a (p. 59-60). Let *a* stand for “sacrifices”, *b* for “*anuyāja*-sacrifices”, *c* for “*ye yajāmahe*” and F(x,y) for “at x one says y”. Disregarding the fact that we are really dealing with injunctions (which would complicate the following expressions in inessential ways), *prasajya-pratiṣedha* may then be expressed by  $F(a,c) \wedge \neg F(b,c)$ , which corresponds to (14), and *pariyudāsa* by  $F(a \wedge \neg b,c)$ , which corresponds to (15). Notice that such expressions are only well-formed if the negation sign and other expressions for logical connectives are defined in combination with terms as well as propositions. It is of course easy to use different signs for term and for proposition negation; but in the present context there is no ambiguity.

Matilal's translations and notes show that Raghunātha argued that the *pariyudāsa* interpretation is not due to the fact that the negative is construed with the following term (which is the traditional view), but to the fact that there is mutual absence between sacrifices and *anuyāja* sacrifices. This suggests an alternative formalization for *pariyudāsa*, viz.,  $F(a,c) \wedge (a \neq b)$ . Matilal discusses several grammatical objections to this view, together with Raghunātha's rebuttals.

At the beginning of this review I said that the overall excellence of Matilal's expositions should be kept in mind throughout the following discussion. I should like to stress this once more and add, that in the preceding pages only very few aspects of Matilal's work could be discussed adequately — mainly passages, moreover, which are open to criticism within the perspective of formal logic. But Matilal's book contains much more. For example, the sketch of the background of Gaṅgeśa's views includes translations from Gautama's *Nyāyasūtra* and from Vātsyāyana's *Nyāyasūtrabhāṣya*. This sketch and the notes on Gaṅgeśa's *Abhāva-vāda* contain translations from several famous works which until now have been hardly or not at all made available or translated: e.g., Nyāya works such as Śrīdhara's *Nyāyakandali* and Jayanta's *Nyāyamañjarī*; but also several Buddhist treatises and Śrīharṣa's *Khaṇḍanakhaṇḍakhādyā*, a celebrated Vedānta work. Professor Matilal is obviously completely at home in the very vast and rich traditions of Indian logic and philosophy, and he combines this knowledge with a very thorough grasp of the problems and methods of contemporary Western logic and philosophy. May the criticisms voiced in the course of this review be taken for no more than what they purport to be: tributes to a scholar who has made some of the thorniest technical texts in Sanskrit more intelligible; and challenges to continue to provide us with more to chew on from these "clichés of Navya-nyāya" as Ingalls called them (1951, p. 2), which "if they are cumbersome in comparison with the symbols of modern logic, are certainly superior as a logical instrument to the language of ordinary discourse".

Berkeley

J. F. Staal

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Bimal K. Matilal, *Epistemology, Logic and Grammar in Indian Philosophical Analysis*, Mouton, The Hague-Paris 1971, pp. 183.

Over the last decade, Professor Matilal has been the leading exponent of the view, that Indian philosophy is not necessarily vague and mysterious, but is 'serious' philosophy in the sense of contemporary Western analytical philosophy. He is the leading exponent of this view not because he is the first to have defended it, but because of the thorough familiarity with Sanskrit philosophical texts which he evinces when defending it. This familiarity does not merely provide him with a wealth of material, but has also prevented him in almost all cases from reading into his sources doctrines which are in fact not there. Whether or not one agrees that such Occidental seriousness is something for which Indian philosophy is to be commended or which makes it more respectable, Matilal's approach highlights important aspects of Indian philosophy which have often been unjustly neglected.

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There are controversial features to such a philosophical approach and one of these may be briefly commented upon here, especially since Matilal himself does not mention it. The most widespread view among professional Orientalists (or should we now say: scientists of the humanities in Asia and North Africa?) is that the only sound approach to texts – whether they are literary, scientific or philosophical – is the philological approach. In the study of Indian philosophy this perspective is accepted without question in the Vienna school. It is also strong in Japan, though there are Japanese scholars who do not adhere to it. On this view, philology is the only neutral tool which can provide a reliable interpretation of a text. Only when such an interpretation is at hand can there be scope for philosophical interpretations and comparisons.

Though it is true that without philology we would be left in the air, there is a simple and devastating objection to this philological apriorism: the interpretations which result from mere philology are themselves not neutral. In the case of philosophical texts, for example, they exhibit the implicit philosophical views and preconceptions of philologists, of which they themselves are most often unaware. In many cases these underlying views reflect a past stage of development of Western philosophy. This can be shown to have affected the interpretation of simple terms (e.g., *pakṣa* in Indian logic: see Staal, 1973) as well as that of entire texts (e.g., the *Yogasūtra*: Staal 1975a, Chapter 6, Section 3). Therefore, Matilal's explicit philosophical perspective, which is that of contemporary Western analytical philosophy, can teach us at least as much as the work of scholars whose orientation is purely philological.

The book under review contains good examples of this philosophical approach. It is largely concerned with the presentation of Indian philosophical views in terms intelligible to modern Western philosophers. The work consists of five chapters: 1. Perception and Language; 2. Individuals, Universals, and Perception; 3. Early Grammarians on Philosophical Semantics; 4. Empty Subject Terms in Logic; 5. Negation and the Mādhyamika Dialectic.

In the chapter on perception and language, Matilal provides succinct and clear surveys of the epistemological doctrines of the Nyāya, the Yogācāra, Bhartṛhari's Śabdādvaīta and of Dinnāga. In spite of brief introductory notes on chronology (which do not cover all the names met with in the text), a Western philosopher will soon be lost unless he makes a list of the *dramatis personae*. A table of the systems, the philosophers, and the main doctrines would indeed have been very useful, but this defect can be turned into a virtue if the reader compiles such a table by way of exercise.

Here as elsewhere in the book, there is no consistency in the reference to Sanskrit terms. Basically, these are limited to technical terms, e.g., *kalpanā*, *abhūtaparikalpa*, etc. But there is no need for the original of such Sanskrit words as *abhrānta*, 'non-erroneous' (p. 38). The term *vyavahāra* is translated as 'worldly affairs', a conventional rendering in many books on Indian philosophy which suggests a contrast between sacred and secular and is entirely inappropriate; in these contexts, the term *vyavahāra* merely means 'communication'.

Matilal's adaptation of the idioms of Western philosophy is not always felicitous; the uses he makes of the terminology of contemporary linguistics are positively misleading. The introduction of 'deep' and 'surface' structures has little to offer to the elucidation of the topics under discussion (pp. 26, 31): to say that a child is aware of deep structures before he is aware of surface structures is wrong (for no speaker of a language need be aware of deep structures) and is curiously similar to the view that a transformational derivation is a psychological picture of the process of speech production – which in turn is like saying that the speaker determines first whether to use the Active or Passive, and subsequently decides what to talk about.

Despite the occurrence of the word 'Grammar' in the title of his book, it is linguistics, Indian as well as Western, which is Matilal's weakest point. This is partly because he has not made use of many recent publications, as we shall see. In the chapter under discussion, his treatment of quotation and of the particle *iti* is vitiated by this lack of information. Thus Matilal says (p. 25) that the distinction, 'made for the first time, between conception (*sic!*) and its phonological realization, may be attributed to Vātsyāyana' (whom Matilal places in the period 300–400 A.D.: p. 21). In fact, the distinction between *use* and *mention* is almost a thousand years older: it goes

back to Pāṇini, as was first shown by Brough (1951) and discussed by the present reviewer in a variety of contexts (last in 1975b).

The surveys on Bhartṛhari and on Dīnāga are very good surveys and raise at the same time many questions. It is true that Bhartṛhari went 'further' than the Buddhist idealists (p. 29); but it remains to be shown how exactly he explained the origin of the idealist position in terms of language. Using contemporary terms, it is probably correct to say that according to Bhartṛhari, our capacity to construct an external world is the same as our capacity to acquire and use language. Matilal wonders in passing (p. 29) whether Bhartṛhari's theory 'is of a piece with the belief of some modern linguists (like Edward Sapir or Benjamin Whorf), that different societies with different languages live in distinct WORLDS . . .' But it is clear from Bhartṛhari's comparison of human language with animal instincts (the bird's capacity to make a nest, the spider's capacity to weave a web, etc.) that he regarded the capacity for language as a general and species-specific human property (which is more in line with Chomsky than with Whorf).

The last sections of the first chapter set up Dīnāga's doctrine mainly as a target for the criticisms which will constitute the topic of the second chapter. While Matilal refers to Hattori's work on Dīnāga, and thus makes indirect use of the Tibetan text, his own account seems generally to follow the characterizations of Dīnāga's doctrine which are met with in the works of his Nyāya critics, especially Uddyotakara. The question naturally arises whether the two pictures of Dīnāga correspond. If Hattori and Matilal would collaborate on this problem, a fascinating article might result.

A second chapter on 'Individuals, Universals, and Perception', addresses itself to the Nyāya and Mīmāṃsā criticisms of Dīnāga. Matilal is now clearly on familiar ground and gives useful summaries of the arguments found in the texts. The discussion centers on the controversy between the phenomenalism of Dīnāga and the physicalism of the Nyāya. It may be useful to put this in a wider perspective by referring not only to pertinent discussions among Western philosophers (as Matilal does) but also to the actual practice adopted in natural sciences such as physics: for the problem is then seen to revolve around the issue of *theory formation*. In physics, there is for example a phenomenalist theory of heat (thermodynamics) which *describes* the different states of matter (solid, liquid and gaseous) which are observed under different temperatures; but there is also a physicalist theory of heat (molecular and atomic theory) which *explains* the facts observed and described in thermodynamics by *postulating* unobserved entities, viz., molecules and atoms. The phenomenalist discipline is in a sense more empiricist and seems therefore more 'scientific'; but the physicalist theory goes beyond *describing* the data: it *explains* them in terms of entities which cannot be directly verified in empirical terms. The advantage of a phenomenalist description is that it is often correct, and that of a physicalist theory that it goes much deeper. The disadvantage of a phenomenalist description is that it does not go very far, and that of a physicalist theory that it is sometimes wrong. This is relevant to the controversies between Dīnāga and his Nyāya critics. My suspicion is that Dīnāga's epistemology thus put in perspective, is right as far as it goes, but not very interesting; the Nyāya epistemology is much more interesting, but probably wrong.

The second chapter of Matilal's book is full of interesting observations. Uddyotakara was aware of the fact that '2' occurs in '2 apples' though it does not occur in either apple by itself (p. 55). Matilal explains the origination of the difficult terms of the later Nyāya *avyāpyavṛtti* 'non-pervasive occurrence' and *avacchedaka* '(de)limitor' (pp. 58–61). On p. 61 he neatly explains why the Navya-nyāya terminology is ontologically neutral. On p. 67, some formalism is introduced, but this leads to pitfalls on the next page. Matilal introduces square brackets to indicate a term as distinct from a proposition. For example, '[Fx]' denotes a term which is *F*, e.g., '[x is a cow]' denotes 'cow'. This implies, presumably, that 'Fx' by itself denotes a proposition. Now, a singular term is introduced by means of the symbols: '[(ix)Fx]'. But here, '(ix)Fx' without the square brackets cannot possibly denote a proposition, and thus the square brackets become redundant. The reason for these curious moves is to pave the way for a confusion between terms and propositions which will be dealt with later (pp. 91–96). But this

formalization itself is confusing, as is borne out by the fact that no further use is made of it. If we wish to analyze these concepts formally I think there is no other way but to use restricted variables (or an equivalent notation: see Staal 1960).

The Nyāya doctrine of universals is summarized in seven points (pp. 73–75). There is an unexplained at least apparent contradiction between two of these, viz., (d) ('A generic property cannot be instantiated in another generic property') and (g) ('Each generic property is NESTED in another generic property'). – On p. 75, there is a brief comment on the connexion between the Nyāya hierarchy of generic properties and the caste system. It is suggestive to point out that the term *saṃkara* is used both for the overlapping of classes which the Naiyāyikas repudiated, and the intermarriage between castes which Hindu orthodoxy prohibits.

The second chapter ends with a discussion of a problem which had been raised by the present reviewer when reviewing Matilal's earlier book on negation in Navya-nyāya (Staal, 1971). On the one hand, Matilal seems to agree with the criticism voiced there, that he, following Navya-nyāya, sometimes confused terms and propositions. But on the other hand, he suggests a way out of this difficulty by ingeniously trying to make use of Frege's analysis of propositions into contents and assertions. I am afraid this will not solve the problem but only introduce further confusion. It is not difficult to see why. Frege expressed the proposition which asserts that  $x$  is  $F$  by writing:

$$\vdash F x. \quad (1)$$

The 'content' ' $F x$ ', unasserted, denotes the thought that  $x$  may be  $F$ , which may be asserted, denied, or entertained as a possibility or a belief. In particular, when it is asserted, this is expressed as ' $\vdash F x$ ', and when it is denied, as ' $\neg F x$ '.

In the aforementioned review I referred to the confusion between the asserted proposition ' $\vdash F x$ ' (in Frege's terms), and the term  $x$  of which  $F$  is asserted, viz., ' $x$  which is  $F$ '. Here,  $x$  is a term. But the 'content' ' $F x$ ', in Frege's use, is not a term. It is a possible state of affairs, or, as Matilal puts it (p. 92), 'a COMPLEX of terms representing a state of affairs'. Frege's distinction between ' $F x$ ' and ' $\vdash F x$ ' does not resolve the ambiguity between either of these expressions and the term  $x$  itself.

The third chapter, 'Grammarians on Philosophical Semantics', provides less than one might expect. It contains lots of interesting information, taken mostly from Patañjali and Bhartṛhari. And yet we now know that the 'philosophical semantics' of the Sanskrit grammarians is vastly more interesting and rich than the relatively simple issues dealt with in this chapter. A more adequate picture could have been drawn if use had been made of numerous publications in recent years by Brough, Kunjunni Raja (whose book on Indian theories of meaning is mentioned only in passing), Ruegg and the present reviewer – to mention only these. If Matilal feels that all of these have been unimportant, he should at least say so.

I shall not try to substantiate this verdict, for this would mean providing a survey of recent work on the philosophical semantics of the Sanskrit grammarians (for such a survey, see Staal, 1969, reprinted 1976). However, one simple example may be in order. Matilal rightly points out, at the end of this chapter, that the Indian grammarians were not primarily interested in logic. Accordingly, they were not interested in the 'substitution of identicals', viz., the principle, in Matilal's words, 'that if a name occurring in a true sentence is replaced by another name with the same nominatum, the sentence remains true' (p. 121). The proper perspective within which to place this, however, is the notion of substitution (*ādeśa*), in all likelihood the most basic notion of Indian linguistics. This notion led to linguistico-philosophical discussions on the organization of grammar, rules, metarules, etc., and also to clarification of the notion of substitution itself. One question which received much attention was the relationship between ' $C(a)$ ' and ' $C(b)$ ', where ' $C(b)$ ' is the result of substituting ' $b$ ' for ' $a$ ' in ' $C(a)$ ' – for example, is ' $C(b)$ ' a transformation of ' $C(a)$ ', or can it only be reached by substituting the entire ' $C(b)$ ' for ' $C(a)$ '? The substitution of identicals now becomes a special case, and the notion of truth is replaced by the notion of well-formedness (as was effectively done in Western theories of truth).

Matilal's lack of familiarity with this area is also apparent in details. For example, Yāska is referred to as: 'probably a pre-Pāṇinian linguist and philosopher of language' (p. 111). It is possible that this was written with a completely ignorant Western reader in mind. But then, Matilal in general rushes forward to dispel such ignorance. Here it would have only been proper to refer to Yāska as author of the *Nirukta*, which is probably the first work on semantics in the Indian tradition. Its relationship to Pāṇini's grammar has been discussed for over a century.

Chapter 4 on 'Empty Subject Terms in Logic' deals with problems which easily become tricky. Matilal's most challenging suggestion is, that the Buddhist logicians employed a kind of 'exclusion-negation', whereas the Naiyāyikas used a kind of 'choice-negation'. Now these two negations are related, respectively, to the principles of the excluded middle and of non-contradiction, and this is nowhere made clear. I am also not sure that Strawson's criticism of Russell in terms of ordinary language is to the point, as Matilal claims (p. 144). And so we are left with an application of very complex notions to the text of Ratnakīrti, which is one of the most difficult texts in Buddhist philosophy. That Ratnakīrti was even more subtle than Matilal seems to indicate is apparent from recent work by McDermott and Mimaki.

If Matilal's suggestion is right, it would be surprising; for the background of these notions, in India and in the West, is entirely different. That the results were in some respects similar is of course possible. Ingalls discovered a bit of intuitionistic logic in Raghunātha, who had arrived at it on grounds that could have nothing in common with the mathematical intuitionism of Brouwer (Ingalls, 1951, p. 68). C. T. K. Chari, on the other hand, detected similarities between intuitionistic logic and the notion of *anirvacanīyatvam* in Advaita-Vedānta (Chari, 1953–1954) which are surely imaginary. It remains to be seen whether Matilal should be ranked in this respect with his Harvard teacher or with his Indian colleague. It is at any rate certain that he has opened up a new and possibly fruitful avenue of investigation.

Matilal, incidentally, uses the Western concepts of middle and major term, to interpret, respectively, the Indian notions of *hetu* and *sādhya* (p. 128–129). This interpretation, though widespread, is erroneous, as I have shown elsewhere (Staal, 1973).

In his fifth and last chapter, 'Negation and the Mādhyamika Dialectic', Matilal returns to the now fashionable Nāgārjuna, whose doctrine of emptiness had been succinctly referred to in Chapter 1 (p. 26). Matilal's approach is serious and sober, as one might expect of him; he avoids the fancy stuff met with in some recent publications on the subject. From a logical point of view, the crux of the matter is the interpretation of the apparent contradictions of the *catuṣkoṭi*. Matilal seeks the solution of this problem in the notion of negation employed in its formulation. In order to introduce this he refers to the distinction between two kinds of negation, *paryudāsa* and *prasajya-pratiśedha* (p. 163), notions which are familiar from the work of the Sanskrit grammarians, the Mīmāṃsā and the Nyāya. The Buddhist logicians of the Mādhyamika school seem to have misunderstood or altered these notions, and in the wake of the ensuing muddle, Matilal seems to have bet on the wrong horse. For he suggests that the principle of non-contradiction does not hold for *prasajya-pratiśedha* negations. Maintaining that this negation is employed in the *catuṣkoṭi*, the apparent contradiction will then disappear (p. 164). But in fact, the principle of non-contradiction *does* apply to *prasajya-pratiśedha* negations. It is *paryudāsa* negations for which it need not hold. Matilal is right that there is no justification for assuming that *paryudāsa* negations were employed in the *catuṣkoṭi*. In fact, all its formulations are clearly in terms of *prasajya-pratiśedha* negations. But then, the contradiction which Matilal sought to explain remains unresolved.

Matilal could have avoided this error if he had read Staal, 1962. In this paper, *prasajya-pratiśedha* negations were written as ' $\neg F(x)$ ', and *paryudāsa* negations as ' $F(\neg x)$ '. The principle of non-contradiction formulated for the former kind of negation, i.e.:

$$\neg[F(x) \wedge \neg F(x)] \quad (2)$$

has the form of the regular principle of non-contradiction (4) and is therefore valid (as was indeed accepted by Nāgārjuna). The principle of non-contradiction formulated for *paryudāsa*

negation, i.e.:

$$\neg[F(x) \wedge F(\neg x)] \quad (3)$$

need not be valid (otherwise it would be incorrect to say, for example, that women as well as men are animals).

Matilal's discussion is further marred by his failure to distinguish between the principle of non-contradiction:

$$\neg(P \wedge \neg P) \quad (4)$$

and the principle of the excluded middle:

$$P \vee \neg P. \quad (5)$$

These principles could have been used to clarify the distinction between choice and exclusion negations, as mentioned before. The resolution of the paradoxes of the *catuṣkoṭi* offered by Matilal are therefore as unhelpful as all other solutions that, to my knowledge, have appeared in the literature (for further discussion see Staal, 1975a, Chapter 2).

It will be clear even from this brief review that Matilal's book is rich in content, full of ideas and opens up new avenues of exploration. It is easily the most interesting and philosophical contribution to the study of Indian philosophy to have appeared in recent years. Given his mastery of the field, one might wish that Matilal had developed some of these investigations in greater detail. The results would have been of more lasting value. As it stands, the book consists of one relatively continuous investigation (roughly the first half of the work) and several reports on work in progress. Let us hope that Professor Matilal will return to all these topics and give them the close attention they deserve. His recent appointment to the Spalding chair at Oxford – for which no better choice could have been made – may provide him with the leisure and opportunity needed to pursue these researches. In the meantime, Western philosophers will probably continue to neglect Indian philosophy. With Matilal's work gaining in clarity and depth, there will be less and less of an excuse for doing so.

Berkeley-Honolulu

FRITS STAAL

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