

Appendix A

Concrete Art Manifesto



In 1930 the Dutch painter Theo van Doesbourg (a pseudonym for Christian Emil Marie Küpper) published the *Manifesto for Concrete Art*, advocating the total freedom of art from the need to describe or represent natural objects or sentiments. The *Manifesto* is reported in Fig. A.1.

The translation of the *Manifesto* is the following one:

BASIS OF CONCRETE PAINTING

We say:

1. Art is universal.
2. A work of art must be entirely conceived and shaped by the mind before its execution. It shall not receive anything of nature's or sensuality's or sentimentality's formal data. We want to exclude lyricism, drama, symbolism, and so on.
3. The painting must be entirely built up with purely plastic elements, namely surfaces and colors. A pictorial element does not have any meaning beyond "itself"; as a consequence, a painting does not have any meaning other than "itself".
4. The construction of a painting, as well as that of its elements, must be simple and visually controllable.
5. The painting technique must be mechanic, i.e., exact, anti-impressionistic.
6. An effort toward absolute clarity is mandatory.

Carlsund, Doesbourg, Héliion, Tutundjian and Wantz.

ART CONCRET

GRUPE ET REVUE FONDÉS EN 1930 A PARIS

PREMIÈRE ANNÉE - NUMÉRO D'INTRODUCTION - AVRIL MIL NEUF CENT TRENTE

BASE DE LA PEINTURE CONCRÈTE

Nous disons :

- 1° L'art est universel.
- 2° L'œuvre d'art doit être entièrement conçue et formée par l'esprit avant son exécution. Elle ne doit rien recevoir des données formelles de la nature, ni de la sensualité, ni de la sentimentalité.
Nous voulons exclure le lyrisme, le dramatisme, le symbolisme, etc.
- 3° Le tableau doit être entièrement construit avec des éléments purement plastiques, c'est-à-dire plans et couleurs. Un élément pictural n'a pas d'autre signification que « lui-même » en conséquence le tableau n'a pas d'autre signification que « lui-même ».
- 4° La construction du tableau, aussi bien que ses éléments, doit être simple et contrôlable visuellement.
- 5° La technique doit être mécanique c'est-à-dire exacte, anti-impressionniste.
- 6° Effort pour la clarté absolue.

Carlsund, Doesbourg, Héliion, Tutundjian, Wantz.

Fig. A.1 *Concrete Art Manifesto*, by Theo van Doesburg (1930)

Appendix B

Cartographic Results for Roads



This appendix shows in Fig. B.1 some more roads and their different representations with and without abstraction. The representations result from:

- a direct *symbolization* (initial),
- the cartographic result produced by the hand-crafted expert system GALBE, specifically developed to generalize road [389, 391],
- the result produced by the set of rules obtained by learning without abstraction,
- the result produced by the set of rules obtained by combining learning and abstraction.





































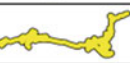



<i>Before symbolization</i>	<i>Direct Symbolization</i>	<i>Hand-crafted expert-system</i>	<i>Ruled learnt without abstraction</i>	<i>Rules learnt with abstraction</i>
				
				
				
				
				
				
				
				

Fig. B.1 Different road generalization results, for different roads. The improvements brought by abstraction are clearly visible

Appendix C

Relational Algebra



In this appendix we recall the basic notions of Relational Algebra for manipulating relational databases. Relational Algebra has been proposed by Ullman [540] as a formal tool for modeling relational database semantics.

Relational databases provide operators for handling *relations* in their extensional form. Given a set \mathcal{X} of variables, a n -ary relation $R(x_1, x_2, \dots, x_n)$ involving the variables in \mathcal{X} is represented as a table with n columns and k rows, where each row describes an n -ple of individuals of \mathcal{X} satisfying R .

The type T of a relation $R(x_1, x_2, \dots, x_n)$ is defined as:

$$T : X_1 \times X_2 \times \dots \times X_n \tag{C.1}$$

where X_1, X_2, \dots, X_n are the domains from which the individuals bound to x_1, x_2, \dots, x_n can be taken. The relation $R(x_1, x_2, \dots, x_n)$ is a subset of its type.

A relational database provides a set of operators that allow one to compute new relations from the existing ones [539]. Operators are usually made available as primitive functions of a query language, which may depend on the specific database implementation. Relational Algebra provides a formal definition of the semantics of these operators, which is independent of the syntax of the query language.

Here, we briefly recall the basic notions of Relational Algebra, whereas a more extensive introduction can be found in [540]. In the following, the list of the basic operators is reported.

Union

Given two relations R_1 and R_2 of the same arity, the union $R = R_1 \cup R_2$ is a relation obtained by taking the union of the tuples occurring either in R_1 or in R_2 .

PhD		
ID	SURNAME	AGE
23	Smith	38
40	Adams	39
132	Ross	32

MANAGERS		
ID	SURNAME	AGE
72	Adams	50
40	Adams	39
132	Ross	32

PhD \cup MANAGERS		
ID	SURNAME	AGE
23	Smith	38
40	Adams	39
132	Ross	32
72	Adams	50

PhD \cap MANAGERS		
ID	SURNAME	AGE
40	Adams	39
132	Ross	32

PhD – MANAGERS		
ID	SURNAME	AGE
23	Smith	38

Fig. C.1 Given the tables corresponding to the relations $R_1 = \text{PhD}$ and $R_2 = \text{MANAGERS}$, we can construct the tables $\text{PhD} \cup \text{MANAGERS}$, $\text{PhD} \cap \text{MANAGERS}$, and $\text{PhD} - \text{MANAGERS}$

Intersection

Given two relations R_1 and R_2 of the same arity, the intersection $R = R_1 \cap R_2$ is a relation obtained by only keeping the tuples occurring in both relations R_1 and R_2 .

Set difference

Given two relations R_1 and R_2 of the same arity, the difference $S = R_1 - R_2$ is obtained by eliminating from R_1 the tuples that occur in R_2 .

In Fig. C.1 examples of the Union, Intersection, and Set Difference operators are reported.

Cartesian product

Let R_1 and R_2 be two relations of arity n and m , respectively. The Cartesian product $R = R_1 \times R_2$ is a relation of arity $n + m$, whose tuples have been obtained by chaining one tuple of R_1 with one tuple of R_2 in all possible ways.

Projection

Let R_1 and R_2 be two relations of arity n and m , respectively, with $n > m$; the relation R_2 will be called a projection of R_1 if it can be generated by taking the distinct tuples obtained by deleting a choice of $(n - m)$ columns in R_1 . The projection is formally written as $R_2 = \pi_{i_1, i_2, \dots, i_m}(R_1)$, where i_1, i_2, \dots, i_m denote the columns of R_1 that are to be kept in R_2 .

Selection

Let R be a n -ary relation. A selection $S = \sigma_\theta(R)$ is obtained by selecting all tuples in R satisfying a condition θ stated as a logical formula, built up using the usual connectives \wedge, \vee, \neg , the arithmetic predicates $<, >, =, \leq, \geq$ and the values of the tuple's components.

ID	SURNAME	AGE
23	Smith	38
40	Adams	39
132	Ross	32

CITY	REGION
Rome	Lazio
Milan	Lombardia
Bergamo	Lombardia

ID	SURNAME	AGE	CITY	REGION
23	Smith	38	Rome	Lazio
23	Smith	38	Milan	Lombardia
23	Smith	38	Bergamo	Lombardia
40	Adams	39	Rome	Lazio
40	Adams	39	Milan	Lombardia
40	Adams	39	Bergamo	Lombardia
132	Ross	32	Rome	Lazio
132	Ross	32	Milan	Lombardia
132	Ross	32	Bergamo	Lombardia

SURNAME	AGE
Smith	38
Adams	39
Ross	32

ID	SURNAME	AGE
23	Smith	38
132	Ross	32

Fig. C.2 Given the relations $R_1 = PhD$ and $R_2 = LOCATION$, the Cartesian product of R_1 and R_2 contains 9 tuples, obtained by concatenating each tuple in R_1 with each tuples in R_2 . Relation $Proj-PhD$ is the projection of relation PhD over the attributes $SURNAME$ and AGE , i.e., $Proj-PhD = \pi_{SURNAME,AGE}(PhD)$. Finally, relation $Sel-PhD$ is obtained by selection from PhD , and contains the tuples that satisfy the condition $AGE \leq 38$, i.e., $Sel-PhD = \sigma_{AGE \leq 38}(PhD)$

FATHER	CHILD
John	Ann
Stuart	Jeanne
Robert	Albert

PARENT	CHILD
John	Ann
Stuart	Jeanne
Robert	Albert

Fig. C.3 Given the relations $R = FATHERHOOD$, we can rename attribute $FATHER$ as $PARENT$, obtaining the new relation $R-FATHERHOOD$, i.e., $R-FATHERHOOD = \rho_{PARENT \leftarrow FATHER}(R)$

In Fig. C.2 examples of the Cartesian product, Projection, and Selection operators are reported.

Renaming

If R is a relation, then $R(B \leftarrow A)$ is the same relation, where column A is now named B . The renaming operation is denoted by $R(B \leftarrow A) = \rho_{B \leftarrow A}(R)$. In Fig. C.3 an example of the Renaming operator is reported.

Natural-join

Let R and S be two relations of arity n and m , respectively, such that k columns A_1, A_2, \dots, A_k in S have the same name as in R . The natural join $Q = R \bowtie S$ is the $(n + m - k)$ arity relation defined as:

$$\pi_{A_1, A_2, \dots, A_{(n+m-k)}} \sigma_{R.A_1=S.A_1 \wedge R.A_2=S.A_2 \wedge \dots \wedge R.A_k=S.A_k} (R \times S).$$

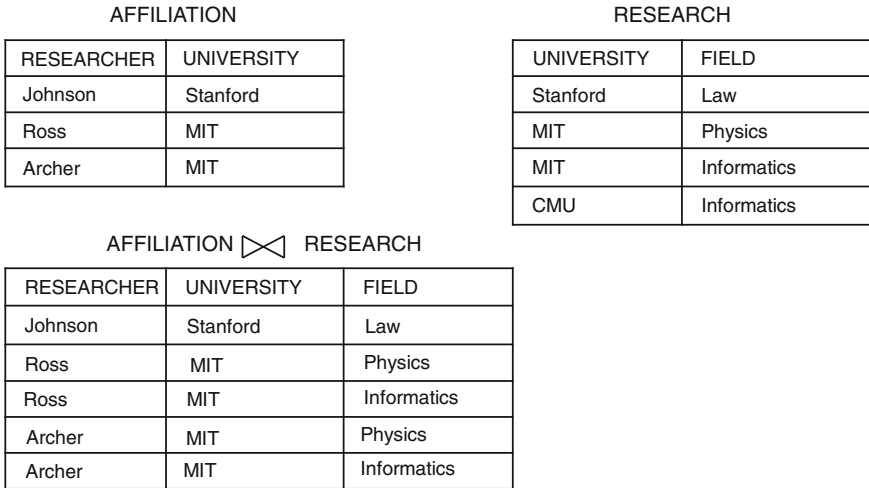


Fig. C.4 Given the two relations *AFFILIATION* and *RESEARCH*, their natural join is obtained by considering all tuples that have the *UNIVERSITY* value in common

In other words, each tuple of *Q* is obtained by merging two tuples of *R* and *S* such that the corresponding values of the shared columns are the same.

In Fig. C.4 an examples of the Natural-Join operator is reported.

Appendix D

Basic Notion of First Order Logics



In this appendix we recall the basic notions of First Order Logic (FOL), in particular those that have been used in this book. Readers interested in a deeper understanding of the topic can find excellent introductions in many textbooks (see, for instance, [496, 545]).

First Order Logic (also known as First Order Predicate Calculus) is a language used in Mathematics, Computer Science, and many other fields, for describing formal reasoning. It is an extension of Propositional Logic to the manipulation of variables. The definition of a logical language has two parts, namely the syntax of the language, and the semantic.

D.1 Syntax

A FOL language \mathcal{L} is a 5-tuple $\langle \mathbb{C}, \mathbb{X}, \mathbb{O}, \mathbb{P}, \mathbb{F} \rangle$, where \mathbb{C} is a set of constants, \mathbb{X} is a set of variables, \mathbb{O} is the set of logical operators, \mathbb{F} is a set of function names and \mathbb{P} is a set of predicate names. All symbols occurring in the definition of \mathcal{L} are partitioned into two sets:

Logical symbols—Logical symbols include:

- *Logical connectives*: \wedge (conjunction), \vee (disjunction), \neg (negation), \rightarrow (implication).
- *Quantifiers*: \forall (universal quantifier), \exists (existential quantifier).
- Parentheses and punctuation symbols.
- An infinite set of variable names. Each variable X takes value in a given domain Ω_X .

Non-logical symbols—Non-logical symbols include:

- *Predicate* symbols. A predicate $p(x_1, \dots, x_n)$, of arity n , describes an elementary property of, or an elementary relation among sets of objects represented by a set of variables.
- *Function* symbols. A function $f(x_1, \dots, x_n)$, of arity n , associates to a tuple of objects, represented by the set of variables, a value or another object.
- *Constants*. These are the identifiers of objects, and can be seen as function symbols of 0-arity.

As an example, let us consider the following Language $\mathcal{L} = \langle \mathbb{C}, \mathbb{X}, \mathbb{O}, \mathbb{P}, \mathbb{F} \rangle$, where:

$\mathbb{C} = \{\text{John, Mary, Ann, Rob, Tom, Billy, Lawrence, Mia}\}$ is a set of constants.

$\mathbb{X} = \{x, y, z, \dots, x_1, x_2, x_3, \dots\}$ is a set of variables.

$\mathbb{O} = \{\wedge, \vee, \neg, \rightarrow, \forall, \exists\}$ is the set of standard logical operators.

$\mathbb{F} = \{\text{mother}(x), \text{father}(x)\}$ is the set of functions. Function $\text{mother}(x)$ ($\text{father}(x)$) assign to x his/her mother (father).

$\mathbb{P} = \{\text{married}(x, y), \text{grandmother}(x, y), \text{siblings}(x, y)\}$ is the set of predicates.

The expression power of the language resides in the possibility of combining the elementary symbols to form complex *terms* and *formulas*.

D.1.1 Formulas

Logical formulas are expressions built up over the dictionary defined by logical and non logical symbols. *Well-formed-formulas* (wffs) are the ones with the syntax recursively defined in the following. We must defined *terms*, and *formulas*.

Terms

- A constant is a term.
- A variable is a term.
- If f is a function symbol of arity n and t_1, \dots, t_n are terms, $f(t_1, t_2, \dots, t_n)$ is a term.

Formulas

- If p is a predicate symbol of arity n , and t_1, t_2, \dots, t_n are terms, then $p(t_1, t_2, \dots, t_n)$ is an atomic formula.
- If φ_1 and φ_2 are formulas, $(\varphi_1 \wedge \varphi_2)$, $(\varphi_1 \vee \varphi_2)$, $(\varphi_1 \rightarrow \varphi_2)$, are formulas.
- If φ is a formula, then $\neg\varphi$ is a formula.
- If φ is a formula and x is a variable occurring in φ , then $\forall x(\varphi)$ and $\exists x(\varphi)$ are formulas.

Only expressions that can be obtained by finitely many applications of the above rules are formulas.

Frequently, in the literature an atomic formulas is called a *literal*. A literal consisting of a non-negated predicate $p(t_1, t_2, \dots, t_n)$ is said a *positive literal*, whereas a negated predicate of the type $\neg p(t_1, t_2, \dots, t_n)$ is said a *negative literal*.

In the language introduced above as an example, terms are, for instance, Marry , y , $\text{mother}(x)$, and $\text{father}(\text{mother}(x))$. Moreover, $\text{sibling}(x, y)$, $\text{married}(x, y) \wedge \text{grandmother}(x, z)$, $\neg \text{married}(y, z)$, $\text{married}(\text{mother}(x), \text{father}(x))$, $\exists x.\text{sibling}(x, y)$, $\forall x \exists y.\text{grandmother}(y, x)$ are all well-formed formulas.

D.2 Semantics

FOL formulas make assertions on generic objects denoted by variables. In order to let a formula assume a precise meaning in the description of the world, it is necessary to define an *interpretation*, in which generic objects, represented by variables, can be mapped to specific individuals.

An interpretation is a *universe* U of individuals, together with a set of rules assigning a meaning to formulas with respect to U . More precisely, for atomic formulas we have:

- Constants identify (are associated to) individuals in U .
- Function symbols are associated to operations in U , which build new objects (or values) starting from the primitive ones. In other words, the semantic of a function $y = f(x_1, \dots, x_n)$ is the set of tuples (x_1, \dots, x_n, y) , where $x_j \in \Omega_j$ ($1 \leq j \leq n$), and $y \in \Omega_y$, such that f associates y to the tuple (x_1, \dots, x_n) .
- 0-arity predicates are mapped to *True* or *False*.
- n -ary predicates are mapped to n -ary relations, *i.e.*, to set of n -ary tuples of objects existing in U and satisfying the predicate.

In other words, objects, operations, and relations are the *extension* of constants, functions, and predicates, respectively. Among formulas, we have to distinguish between *open* and *closed* formulas. Open formulas are those that contain at least one *free variable*, namely a variable that is not assigned to a specific value. Closed formulas are those that do not contain free variables. A free variable can be closed by either assigning to it a specific constant, or attaching to it a quantifier. For instance, the formula $\text{married}(x, y)$ is open, whereas $\text{siblings}(\text{John}, \text{Mary})$ and $\exists x.\text{sibling}(x, \text{Ann})$ are closed ones. Open formulas (called “concepts” by Frege) have an extension associated to them, whereas closed formulas (called “sentences” by Frege) have associated a truth value.

Replacing a variable x by a constant A is called a *substitution* $\theta = x/A$. An atomic formula $q(x/A)$ is true in U if the constant A belongs to the unary relation R_q , corresponding to predicate q .¹ In analogous way, the atomic formula $p(y/B, z/C)$ is

¹ With $x/A, x/B, y/C$ we mean that the variables x, y , and z are replaced by the constant values A, B , and C , respectively.

AND		
φ_1	φ_2	$\varphi_1 \wedge \varphi_2$
F	F	F
F	T	F
T	F	F
T	T	T

OR		
φ_1	φ_2	$\varphi_1 \vee \varphi_2$
F	F	F
F	T	T
T	F	T
T	T	T

NOT	
φ	$\neg\varphi$
F	T
T	F

Implication		
φ_1	φ_2	$\varphi_1 \rightarrow \varphi_2$
F	F	T
F	T	T
T	F	F
T	T	T

Bi-Implication		
φ_1	φ_2	$\varphi_1 \leftrightarrow \varphi_2$
F	F	T
F	T	F
T	F	F
T	T	T

Fig. D.1 Semantics of logical connectives AND (\wedge), OR(\vee), NOT (\neg), Implication (\rightarrow), and BI-Implication (\leftrightarrow)

true iff the tuple (B, C) belongs to the table defining the binary relation R_p , associated to predicate p .

The truth of complex formulas can be evaluated in a universe U by combining the truth of the single atomic formulas according to the classical semantics of the logical connectives (see Fig. D.1). For instance, the formula $\varphi(x, y) = q(x/A) \wedge p(x/A, y/B)$ is true iff A belongs to relation R_q and (A, B) belongs to relation R_p .

By referring to the truth tables reported in Fig. D.1, it is easy to prove that, among the five connectives \wedge , \vee , \neg , \rightarrow , and \leftrightarrow , only three of them are essential because *implication* and *bi-implication* can be expressed as a combination of the others. For instance, formula $\varphi \rightarrow \psi$ (φ implies ψ), is semantically equivalent to $\neg\varphi \vee \psi$, while formula $\varphi \leftrightarrow \psi$ (φ implies ψ and ψ implies φ) is semantically equivalent to $(\neg\varphi \vee \psi) \wedge (\neg\psi \vee \varphi)$.

D.3 Clausal Form

In wffs quantifiers can be nested arbitrarily. However, it can be proved that any wff can be syntactically transformed in such a way that all quantifiers occur only at the most external level, while preserving the formula’s semantics. This syntactic form is called *prenexed* form. Moreover, the existential quantifier can be eliminated by introducing the so called *Skolem function*.

The prenexed form of a formula can be a universally quantified formula of the type $\forall_{x_1, x_2, \dots, x_n} \varphi(x_1, x_2, \dots, x_n)$, where φ is a formula with only free variables, which is built up by means of the connectives \wedge , \vee , \neg , and, possibly, \rightarrow and \leftrightarrow . Finally, any formula, built up through the connectives \vee , \wedge and \neg , can be represented in *Conjunctive Normal Form* (CNF), i.e., as a conjunction of disjunctions of atoms (literals). In particular, any FOL sentence can always be written as in the following:

$$\forall_{x_1, x_2, \dots, x_n} [(L_{11} \vee L_{12} \vee \dots \vee L_{1k_1}) \wedge (L_{21} \vee L_{22} \vee \dots \vee L_{2k_2}) \wedge \dots \wedge (L_{m1} \vee L_{m2} \vee \dots \vee L_{mk_m})] \tag{D.1}$$

where L_{ij} denotes a positive or negative literal, with any subset of the variables x_1, x_2, \dots, x_n as arguments.

Form (D.1) is usually referred to as *clausal form* (the word *clause* denotes a disjunction of literals), and is the one most widely used for representing knowledge in Relational Machine Learning.

For the sake of simplicity, notation (D.1) is usually simplified as follows:

- Universal quantification is implicitly assumed, and the quantifier symbol is omitted.
- Symbol \wedge denoting conjunction is replaced by “;” or implicitly assumed.

Horn clauses. A Horn clause is a clause with at most one positive literal. Horn clauses are named after the logician Alfred Horn [262], who investigated the mathematical properties of similar sentences in the non-clausal form of FOL. The general form of Horn clauses is then:

$$\neg L_1 \vee \neg L_2 \vee \dots \vee \neg L_{k-1} \vee L_k, \quad (\text{D.2})$$

which can be equivalently rewritten as

$$\neg(L_1 \wedge L_2 \wedge \dots \wedge L_{k-1}) \vee L_k \quad \equiv \quad L_1 \wedge L_2 \wedge \dots \wedge L_{k-1} \rightarrow L_k \quad (\text{D.3})$$

Horn clauses play a basic role in Logic Programming [299] and are important for Machine Learning [382]. A Horn clause with exactly one positive literal is said a *definite clause*. A definite clause with no negative literals is also called a *fact*.

DATALOG. *DATALOG* is a subset of a Horn clause language designed for querying databases. It imposes several further restrictions to the clausal form:

- It disallows complex terms as arguments of predicates. Only constants and variables can be terms of a predicate.
- Variables are range restricted, i.e., each variable in the conclusion of a clause must also appear in a non negated literal in the premise.

Appendix E

Abstraction Operators



All operators that we have defined so far are summarized in Table E.1. They are grouped according to the elements of the description frame they act upon, and their abstraction mechanism. Even though there is quite a large number of them, several operators can be “technically” applied in the same way, exploiting synergies. For instance, equating values of a variable can be implemented with the same code for attributes, argument values in functions and relations, and in a function’s co-domain. Nevertheless, we have kept them separate, because they differ in meaning, and also in the impact they have on the Γ ’s.

As it was said at the beginning, the listed operators are defined at the level of description frames, because they correspond to abstracting the observations that are obtained from the sensors used to analyze the world. To each one of them a corresponding method is associated, which acts on specific \mathcal{P} -Sets according to rules that guide the actual process of abstraction.

E.1 Some More Operators

In this appendix some operators are described in addition to those introduced in Chap. 7. The complete set of available operators can be found in the book’s companion site.

The introduced operators are by no means intended to exhaust the spectrum of abstractions that can be thought of. However they are sufficient to describe most of the abstractions proposed in the past in a unified way. Moreover, they provide a guide for defining new ones, better suited to particular fields.

Table E.1 Summary of the elementary abstraction and approximation operators, classified according to the elements of the description frame they act upon and their mechanism

Operators	Elements	Arguments	Values
Hiding	ω_{hobj} , $\omega_{h\text{type}}$,	$\omega_{h\text{funarg}}$, $\omega_{h\text{relarg}}$	$\omega_{h\text{attrval}}$
	$\omega_{h\text{attr}}$, $\omega_{h\text{rel}}$,		$\omega_{h\text{funargval}}$
	$\omega_{h\text{fun}}$		$\omega_{h\text{funcodom}}$
			$\omega_{h\text{relargval}}$
Equating	ω_{eqobj} , $\omega_{eq\text{type}}$,	$\omega_{eq\text{funarg}}$	$\omega_{eq\text{attrval}}$
	$\omega_{eq\text{attr}}$, $\omega_{eq\text{fun}}$,		$\omega_{eq\text{funargval}}$
	$\omega_{eq\text{rel}}$		$\omega_{eq\text{funcodom}}$
			$\omega_{eq\text{relargval}}$
Building hierarchy	$\omega_{hier\text{attr}}$, $\omega_{hier\text{fun}}$,		$\omega_{hier\text{attrval}}$
	$\omega_{hier\text{rel}}$, $\omega_{hiertype}$		$\omega_{hier\text{funcodom}}$
Combining	ω_{coll} , ω_{aggr} , ω_{group}	ω_{constr}	
Approximating	ρ_{repl}	ρ_{repl}	ρ_{repl}
	ρ_{idobj} , $\rho_{id\text{type}}$,	$\rho_{id\text{funarg}}$	$\rho_{id\text{attrval}}$
	$\rho_{id\text{attr}}$, $\rho_{id\text{fun}}$,	$\rho_{id\text{relarg}}$	$\rho_{id\text{funargval}}$
	ρ_{idrel}		$\rho_{id\text{funcodom}}$
			$\rho_{id\text{relargval}}$

E.1.1 Operator that Hides a Type: $\omega_{h\text{type}}$

If $X^{(g)} = \Gamma_{TYPE}^{(g)}$ and $y = \tau$, type τ cannot be anymore observed in a system, and objects that were previously of this type become of type `obj`. We define:

$$\omega_{h\text{type}}(\tau) \stackrel{\text{def}}{=} \omega_h(\Gamma_{TYPE}^{(g)}, \tau)$$

and we obtain:

$$\Gamma_{TYPE}^{(a)} = \Gamma_{TYPE}^{(g)} - \{\tau\} \cup \{\text{obj}\}$$

The corresponding method $\text{meth}_{h\text{type}}(\mathcal{P}_g, \tau)$, applied to an observed \mathcal{P}_g , replaces with `obj` the type of all objects of type τ .

E.1.2 Operator that Hides a Value from a Function's Codomain: $\omega_{h\text{funcodom}}$

If $X^{(g)} = \Gamma_{\mathcal{F}}^{(g)}$, $y = (f_h, CD(f_h))$, $\forall \in CD(f_h)$, then the operator

$$\omega_{h\text{funcodom}}(f_h, CD(f_h), \forall) \stackrel{\text{def}}{=} \omega_h(\Gamma_{\mathcal{F}}^{(g)}, (f_h, CD(f_h)), \forall)$$

removes some value v from the codomain of f_h . Then an abstract function is created, whose codomain is given by:

$$CD(f_h^{(a)}) = CD(f_h) - \{v\},$$

and

$$\Gamma_{\mathcal{F}}^{(a)} = \Gamma_{\mathcal{F}}^{(g)} - \{f_h\} \cup \{f_h^{(a)}\}$$

For instance, let us consider the function *Price*, with codomain $CD(Price) = \{\text{cheap, moderate, fair, costly, very-costly}\}$; if we want to remove the value *very-costly*, we have to specify, in method

$$\text{meth}_{hf\text{uncodomain}}(\mathcal{P}_g, Price, CD(Price), \text{very-costly}),$$

what happens for those tuples in $FCOV(f_h^{(a)})$ that contain v . One possibility is that the value is turned into UN.

E.1.3 Operator that Builds Equivalence Classes of Relations: ω_{eqrel}

If $X^{(g)} = \Gamma_{\mathcal{R}}^{(g)}$ and $y^{(a)} = R^{(a)}$, the operator makes indistinguishable all relations R satisfying $\varphi_{eq}(R_1, \dots, R_k)$. Let

$$\Gamma_{\mathcal{R},eq} = \{(R_1, \dots, R_k) \mid \varphi_{eq}(R_1, \dots, R_k)\}$$

be the set of indistinguishable relations. We define:

$$\omega_{eqrel}(\varphi_{eq}(R_1, \dots, R_k), R^{(a)}) \stackrel{\text{def}}{=} \omega_{eqelem}(\Gamma_{\mathcal{R}}^{(g)}, \varphi_{eq}(R_1, \dots, R_k), R^{(a)})$$

The operator $\omega_{eqrel}(\varphi_{eq}(R_1, \dots, R_k), R^{(a)})$ generates first the set $\Gamma_{\mathcal{R},eq}$, obtaining:

$$\Gamma_{\mathcal{R}}^{(a)} = \Gamma_{\mathcal{R}}^{(g)} - \Gamma_{\mathcal{R},eq} \cup \{R^{(a)}\}$$

It is the method $\text{meth}_{eqrel}(\mathcal{P}_g, \varphi_{eq}(R_1, \dots, R_k), R^{(a)})$ that specifies how the cover of $R^{(a)}$ has to be computed.

As an example, let us suppose that the set of relations to be made indistinguishable be define extensionally, as in the case of functions. For instance, let

$$\Gamma_{\mathcal{R},eq} = \{R_{IsMotherof}, R_{IsStepMotherof}\},$$

where:

$$\begin{aligned} R_{IsMotherof} &\subseteq \Gamma_{\text{women}}^{(g)} \times \Gamma_{\text{people}}^{(g)} \\ R_{IsStepMotherof} &\subseteq \Gamma_{\text{women}}^{(g)} \times \Gamma_{\text{people}}^{(g)} \end{aligned}$$

If we state the equivalence between the two relations, we may keep only $R^{(a)}$ in place of the two. Again, method $\text{meth}_{\text{eqrel}}(\mathcal{P}_g, \varphi_{\text{eq}}(R_1, \dots, R_k), R^{(a)})$ shall specify how the cover $RCOV(R_{IsStepMotherof})$ must be computed.

E.1.4 Operator that Builds Equivalence Classes of Values in a Function's Codomain: $\omega_{\text{eqfuncodom}}$

If $X^{(g)} = \Gamma_{\mathcal{F}}^{(g)}$, $Y = (f_h, CD(f_h))$, $V_{\text{eq}} \subseteq CD(f_h)$, then the operator equates values of the codomain of function f_h and set all equal to $\forall^{(a)}$. We define:

$$\omega_{\text{eqfuncodom}}(f_h, CD(f_h), V_{\text{eq}}, \forall^{(a)}) \stackrel{\text{def}}{=} \omega_{\text{eqval}}(\Gamma_{\mathcal{F}}^{(g)}, (f_h, CD(f_h)), V_{\text{eq}}, \forall^{(a)})$$

An abstract function is defined:

$$f_h^{(a)} \subseteq \underbrace{\Gamma_{\mathcal{O}}^{(g)} \times \dots \times \Gamma_{\mathcal{O}}^{(g)}}_{1 \dots t_h} \rightarrow CD(f_h) - V_{\text{eq}} \cup \{\forall^{(a)}\}$$

Then:

$$\Gamma_{\mathcal{F}}^{(a)} = \Gamma_{\mathcal{F}}^{(g)} - \{f_h\} \cup \{f_h^{(a)}\}$$

Method $\text{meth}_{\text{eqfuncodom}}(\mathcal{P}_g, (f_h, CD(f_h)), V_{\text{eq}}, \forall^{(a)})$ handles the cover of $f_h^{(a)}$ by replacing in $FCOV(f_h^{(a)})$ all occurrences of members of V_{eq} with $\forall^{(a)}$.

For the sake of exemplification, let us consider a gray-level picture, in which the attribute *Intensity* of a pixel x can take on a value in the integer interval $[0, 255]$. Let τ be a threshold, such that:

$$I^{(a)}(x) = \begin{cases} 255 & \text{if } I(x) > \tau, \\ I(x) & \text{otherwise.} \end{cases} \quad (\text{E.1})$$

In Eq. (E.1) all values greater than the threshold are considered equivalent. An example is reported in Fig. E.1.

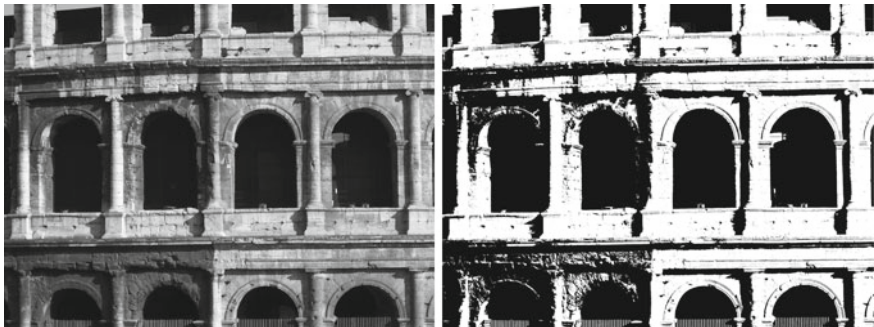


Fig. E.1 Example of method $\text{meth}_{\text{eqfincodom}}(\mathcal{P}_g, (f_h, CD(f_h)), V_{eq}, \vee^{(a)})$. The picture on the left is a 256-level gray picture. By a thresholding operation, all pixels whose intensity is greater than τ are considered white

E.1.5 Operator that Builds a Hierarchy of Attributes: ω_{hierattr}

If $X^{(g)} = \Gamma_{\mathcal{A}}^{(g)}$, $Y = \emptyset$, $Y_{\text{child}} = \Gamma_{\mathcal{A}, \text{child}}^{(g)}$, and $y^{(a)} = (A^{(a)}, \Lambda^{(a)})$, then the operator works on an attribute hierarchy, where a set of nodes, those contained in $\Gamma_{\mathcal{A}, \text{child}}^{(g)}$ are replaced by $(A^{(a)}, \Lambda^{(a)})$. We define:

$$\omega_{\text{hierattr}} \left(\Gamma_{\mathcal{A}, \text{child}}^{(g)}, (A^{(a)}, \Lambda^{(a)}) \right) \stackrel{\text{def}}{=} \omega_{\text{hier}} \left(\Gamma_{\mathcal{A}}^{(g)}, \Gamma_{\mathcal{A}, \text{child}}^{(g)}, (A^{(a)}, \Lambda^{(a)}) \right)$$

and we obtain:

$$\Gamma_{\mathcal{A}}^{(a)} = \Gamma_{\mathcal{A}}^{(g)} - \Gamma_{\mathcal{A}, \text{child}}^{(g)} \cup \{(A^{(a)}, \Lambda^{(a)})\}.$$

The method $\text{meth}_{\text{hierattr}}(\mathcal{P}_g, \Gamma_{\mathcal{A}, \text{child}}^{(g)}, (A^{(a)}, \Lambda^{(a)}))$ states how the values in $\Lambda^{(a)}$ must be derived from those in the domains of the attributes in $\Gamma_{\mathcal{A}, \text{child}}^{(g)}$.

As an example, let us consider the attributes *Length* and *Width*. We introduce the abstract attribute $\text{LinearSize}^{(a)}$, such that *Length is-a LinearSize^(a)* and *Width is-a LinearSize^(a)*. We have then, $\Gamma_{\mathcal{A}, \text{child}} = \{\text{Length}, \text{Width}\}$, and $A^{(a)} = \text{LinearSize}^{(a)}$. The values of the attribute $\text{LinearSize}^{(a)}$ are to be defined; for instance, we may assume that, for an object x ,

$$\text{LinearSize}^{(a)}(x) = \text{Max}[\text{Length}(x), \text{Width}(x)].$$

The original attribute do not enter Γ_a .

E.1.6 Operator that Builds a Hierarchy of Relations: $\omega_{hierrel}$

If $X^{(g)} = \Gamma_{\mathcal{R}}^{(g)}$, $Y = \emptyset$, $Y_{child} = \Gamma_{\mathcal{R},child}^{(g)}$ and $y^{(a)} = R^{(a)}$, then the operator works on a relation hierarchy, where a set of nodes, those contained in $\Gamma_{\mathcal{R},child}^{(g)}$, are replaced by $R^{(a)}$. We define:

$$\omega_{hierrel} \left(\Gamma_{\mathcal{R},child}^{(g)}, R^{(a)} \right) \stackrel{def}{=} \omega_{hier} \left(\Gamma_{\mathcal{R}}^{(g)}, \Gamma_{\mathcal{R},child}^{(g)}, R^{(a)} \right)$$

and we obtain:

$$\Gamma_{\mathcal{R}}^{(a)} = \Gamma_{\mathcal{R}}^{(g)} - \Gamma_{\mathcal{R},child}^{(g)} \cup R^{(a)}$$

The method $\text{meth}_{hierrel} \left(\mathcal{P}_g, \Gamma_{\mathcal{R},child}^{(g)}, R^{(a)} \right)$ states how the cover of $R^{(a)}$ must be computed starting from those of the relations in $\Gamma_{\mathcal{R},child}^{(g)}$.

As an example, let $R_{HorizAdjacent} \subseteq \Gamma_{\mathcal{O}}^{(g)} \times \Gamma_{\mathcal{O}}^{(g)}$ and $R_{VertAdjacent} \subseteq \Gamma_{\mathcal{O}}^{(g)} \times \Gamma_{\mathcal{O}}^{(g)}$ be two relations over pairs of objects. The former is verified when two objects touch each other horizontally, whereas the latter is verified when two objects touch each other vertically. We introduce the abstract relation $R_{Adjacent}^{(a)} \subseteq \Gamma_{\mathcal{O}} \times \Gamma_{\mathcal{O}}$, which does not distinguish the modality (horizontal or vertical) of the adjacency. In this case we have $\Gamma_{\mathcal{R},child} = \{R_{HorizAdjacent}, R_{VertAdjacent}\}$ and the new relation $R^{(a)} = R_{Adjacent}^{(a)}$. Operator $\omega_{hierrel}^{(\Psi)}(\mathcal{P}_g, \Gamma_{\mathcal{R},child}^{(g)}, R^{(a)})$ will establish that, for instance:

$$FCOV(R_{Adjacent}^{(a)}) = FOCV(R_{HorizAdjacent}) \cup FCOV(R_{VertAdjacent})$$

The original relations are hidden in the abstract space.

E.2 Approximation Operators

In this section we illustrate some additional approximation operators.

E.2.1 Operator that Identifies Types: ω_{idtype}

If $X^{(g)} = \Gamma_{TYPE}^{(g)}$ and $y^{(a)} = \tau^{(a)}$, the operator makes all types satisfying $\varphi_{id}(\tau_1, \dots, \tau_k)$ indistinguishable. Then type $\tau^{(a)}$ is applied to all objects in the equivalence class. We define:

$$\omega_{idtypes}(\varphi_{id}(\tau_1, \dots, \tau_k), \tau^{(a)}) \stackrel{def}{=} \omega_{idelem}(\Gamma_{TYPE}^{(g)}, \varphi_{id}(\tau_1, \dots, \tau_k), \tau^{(a)})$$

The operator $\omega_{idtype}(\varphi_{id}(\tau_1, \dots, \tau_k), \tau^{(a)})$ generates first the set of $\Gamma_{TYPE, id}$ of indistinguishable types, and then it applies $\tau^{(a)}$ to the obtained class. All types in $\Gamma_{TYPE, id}$ become $\tau^{(a)}$, obtaining:

$$\Gamma_{TYPE}^{(a)} = \Gamma_{TYPE}^{(g)} - \Gamma_{TYPE, id} \cup \{\tau^{(a)}\}$$

It is the method $\text{meth}_{idtype}(\mathcal{P}_g, \varphi_{id}(\tau_1, \dots, \tau_k), \tau^{(a)})$ that specifies what properties are to be assigned to $\tau^{(a)}$, considering the ones of the equated types. For instance, if the types in $\Gamma_{TYPE, id}$ have different sets of attributes, $\tau^{(a)}$ could have the intersection of these sets, or the union, by setting some values to NA, depending on the choice of the user.

As an example, we can consider the types `chair` and `armchair` and we can equate them to be both `chair`^(a).

E.2.2 Operator that Approximates Attribute Values: $\omega_{idattrval}$

If $X^{(g)} = \Gamma_{\mathcal{A}}^{(g)}$, $Y = (A, \Lambda_A)$, and $V_{id} = \Lambda_{A, id} \subseteq \Lambda_A$, then the operator makes indistinguishable a subset $\Lambda_{A, id}$ of the domain Λ_A of A . We define:

$$\omega_{idattrval}((A, \Lambda_A), \Lambda_{A, id}, v^{(a)}) \stackrel{def}{=} \omega_{idval}(\Gamma_{\mathcal{A}}^{(g)}, (A, \Lambda_A), \Lambda_{A, id}, v^{(a)})$$

We obtain an approximate attribute $A^{(a)}$ such that $\Lambda_{A^{(a)}} = \Lambda_A - \Lambda_{A, id} \cup \{v^{(a)}\}$, and

$$\Gamma_{\mathcal{A}}^{(a)} = \Gamma_{\mathcal{A}}^{(g)} - \{(A, \Lambda_A)\} \cup \{(A^{(a)}, \Lambda_{A^{(a)}})\}$$

For the sake of exemplification, let us consider an attribute, say *Color*, which takes values in the set:

{white, yellow, olive-green, sea-green, lawn-green, red, pink, light-green, dark-green, blue, light-blue, aquamarine, orange, magenta, cyan, black}.

We might consider equivalent all the shades of green, and identify them with $v^{(a)} = \text{sea-green}$. In this case, the true shade of green is no more known (see Fig.E.2).

As another important example, let us consider the discretization of real numbers. Let us consider the interval [0, 100), and let us divide it into 10 subintervals $\{[10k, 10(k+1)) \mid 0 \leq k \leq 9\}$. Numbers falling inside one of the intervals are considered all equal to the mean value $10k + 0.5$.

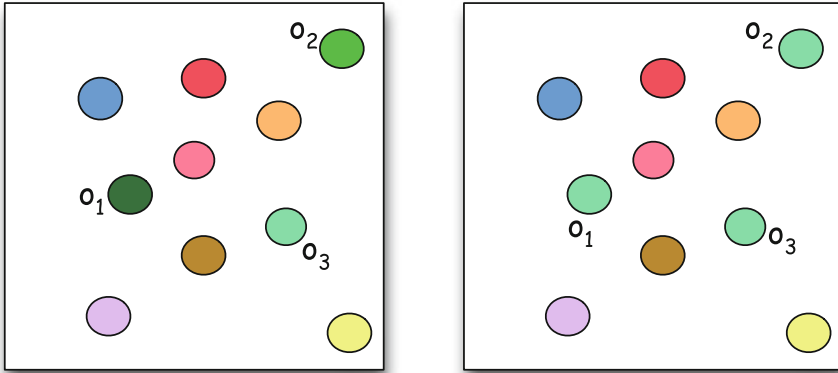


Fig. E.2 Application of method $\text{meth}(\mathcal{P}_g, \omega_{\text{eqattrval}}((\text{Color}, \Lambda_{\text{Color}}), V_{id}, v^{(a)}))$ to the figure on the left. Let $V_{id} = \{\text{olive-green}, \text{sea-green}, \text{lawn-green}, \text{light-green}, \text{dark-green}\}$. Objects o_1, o_2 , and o_3 have color dark-green, lawn-green, and sea-green, respectively. After equating all shades of green to sea-green, the color of all three objects becomes sea-green. [A color version of the figure is reported in Fig.H.16 of Appendix H]

E.3 Some More Methods

In Chap.7 the methods associated to some operators have been described. In this section we give some additional examples, whereas the complete set of methods is provided in the book's companion site.

Let us consider the operators that hide an attribute, or a function, or a relation, i.e., $\omega_{\text{hattr}}((A_m, \Lambda_m))$, $\omega_{\text{hfun}}(f_h)$, and $\omega_{\text{hrel}}(R_k)$. Hiding an attribute or a function or a relation are all instantiations of the same PDT introduced in Sect.7.2.1, and then we group them together in Table E.2, whereas their bodies are reported in Tables E.3, E.4, and E.5, respectively.

Also at the description level the operators $\omega_{\text{hattr}}((A_m, \Lambda_M))$, $\omega_{\text{hfun}}(f_h)$, and $\omega_{\text{hrel}}(R_k)$ are similar; in fact, they simply hide from the appropriate set the concerned element (attribute, function, or relation), as it was illustrated in Sect.7.2.1. But when we must apply them to a specific \mathcal{P}_g , some complication may arise. Let us look first at Table E.2.

Operator $\omega_{\text{hattr}}((A_m, \Lambda_m))$ hides the attribute from the set of available ones, and, as a consequence, $\text{meth}(\mathcal{P}_g, \omega_{\text{hattr}}(A_m, \Lambda_m))$ hides the value of that attribute in each object in \mathcal{P}_g . Hiding an attribute may cause the descriptions of some objects to become identical. However, as each object has a unique identity, they remain distinguishable.

As both functions and relations cannot have an attribute as an argument, removing A_m does not have any further effect. For the hidden information, it is not necessary to store all the tuples hidden in \mathcal{A}_g , but only the value of A_m for each object.

Table E.2 Summary of methods $\text{meth}(\mathcal{P}_g, \omega_{\text{hattr}}(A_m, \Lambda_m))$, $\text{meth}(\mathcal{P}_g, \omega_{\text{hfun}}(f_h))$, and $\text{meth}(\mathcal{P}_g, \omega_{\text{hrel}}(R_k))$

NAME	$\text{meth}(\mathcal{P}_g, \omega_{\text{hattr}})$	$\text{meth}(\mathcal{P}_g, \omega_{\text{hfun}})$	$\text{meth}(\mathcal{P}_g, \omega_{\text{hrel}})$
INPUT	$\mathcal{P}_g, (A_m, \Lambda_m)$	\mathcal{P}_g, f_h	\mathcal{P}_g, R_k
OUTPUT	\mathcal{P}_a	\mathcal{P}_a	\mathcal{P}_a
APPL-CONDITIONS	$A_m \in \mathcal{A}_g$	$f_h \in \mathcal{F}_g$	$R_k \in \mathcal{R}_g$
PARAMETERS	\emptyset	\emptyset	\emptyset
MEMORY	$\Delta^{(\mathcal{P})}$	$\Delta^{(\mathcal{P})}$	$\Delta^{(\mathcal{P})}$
BODY	See Table E.3	See Table E.4	See Table E.5

Table E.3 Pseudo-code for the method $\text{meth}(\mathcal{P}_g, \omega_{\text{hattr}}(A_m, \Lambda_m))$

```

METHOD  $\text{meth}(\mathcal{P}_g, \omega_{\text{hattr}}(A_m, \Lambda_m))$ 
  Let  $|\mathcal{O}_g| = N$ 
   $\Delta^{(\mathcal{P})} = \emptyset$ 
  for  $n = 1, N$  do
     $\mathcal{A}_a = \mathcal{A}_g - \{(\circ_n, A_1(\circ_n), \dots, A_m(\circ_n), \dots, A_M(\circ_n))\}$ 
     $\Delta^{(\mathcal{P})} = \Delta^{(\mathcal{P})} \cup \{(\circ_n, A_m(\circ_n))\}$ 
     $\mathcal{A}_a = \mathcal{A}_a \cup \{(\circ_n, A_1(\circ_n), \dots, A_{m-1}(\circ_n), A_{m+1}(\circ_n), \dots, A_M(\circ_n))\}$ 
  end

```

Table E.4 Pseudo-code for the method $\text{meth}(\mathcal{P}_g, \omega_{\text{hfun}}(f_h))$

```

METHOD  $\text{meth}(\mathcal{P}_g, \omega_{\text{hfun}}(f_h))$ 
   $\Delta_{\mathcal{F}}^{(\mathcal{P})} = \{f_h\}$ 
   $\mathcal{O}_a = \mathcal{O}_g$ 
   $\mathcal{A}_a = \mathcal{A}_g$ 
  if  $CD(f_h) = \Gamma_{\mathcal{O}}^{(g)}$ 
    then  $\mathcal{F}_a = \mathcal{F}_g - \{FCOV(f_h)\}$ 
    forall  $f_j(x_1, \dots, x_{i_j}) \mid \exists x_i = f_h$  do
      Define  $f_j^{(a)}(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{i_j})$ 
       $\mathcal{F}_a = \mathcal{F}_a - FCOV(f_j) \cup FCOV(f_j^{(a)})$ 
       $\Delta_{\mathcal{F}}^{(\mathcal{P})} = \Delta_{\mathcal{F}}^{(\mathcal{P})} \cup \{(f_j, x_i)\}$ 
    end
  end
   $\mathcal{R}_a = \mathcal{R}_g$ 
   $\Delta_{\mathcal{R}}^{(\mathcal{P})} = \emptyset$ 
  forall  $R_k(x_1, \dots, x_{i_k}) \mid \exists x_i = f_h$  do
    Define  $R_k^{(a)}(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{i_k})$ 
     $\mathcal{R}_a = \mathcal{R}_a - FCOV(R_k) \cup FCOV(R_k^{(a)})$ 
     $\Delta_{\mathcal{R}}^{(\mathcal{P})} = \Delta_{\mathcal{R}}^{(\mathcal{P})} \cup \{(R_k, x_i)\}$ 
  end
endif

```

Table E.5 Pseudo-code for the method $\text{meth}(\mathcal{P}_g, \omega_{hrel}(R_k))$

METHOD $\text{meth}(\mathcal{P}_g, \omega_{hrel}(R_k))$
$\mathcal{O}_a = \mathcal{O}_g$
$\mathcal{A}_a = \mathcal{A}_g$
$\mathcal{F}_a = \mathcal{F}_g$
$R_a = \mathcal{R}_g - \{RCOV(R_k)\}$
$\Delta_{\mathcal{R}}^{(\mathcal{P})} = RCOV(R_k)$

Hiding a function is a simple operation, *per se*, but it may have indirect effects on the set of functions and relations. In fact, if the co-domain of f_h is the set of objects, there may be in $\Gamma_{\mathcal{F}}^{(g)}$ or $\Gamma_{\mathcal{R}}^{(g)}$ some function or relation that has f_h as one of its arguments. Then, hiding f_h , these arguments disappear and new abstract functions or relations, with one less argument, are to be defined, increasing thus the degree of abstraction. Hiding a relation has no side-effects.

E.4 Complete List of Operators

As a conclusion, we report here the complete list of the domain-independent operators available so far.

Table E.6 List of currently available operators, organized according to their nature, elements acted upon, and abstraction mechanism

Operator	Type	Arguments	Effects	Comments
ω_{hobj}	Abstr	\circ	Hides object \circ	All objects of type τ are hidden in every \mathcal{P}_g
ω_{htype}	Abstr	τ	Hides type τ	
ω_{hatr}	Abstr	(A_m, Λ_m)	Hides attribute A_m with domain Λ_m	Values of attribute A_m are hidden in all objects
ω_{hfun}	Abstr	f_h	Hides function f_h	Value v_i is replaced by UN in all \mathcal{P}_g
ω_{hrel}	Abstr	R_k	Hides relation R_k	
$\omega_{hatrval}$	Abstr	$(A_m, \Lambda_m), v_i$	Hides value $v_i \in A_m$	Value \circ assumed by x_j is replaced by UN in the $FCOV(f_h)$ of all \mathcal{P}_g
$\omega_{hfunaryval}$	Abstr	f_h, x_j, \circ	Hides the value \circ from the domain of argument x_j of function f_h	Value \circ assumed by x_j is replaced by UN in the $RCOV(R_k)$ of all \mathcal{P}_g
$\omega_{hrelaryval}$	Abstr	R_k, x_j, \circ	Hides the value \circ from the domain of argument x_j of relation R_k	Value v assumed by f_h is replaced by UN in all \mathcal{P}_g
$\omega_{hfuncodom}$	Abstr	$f_h, CD(f_h), v$	Hides value v from the codomain of f_h	Arity t_h of f_h is reduced by 1
$\omega_{hfunarg}$	Abstr	f_h, x_j	Argument x_j of function f_h is hidden	Arity t_k of R_k is reduced by 1
$\omega_{hrelarg}$	Abstr	R_k, x_j	Argument x_j of function R_k is hidden	$\circ^{(a)}$ is a generic name denoting the class
ω_{eqobj}	Abstr	$\varphi_{eq}, \circ^{(a)}$	Builds up an equivalence class with the objects satisfying φ_{eq}	$A^{(a)}$ is a generic name denoting the class
ω_{eqattr}	Abstr	$\varphi_{eq}, (A^{(a)}, \Lambda_{A^{(a)}})$	Builds up an equivalence class with the attributes satisfying φ_{eq}	$\mathcal{F}^{(a)}$ is a generic name denoting the class
ω_{eqfun}	Abstr	$\varphi_{eq}, f^{(a)}$	Builds up an equivalence class with the functions satisfying φ_{eq}	(continued)

Table E.6 continued

Operator	Type	Arguments	Effects	Comments
ω_{eqrel}	Abstr	φ_{eq} $R^{(a)}$	Builds up an equivalence class with the relations satisfying φ_{eq}	$R^{(a)}$ is a generic name denoting the class
ω_{eqtype}	Abstr	$\varphi_{eq}(\tau_1, \dots, \tau_k)$ $\tau^{(a)}$	Builds up an equivalence class with the types satisfying φ_{eq}	$\tau^{(a)}$ is a generic name denoting the class
$\omega_{eqattrval}$	Abstr	$(A, \Lambda_A), \Lambda_{A,eq}, \nabla^{(a)}$	All values in $\Lambda_{A,eq}$ form an equivalence class $\nabla^{(a)}$	$\nabla^{(a)}$ is a generic name denoting the class
$\omega_{eqfunaryval}$	Abstr	$f_h, x_j, \Gamma_{O,eq}, o^{(a)}$	All values in $\Gamma_{O,eq}$ form an equivalence class $o^{(a)}$	$o^{(a)}$ is a generic name denoting the class
$\omega_{eqrelargval}$	Abstr	$R_k, x_j, \Gamma_{O,eq}, o^{(a)}$	All values in $\Gamma_{O,eq}$ form an equivalence class $o^{(a)}$	$o^{(a)}$ is a generic name denoting the class
$\omega_{eqfuncodom}$	Abstr	$f_h, CD(f_h), V_{eq}, \nabla^{(a)}$	All values in V_{eq} form an equivalence class $o^{(a)}$	$o^{(a)}$ is a generic name denoting the class
$\omega_{eqfunary}$	Abstr	$f_h, Z_{eq}, z^{(a)}$	All values in Z_{eq} form an equivalence class $z^{(a)}$	$z^{(a)}$ is a generic name denoting the class
$\omega_{eqrelarg}$	Abstr	$R_k, Z_{eq}, z^{(a)}$	All values in Z_{eq} form an equivalence class $z^{(a)}$	$z^{(a)}$ is a generic name denoting the class
$\omega_{hierattrval}$	Abstr	$(A_m, \Lambda_m),$ $\Lambda_{m,child}, \nabla^{(a)}$	The values of attribute A_m belonging to $\Lambda_{m,child}$ are hidden, and a new node $\nabla^{(a)}$ is created, such that $\forall \tau_i \in \Lambda_{m,child} : \tau_i \mathbf{is} - \mathbf{a} \nabla^{(a)}$	A node of higher level is created in a hierarchy of attribute's values
$\omega_{hiertype}$	Abstr	$\Gamma_{TYPE,child}^{(a)}, \tau^{(a)}$	Types $\tau \in \Gamma_{TYPE,child}^{(a)}$ are hidden, and a new type $\tau^{(a)}$ is created, such that $\tau \mathbf{is} - \mathbf{a} \tau^{(a)}$	In each \mathcal{P}_g , all objects of a type $\tau \in \Gamma_{TYPE,child}^{(a)}$ are hidden and replaced by a corresponding object of type $\tau^{(a)}$

(continued)

Table E.6 continued

Operator	Type	Arguments	Effects	Comments
$\omega_{hierfuncdom}$	Abstr	$(f_h, CD(f_h)), CD(f_h)_{child}, \mathbf{v}^{(a)}$	The values of $CD(f_h)$ belonging to $CD(f_h)_{child}$ are hidden, and a new node $\mathbf{v}^{(a)}$ is created, such that $\forall v_i \in CD(f_h) : v_i \mathbf{is} - \mathbf{a} \mathbf{v}^{(a)}$	A node of higher level is created in a hierarchy of values in a function's codomain
$\omega_{hierattr}$	Abstr	$\Gamma_{\mathcal{A},child}^{(g)}, (A^{(a)}, \Lambda^{(a)})$	The attributes contained in $\Gamma_{\mathcal{A},child}^{(g)}$ are replaced by a new attribute, $(A^{(a)}, \Lambda^{(a)})$, such that $\forall A_i \in \Gamma_{\mathcal{A},child}^{(g)} : A_i \mathbf{is} - \mathbf{a} A^{(a)}$	Values in Λ_i , for each $A_i \in \Gamma_{\mathcal{A},child}^{(g)}$ are linked to corresponding values in $\Lambda^{(a)}$
$\omega_{hierfun}$	Abstr	$\Gamma_{\mathcal{F},child}^{(g)}, f^{(a)}$	The functions contained in $\Gamma_{\mathcal{F},child}^{(g)}$ are replaced by a new function, $f^{(a)}$ such that $\forall f_i \in \Gamma_{\mathcal{F},child}^{(g)} : f_i \mathbf{is} - \mathbf{a} f^{(a)}$	Values in $CD(f_i)$, for each $f_i \in \Gamma_{\mathcal{F},child}^{(g)}$ are linked to corresponding values in $CD(f^{(a)})$
$\omega_{hierrel}$	Abstr	$\Gamma_{\mathcal{R},child}^{(g)}, R^{(a)}$	The relations contained in $\Gamma_{\mathcal{R},child}^{(g)}$ are replaced by a new relation, $R^{(a)}$ such that $\forall R_i \in \Gamma_{\mathcal{R},child}^{(g)} : R_i \mathbf{is} - \mathbf{a} R^{(a)}$	
ω_{coll}	Abstr	$\mathbf{t}, \mathbf{t}^{(a)}$	Makes a single, collective type $\mathbf{t}^{(a)}$, starting with a number of elements of type \mathbf{t}	
ω_{aggr}	Abstr	$(\mathbf{t}_1, \dots, \mathbf{t}_s), \mathbf{t}^{(a)}$	Makes a composite type $\mathbf{t}^{(a)}$, starting with a set of objects of different types $\mathbf{t}_1, \dots, \mathbf{t}_s$	
ω_{group}	Abstr	$\varphi_{group}, G^{(a)}$	Forms a group with name $G^{(a)}$ with the set of objects satisfying φ_{group}	The group can be defined extensionally. The operator acts on <i>objects</i> , not types

(continued)

Table E.6 continued

Operator	Type	Arguments	Effects	Comments
ω_{constr}	Abstr	Constr	Defines a new description element (attribute, function, relation) $Constr : \Gamma_{A,\mathcal{F},g}^{(g)} \rightarrow \Gamma_{A,\mathcal{F},g}^{(a)}$ $\Gamma_{A,\mathcal{F},\mathcal{R}}^{(g)} = \Gamma_{A,\mathcal{F}}^{(g)} \cup \Gamma_{\mathcal{R}}^{(g)}$ $\Gamma_{A,\mathcal{F},\mathcal{R}}^{(a)} = \Gamma_{A,\mathcal{F}}^{(a)} \cup \Gamma_{\mathcal{R}}^{(a)}$ Value v_i of attribute A_m is replaced with value $v^{(p)}$	Most often used to construct a new attribute
$\rho_{replattrval}$	Approx	$(A_m, \Delta_m), v_i, v^{(p)}$	Function f_h is replaced with function $g_h^{(p)}$	$v^{(p)}$ is a specific value, different from v_i
$\rho_{replfun}$	Approx	$f_h, g_h^{(p)}$	Relation R_k is replaced with relation $R_k^{(p)}$	$g_h^{(p)}$ is a specific function, different from f_h
$\rho_{replrel}$	Approx	$R_k, R_k^{(p)}$	Set of objects satisfying φ_{id} form an equivalence class	$R_k^{(p)}$ is a specific relation, different from R_k
ρ_{idobj}	Approx	$\varphi_{id}(\circ_1, \dots, \circ_k), \circ^{(p)}$	All objects are equated to $\circ^{(p)}$	$\circ^{(p)}$ is one of the objects in the equivalence class
ρ_{idtype}	Approx	$\varphi_{id}(t_1, \dots, t_k), t^{(p)}$	Set of types satisfying φ_{id} form an equivalence class	$t^{(p)}$ is one of the types in the equivalence class
$\rho_{idattrval}$	Approx	$(A, \Delta_A), \Delta_{A,id}, v^{(p)}$	All objects of the equated types become of type $t^{(p)}$	$v^{(p)}$ is one of the values in $\Delta_{A,id}$
$\rho_{idfuncodom}$	Approx	$f_h, CD(f_h), V_{id}, v^{(p)}$	Values in V_{id} become equal to $v^{(p)}$	$v^{(p)}$ is one of the values in V_{id}
$\rho_{idfunarg}$	Approx	$f_h, Z_{id}, z^{(a)}$	All arguments in Z_{id} are equated to $z^{(a)}$	$z^{(a)}$ is an element of Z_{id}
$\rho_{idrelarg}$	Approx	$R_k, Z_{id}, z^{(a)}$	All arguments in Z_{id} are equated to $z^{(a)}$	$z^{(a)}$ is an element of Z_{id}

Type “Abstr” stands for abstraction operator, whereas “Approx” stands for approximation operator

Appendix F

Abstraction Patterns



In this appendix two more abstraction patterns are described, for the sake of illustration. The complete set, corresponding to the full set of operators listed in Appendix E, can be found in the book’s companion Web site.

In Table F.1 the pattern referring to *hiding an argument* of a function or relation is reported.

Table F.1 HIDING-ARGUMENT—Abstraction Pattern that hides an argument of a function or relation

NAME	HIDING-ARGUMENT
ALSO KNOWN	Described by Plaisted [419] as “propositionalization”. As it requires a structured representation, it is less popular than hiding an element. In Machine Learning it may correspond to the task of “propositionalization”.
GOAL	In Problem Solving and Automated Reasoning it is meant to speed up inference by providing a sketch of a proof without variables.
TYPICAL APPLICATIONS and KNOWN USE	Very much used in databases, where it corresponds to the <i>projection</i> operation in relational algebra.
IMPLEMENTATION ISSUES	Problems with this operator may arise when the unique argument of a univariate function has to be hidden. In this case the function becomes a constant. A relation, whose arguments are all hidden, becomes a Boolean variable with an empty cover.
KNOWN USES	Machine Learning, CSP, Problem Solving, Theorem Proving.
SIMILAR PATTERNS	This pattern is related to the Equating Arguments Pattern, and to Building a hierarchy of arguments.

In Table F.2 we provide the template for aggregating objects.

Table F.2 AGGREGATION—Aggregation Pattern that forms new objects starting from existing ones

NAME	AGGREGATION
ALSO KNOWN	In Machine Learning the <i>aggregation</i> operator is known as “predicate invention”, “predicate construction”, or “term construction”, whereas in Data Mining it is related to “motif discovery”. In general, it is the basis of the “constructive induction” approach to learning. In Planning, Problem Solving, and Reinforcement Learning it includes “state aggregation” and “spatial and/or temporal aggregation”.
GOAL	It aims at working, in any field, with “high level” constructs in the description of data and in theories, in order to reduce the computational cost and increasing the meaningfulness of the results.
TYPICAL APPLICATIONS	Finding regions and objects in the visual input, representing physical apparatus at various levels of details by introducing composite components.
IMPLEMENTATION ISSUES	Implementing the grouping operator may require even complex algorithms, and the cost of aggregation has to be weighted against the advantages in the use of the abstract representation.
KNOWN USES	Even though not always under the name of abstraction, aggregation and feature construction is very much used in computer vision, description of physical systems, Machine Learning, Data Mining, and Artificial Intelligence in general.

Appendix G

Abstraction of Michalski’s “Train” Problem



In this appendix the detailed application of the introduce operators in Michalski’s “train” problem are reported. The results of the method are described in Chap. 9. In Table G.1 the method $\text{meth}(\mathcal{P}_g, \omega_{\text{aggr}}(\{\text{car}, \text{load}\}, \text{loadedcar}))$ is reported.

The parameters, which are listed in Table G.2, specify how objects are actually aggregated and how attributes and relations change as a consequence.

Finally, Table G.3 describes the actual algorithm performing the aggregation abstraction.

Table G.1 Method $\text{meth}(\mathcal{P}_g, \omega_{\text{aggr}}(\{\text{car}, \text{load}\}, \text{loadedcar}))$

NAME	$\text{meth}(\mathcal{P}_g, \omega_{\text{aggr}}(\{\text{car}, \text{load}\}, \text{loadedcar}))$
INPUT	$\mathcal{P}_g, \{\text{car}, \text{load}\}, \text{loadedcar},$ $g : \mathcal{O}_{\text{car}} \times \mathcal{O}_{\text{load}}^n \rightarrow \mathcal{O}_{\text{loadedcar}} \quad (n \geq 0)$ $g(y, x_1, \dots, x_n) = \text{if } [y \in \mathcal{O}_{\text{car}}] \wedge [x_1, \dots, x_n \in \mathcal{O}_{\text{load}}] \wedge$ $[(x_i, y) \in \text{RCOV}(R_{\text{Inside}}) \quad (1 \leq i \leq n)] \wedge$ $[y, x_1, \dots, x_n] \text{ are labelled with the same example} \text{ then } z$
OUTPUT	$\mathcal{P}_a, R_{\text{Partof}} \subseteq (\mathcal{O}_{\text{load}} \cup \mathcal{O}_{\text{car}}) \times \mathcal{O}_{\text{loadedcar}}$
APPL-CONDITIONS	$\exists c \in \mathcal{O}_{\text{car}}$ $\exists (\text{different}) \ell_1, \dots, \ell_n \in \mathcal{O}_{\text{load}}$ $c, \ell_1, \dots, \ell_n \text{ are labelled with the same example}$ $(\ell_i, c) \in \text{RCOV}(R_{\text{Inside}}) \quad (1 \leq i \leq n)$
PARAMETERS	See Table G.2
MEMORY	$\Delta^{(\mathcal{P})}, \text{RCOV}(R_{\text{Partof}})$
BODY	See Table G.3

Table G.2 Parameters of the method `meth` ($\mathcal{P}_g, \omega_{\text{aggr}}(\{\text{car}, \text{load}\}, \text{loadedcar})$)

$\alpha(x, y) \Rightarrow$	$LCshape^{(a)}(z) = Cshape(y)$ $LClength^{(a)}(z) = Clength(y)$ $LCwall^{(a)}(z) = Cwall(y)$ $LCwheels^{(a)}(z) = Cwheels(y)$
$\gamma(x_1, x_2) \Rightarrow$	R_{Inside} is NA if $\exists y'$ s.t. $(y', y) \in RCOV(R_{\text{Infrontof}})$ then $(y', z) \in RCOV(R_{\text{Infrontof}}^{(a)})$ if $\exists y'$ s.t. $(y, y') \in RCOV(R_{\text{Infrontof}})$ then $(z, y') \in RCOV(R_{\text{Infrontof}}^{(a)})$

Table G.3 Pseudo-code for the method `meth` ($\mathcal{P}_g, \omega_{\text{aggr}}(\{\text{car}, \text{load}\}, \text{loadedcar})$)

METHOD `meth` ($\mathcal{P}_g, \omega_{\text{aggr}}(\{\text{car}, \text{load}\}, \text{loadedcar})$)
Let $R_{\text{partof}} \subseteq (\mathcal{O}_{\text{load}} \cup \mathcal{O}_{\text{car}}) \times \mathcal{O}_{\text{loadedcar}}$ *be a new predicate*
Let $\sigma = \{\ell_1, \dots, \ell_n \mid \ell_i \in \mathcal{O}_{\text{load}}, (\ell_i, c) \in RCOV(R_{\text{Inside}}) (1 \leq i \leq n)\}$
 $\mathcal{O}_a = \mathcal{O}_g, \mathcal{A}_a = \mathcal{A}_g, \mathcal{R}_a = \mathcal{R}_g$
 $\Delta_{\mathcal{O}}^{(P)} = \Delta_{\mathcal{A}}^{(P)} = \Delta_{\mathcal{R}}^{(P)} = \emptyset$
 $R_{\text{partof}} = \emptyset$
Build up $\bar{d} = g(c, \ell_1, \dots, \ell_n)$
 $R_{\text{partof}} = R_{\text{partof}} \cup \{(c, \bar{d})\}$
for $i = 1, n$ **do**
 $R_{\text{partof}} = R_{\text{partof}} \cup \{(\ell_i, \bar{d})\}$
end
 $\mathcal{O}_a = \mathcal{O}_a - \{c, \ell_1, \dots, \ell_n\} \cup \{\bar{d}\}$
 $\Delta_{\mathcal{O}}^{(P)} = \{c, \ell_1, \dots, \ell_n\}$
 $\mathcal{A}_a = \mathcal{A}_a - \{(c, \text{car}, Cshape(c), Clength(c), Cwall(c), Cwheels(c))\} -$
 $\{(\ell_i, \text{load}, Lshape(\ell_i)) \mid (1 \leq i \leq n)\}$
 $\mathcal{A}_a = \mathcal{A}_a \cup \{(\bar{d}, \text{loadedcar}, Cshape(c), Clength(c), Cwall(c), Cwheels(c))\},$
 $\Delta_{\mathcal{A}}^{(P)} = \Delta_{\mathcal{A}}^{(P)} \cup \{(c, \text{car}, Cshape(c), Clength(c), Cwall(c), Cwheels(c))\} \cup$
 $\{(\ell_i, \text{load}, Lshape(\ell_i)) \mid (1 \leq i \leq n)\}$
forall $(y', c) \in RCOV(R_{\text{Infrontof}})$ **do**
 $R_{\text{Infrontof}}^{(a)} = R_{\text{Infrontof}} - \{(y', c)\} \cup \{(y', \bar{d})\}$
 $\Delta_{\mathcal{R}}^{(P)} = \Delta_{\mathcal{R}}^{(P)} \cup \{(y', c)\}$
end
forall $(c, y') \in RCOV(R_{\text{Infrontof}})$ **do**
 $R_{\text{Infrontof}}^{(a)} = R_{\text{Infrontof}} - \{(c, y')\} \cup \{(\bar{d}, y')\}$
 $\Delta_{\mathcal{R}}^{(P)} = \Delta_{\mathcal{R}}^{(P)} \cup \{(c, y')\}$
end
 $\Delta^{(P)} = \Delta_{\mathcal{O}}^{(P)} \cup \Delta_{\mathcal{A}}^{(P)} \cup \Delta_{\mathcal{R}}^{(P)} \cup RCOV(R_{\text{partof}})$

Appendix H

Color Figures



In this appendix, some of the figures appearing in the book are reported with their original colors.

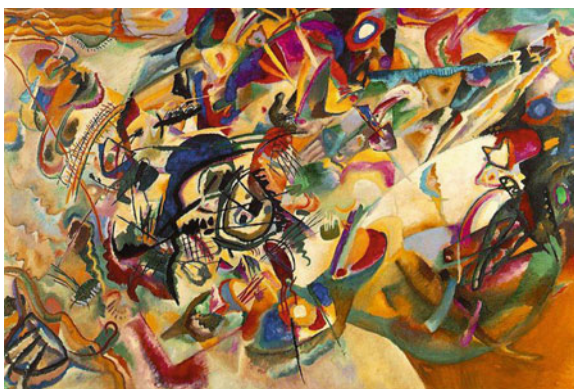


Fig. H.1 Vasilij Kandinsky, Composition VII, 1913. The Tretyakov Gallery, Moscow

Fig. H.2 *Nocturne in Black and Gold* by J. McNeill Whistler (1875). It is considered as a first step toward abstraction in painting

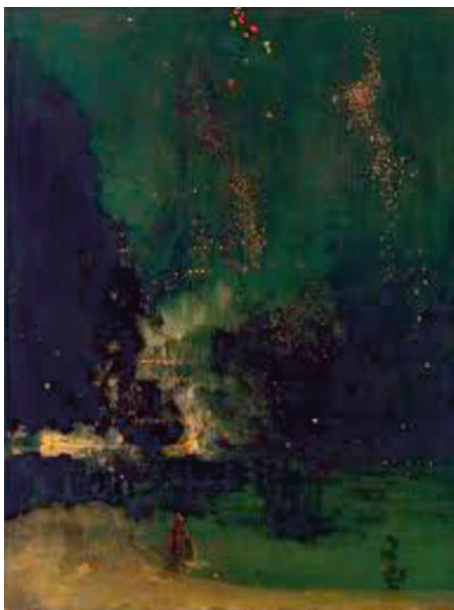


Fig. H.3 K. Malevich's *Portrait of Ivan Klioune* (1911). The State Russian Museum, St. Petersburg



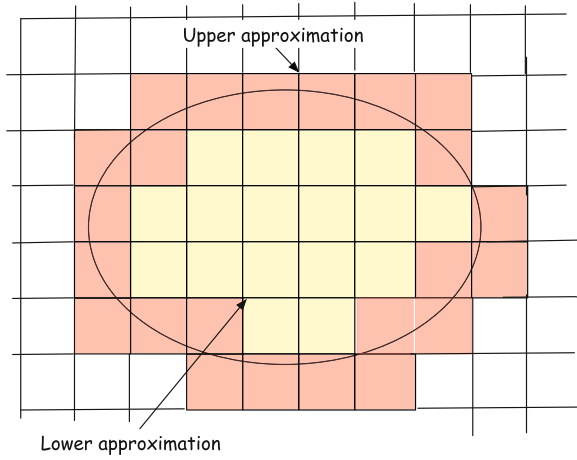


Fig. H.4 Upper (pink + yellow regions) and lower (yellow region) approximations of a concept \mathcal{X} = Oval, defined as a region in the 2D plane



Fig. H.5 Incas used *quipus* to memorize numbers. A quipu is a cord with nodes that assume position-dependent values. An example of the complexity a quipu may reach. (Reprinted with permission from *Museo Larco*, Pueblo Libre, Lima, Peru.)

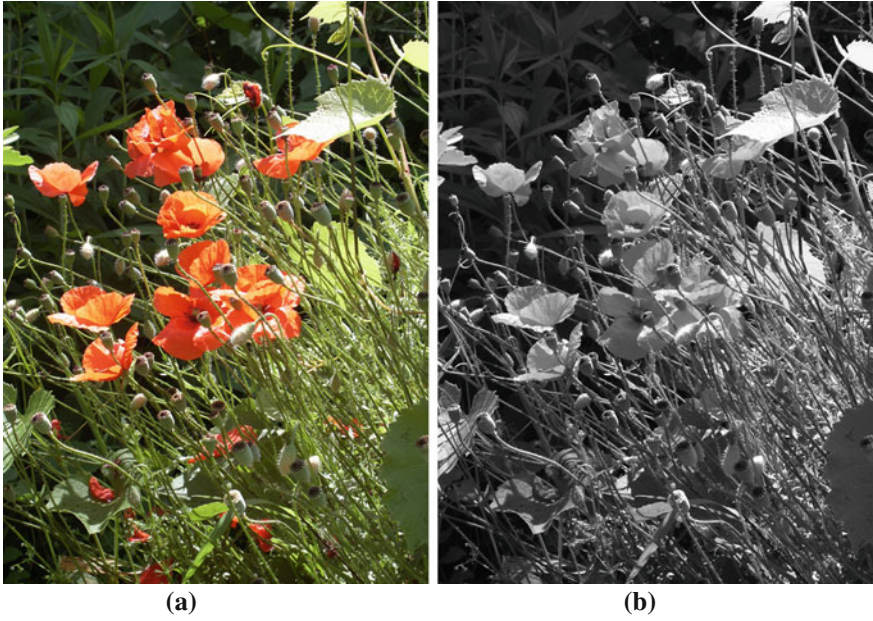


Fig. H.6 **a** Picture of a poppy field. If we only have this picture, it is impossible to say whether it is concrete or abstract. **b** The same picture in black and white. By comparison, this last is less informative than the colored one, because the information referring to the color has been removed; then picture **b** is more abstract than picture **a**



Fig. H.7 A color picture has been transformed into a *black and white* one. If the color is added again, there is no clue for performing this addition correctly, if it is not know how the color was originally removed

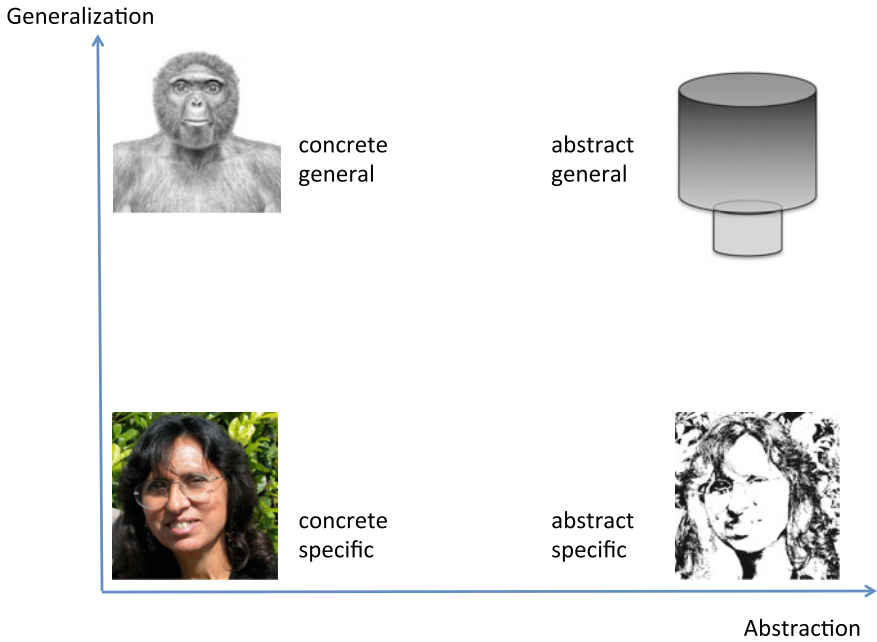


Fig. H.8 Abstraction and generalization can be combined in every possible way. In the *left-bottom* corner there is picture of one of the authors, which is specific (only one instance) and concrete (all the skin, hair, face, ... details are visible). In the *right-bottom* corner there is a version of the picture which is specific (only one instance, as the person is still recognizable) and abstract (most details of the appearance are hidden). In the *top-left* corner the chimpanzee–human last common ancestor is represented with many physical details, making thus the picture still concrete; however many monkeys and hominides satisfy the same description, so that this is an example of a concrete but general concept. Finally, in the *top-right* corner there is a representation of a human head according to Marr [353] (see Fig. 2.13); the head is abstract (very few details of the appearance) and general (any person could be an instance)

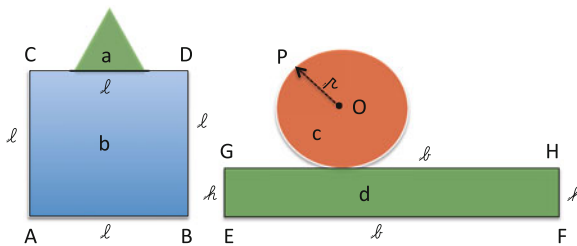


Fig. H.9 A geometrical scenario with various geometrical elements



Fig. H.10 Example of method $\text{meth}(\mathcal{P}_g, \omega_{\text{hattr}}((A_m, A_m)))$. The attribute $A_m = \text{Color}$ is hidden from the *left* picture giving a *gray-level* picture (*right*). Each pixel shows a value of the light intensity, but this last is no more distributed over the R,G,B channels

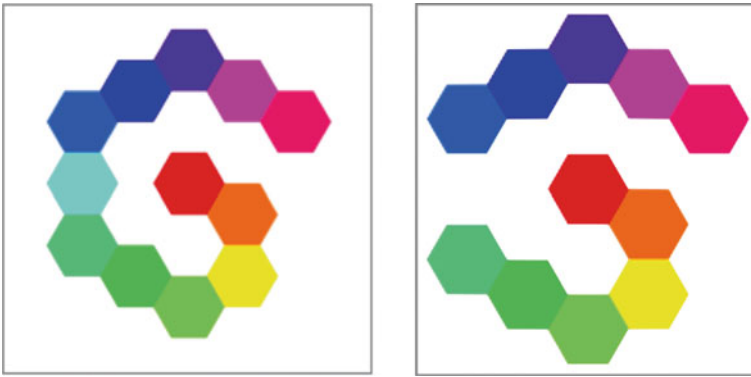


Fig. H.11 Example of application of the method $\text{meth}[\mathcal{P}_g, \omega_{\text{hattrval}}((\text{Color}, \Lambda_{\text{Color}}), \text{turquoise})]$. The value turquoise is *hidden* from the left picture; a less colorful picture is obtained (*right*), where objects of color turquoise become transparent (UN)

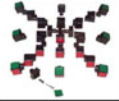




	<p>The Description Frame: the 26 small cubes of the Rubik's Cube</p> $\Gamma(\Sigma)$
	<p>The Configuration Space: all 43 252 003 274 489 856 000 configs.</p> $\Psi(\Sigma)$
	<p>A given configuration : one of the Cube configuration</p> $\psi \in \Psi$
	<p>An observed configuration : one cube's face (9 small cubes) (corresponding to 3344302080 consistent configurations)</p> $\mathcal{P}\text{-Set } \subset \Psi$
	<p>A query : in this case a particular configuration to reach</p> Q

Fig. H.12 The Rubik's cube can be described in terms of the 26 small component cubes, which give rise to the description frame Γ . Each arrangement of the cubes generates a specific configuration ψ ; the configuration set, Ψ , is very large. A configuration is a complete description of the positions of the small cubes, so that it is unique. If Rubik's cube is observed only partially, for instance by looking only at one face, the observation corresponds to many configurations, each one obtained by completing the invisible faces of the cube in a different way; in this case we have a \mathcal{P} -Set \mathcal{P} , which is a set of configurations. The query Q can be represented by a particular configuration to be reached starting from an initial one

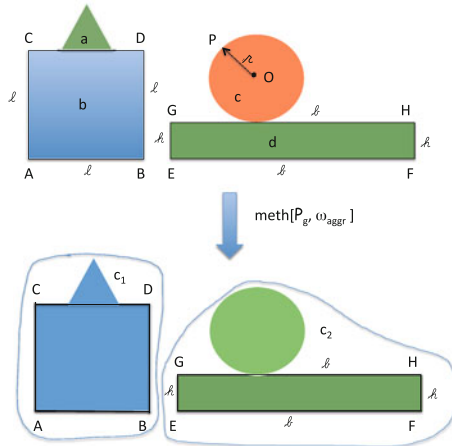


Fig. H.13 Application of method $\text{meth}[P_g, \omega_{aggr}(\text{figure}, \text{figure}, \text{tower})]$. Objects a and b are aggregated to obtain object c_1 , and objects c and d are aggregated to obtain object c_2 . The color of c_1 is blue, because b is larger than a, whereas the color of c_2 is green. Both composite objects are large. The new object c_1 is at the left of c_2

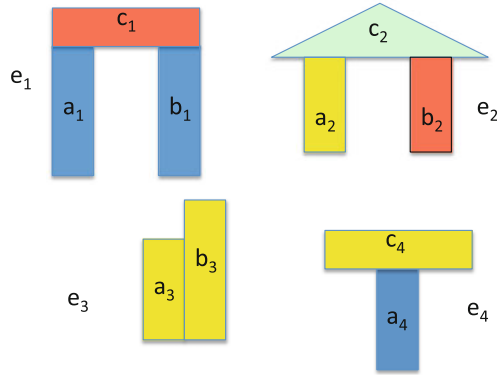


Fig. H.14 Examples of four structured objects, used to learn the concept of an “arch”. Each component has a shape (rectangle or triangle) and a color (*blue, red, yellow, or green*). They are linked by two relations, namely R_{ontop} and $R_{adjacent}$



Fig. H.15 **a** Part of a map at 1/25000 scale. **b** A 16-fold reduction of the map. **c** Cartographic generalization of the map at the 1/100 000 scale. By comparing **b** and **c** the differences between simply reducing and generalizing are clearly apparent

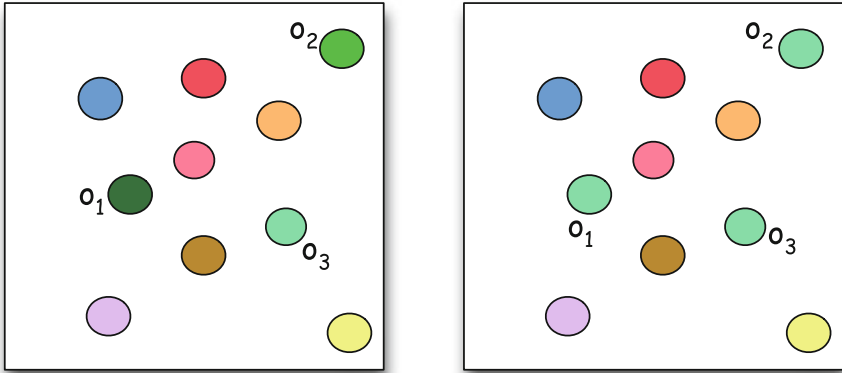


Fig. H.16 Application of method $\text{meth}(\mathcal{P}_g, \omega_{\text{equival}}((Color, \Lambda_{Color}), V_{id}, v^{(a)}))$ to the figure on the left. Let $V_{id} = \{\text{olive-green, sea-green, lawn-green, light-green, dark-green}\}$. Objects $o_1, o_2,$ and o_3 have color dark-green, lawn-green, and sea-green, respectively. After equating all shades of green to sea-green, the color of all three considered objects becomes sea-green

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