№055

Ada Lovelace

Introduction / Einführung:
Joasia Krysa

100 Notes – 100 Thoughts / 100 Notizen – 100 Gedanken | №055

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Introduction

Joasia Krysa

There Never Was a Note G

In his letter to Ada Augusta Lovelace of July 2, 1843, Charles Babbage writes: "I like much the improved form of the Bernoulli Note but can judge of it better when I have the Diagram and Notation."[1] He is referring to the last in a set of notes written by Lovelace that interpreted the Analytical Engine, the first fully automatic and universal computer, invented by Babbage in 1834, although never actually completed during his lifetime. She appended these notes to her translation of an article written by Luigi Federico Menabrea after he had heard Babbage present a paper on the Engine. Her translation, together with her extensive notes (three times the length of the original article), were published in 1843 and signed A.A.L.[2]

In the same letter, Babbage recounts the order of the notes in preparation for submitting them to the publisher:

A Sent to Lady L.
B With C.B.
C Ditto
D Sent to Lady L.
E With C.B.
F Retained by Lady L.
G Where is it gone?
H With C.B.

In response to his question about the missing Note G, Lovelace writes: "There never was a Note G. I do not know why I chose H instead of G, & thus insulted the latter worthy letter."[3] In the final published version of the work, Note H becomes Note G and subsequently, along with Babbage's machine and the rest of Lovelace's notes, a key reference point in the history of modern computing.[4]

1 | Letter from Charles Babbage to Ada Augusta Lovelace, July 2, 1843, typed copy, British Library, Additional ADD 54089.

2 | Luigi Federico Menabrea, "Notions sur la machine analytique de Charles Babbage," *Bibliothèque Universelle de Genève* 82 (October 1842). The translated article with Lovelace's notes was published as "Sketch of the Analytical Engine Invented by Charles Babbage, Esq. By L. F. Menabrea, of Turin, Officer of the Military Engineers. With Notes by the Translator," *Scientific Memoirs* 3 (1843), pp. 666–731.

3 | Letter from Ada Augusta Lovelace to Charles Babbage, July 2, 1843, handwritten, British Library, Additional ADD 37192.

4 | This came almost one hundred years in advance of the work of Konrad Zuse, Alan Turing, Howard Aiken, and Grace Hopper and the invention of the machines that came to symbolize the age of modern computing (Z1, Baby Manchester Computer, Harvard Mark I, and ENIAC).

The significance of Note G is that it provides a description of method and a diagram of an algorithm for setting up the engine to compute the Bernoulli numbers.[5] The diagram is widely referred to as the first computer program, and the notes the first expression of computer theory. Together, they can be considered what we would describe in contemporary terms as the software required to operate the hardware of Babbage's machine, which did not yet exist.

However, at the time there remained uncertainty over the significance of Babbage's invention, and the future potential of computation per se. Note G, reproduced in full in this notebook from its first published version of 1843, contains Lovelace's reservations in this respect:

> It is desirable to guard against the possibility of exaggerated ideas that might arise as to the powers of the Analytical Engine. . . . The Analytical Engine has no pretensions whatever to *originate* anything. It can do whatever we *know how to order it* to perform. It can *follow* analysis; but it has no power of *anticipating* any analytical relations or truths. Its province is to assist us in making *available* what we are already acquainted with.[6]

While Note G advises caution as to the potential of the Engine, and computational machines more generally, to demonstrate independent thinking (artificial intelligence), Lovelace recognizes the particular significance of the Engine in marking a transition from calculation to general-purpose computing—from a machine merely able to tabulate numbers to a programmable universal machine capable of manipulating symbols according to rules and of generating anything at all, whether music, poetry, or images.[7] In Note A she writes:

> The Analytical Engine, on the contrary, is not merely adapted for *tabulating* the results of one particular function and of no other, but *for developing and tabulating* any function whatever. In fact the engine may be described as being the material expression of any indefinite function of any degree of generality and complexity. . . . The operating mechanism can even be thrown into action independently of any object to operate upon.[8]

The distinctive feature of the machine is its use of punch cards for programming the Engine, adopted by Babbage after they were first introduced by Joseph Jacquard to instruct the loom to automate and regulate weaving patterns in 1804. Lovelace remarks that the Engine "*weaves algebraic patterns* just as the Jacquard-loom weaves flowers and leaves."[9] Furthermore, in expressive terms that combine poetry and mathematics (as befits the daughter of the poet Lord Byron), she imagines that the Engine might "compose elaborate and scientific pieces of music of any degree of complexity or extent,"[10] preempting Donald Knuth's comments in 1968 that computer programming need not be considered to be merely functional, but "an esthetic experience much like composing poetry or music."[11] Lovelace points to the poetic and metaphysical dimensions of technological invention, something to which Geoffrey Batchen draws attention in his article describing the parallel history of the Analytical Engine and lace in connection with William Henry Fox Talbot's lace contact print of 1845.[12]

In keeping with Babbage's description of Lovelace as the "Enchantress of Numbers," it is the speculative nature of her work that continues to intrigue, such as her childhood dreams of writing a book about "Flyology" that would set out a method of flying (predating William Henson's design for an aerial steam carriage of 1842):

5 | In Note G, Lovelace explains that her choice of Bernoulli numbers—a sequence of rational numbers—to demonstrate the computing powers of the Engine is "a rather complicated example" ("Sketch of the Analytical Engine Invented by Charles Babbage" [see note 2], Note G, p. 724), but useful in that it allows one to highlight the contrast between Babbage's earlier machine, the Difference Engine, as a mere *calculating* machine, and his more advanced Analytical Engine as a universal *computing* machine.

6 | Ibid., p. 722.

7 | In time, this also became contestable. In 1950, Alan Turing, mathematician and computer scientist, wrote his seminal paper on "Computing Machinery and Intelligence" to question the perceived limitations of machines for independent thinking (artificial intelligence), against the reservations expressed by Lovelace some hundred years earlier. He famously asked: "Can machines think?"

8 | "Sketch of the Analytical Engine Invented by Charles Babbage" [see note 2], Note A, pp. 691, 694.

9 | Ibid., p. 696.

10 | Ibid., pp. 696, 694.

11 | Donald E. Knuth, *The Art of Computer Programming* (Reading, Mass.: Addison-Wesley Publishing Company, 1981 [orig. 1968]). It is also interesting to note that there is an object-oriented programming language named Ada in recognition of Lovelace's contribution to programming.

12 | Geoffrey Batchen, "Electricity Made Visible," in *New Media Art: Context and Practice in the UK 1994–2004*, ed. Lucy Kimbell (London and Manchester: Arts Council England and Cornerhouse Publications, 2004), pp. 27–44. In 1999 Hüseyin Alptekin made an embroidered canvas that connects many of these threads and named it *Lovelace*. He explained: "Lace, which denotes a completion of love, which is remembered along with love, purveys meaning both in the sense of being a sucker as well as in terms of embroidery. Besides this, lovelace is once again a borrowing from disparate references and is a send off to different contexts. Like the famous porno star Linda Lovelace, it is also a borrowing of the famous mathematician Ada Byron Lovelace whose name was given to an infobiogen server. Tulle, curtaining, veiling, removing, stitching. . . . Fantasy deals with postponing the jouissance,

> As soon as I have brought flying to perfection, I have got a scheme about a . . . steamengine which, if ever I effect it, will be more wonderful than either steampackets or steamcarriages, it is to make a thing in the form of a horse with the steamengine in the inside so contrived as to move an immense pair of wings, fixed on the outside of the horse, in such a manner as to carry it up into the air while a person sits on its back.[13]

In her work, as well as in her life, Lovelace managed to combine scientific rationalism with subjective imagination, influenced by her experience of the Industrial Revolution and the many technological innovations at that time.[14] However, in a departure from the discipline-based systems of thinking and acquiring knowledge reinforced by the industrial period, she strongly believed in the need for connecting *all* disciplines. This attempt to go beyond the separation of fields of knowledge has since become a common thread in contemporary thinking, as for instance in the work of cybernetician Heinz von Foerster, who argued that in an increasingly complex world, it is no longer possible to maintain traditional science as the dominant structure of thinking. Consequently, there is a shift toward what he described as "systemics," an approach that sees *things together* in complex connections and interrelations.[15] Lovelace's term for this was "Poetical Science," and her Note G anticipates the indefinite potential of machines to express complexity.[16]

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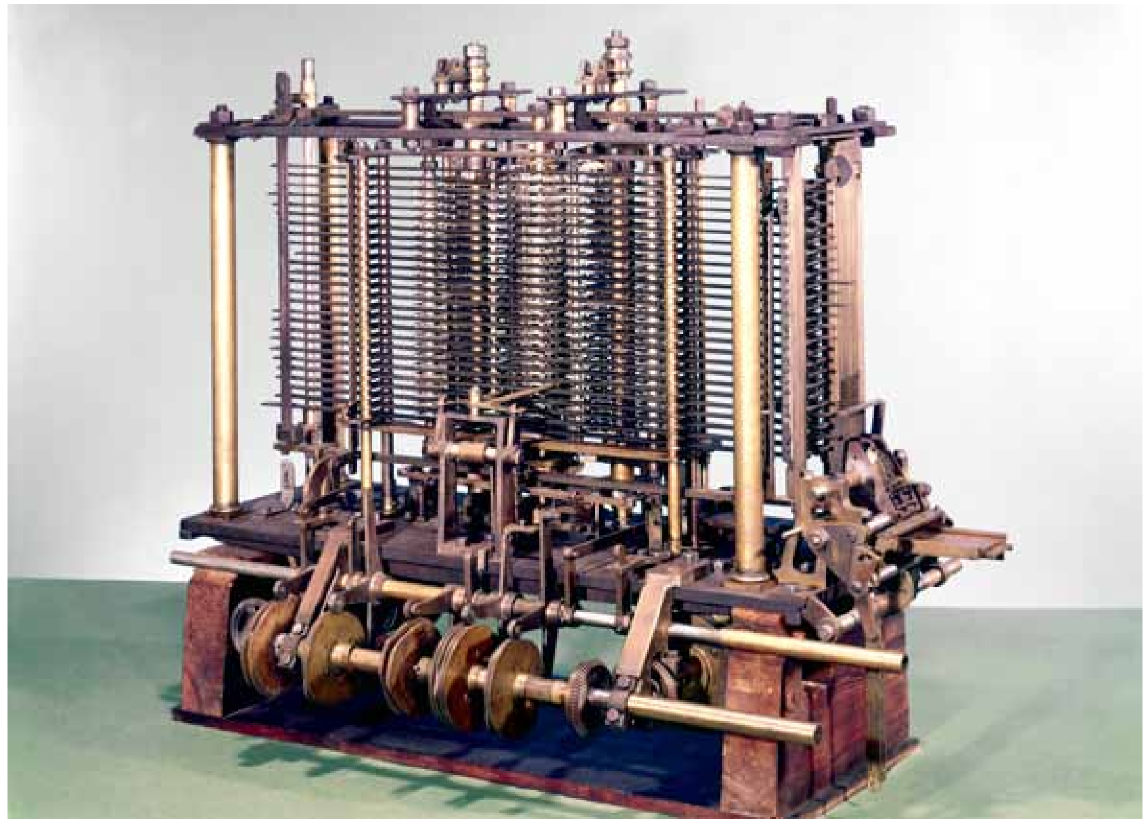
inasmuch as the truth is embroidered in the dance of seven veils." Quoted from: www.clubmedia.de/clubmedia99/hba_html/hba_set_12.html (accessed October 2011).

13 | Quoted from a letter from Ada Lovelace to her mother Lady Byron dated Monday, April 7, 1828, in Betty A. Toole, *Ada, The Enchantress of Numbers* (Mill Valley, Cal.: Strawberry Press, 1992), p. 34.

14 | Most importantly Charles Wheatstone's telegraph, Joseph Jacquard's loom and punch cards, Sir David Brewster's kaleidoscope, Michael Faraday's first induction of electric current, and Mary Somerville's scientific writings *On the Connexion of the Physical Sciences* (1934).

15 | Interview with Heinz von Foerster in Lutz Dammbeck's documentary film *Das Netz* (2003).

16 | The author would like to thank those who assisted in various ways with research in preparation of the material for this notebook: Colin Harris, Mary Clapinson and Alan Brown at the Bodleian Library, University of Oxford; Adrian Shindler and Justin Clegg at the British Library, London; Matthew Connell and Paul Wilson at Powerhouse Museum, Sydney; Nicola Thwaite at the National Trust, London; Jo Francis and John Fuegi; and Lucia Pietroiusti.



Charles Babbage,
Analytical Engine, 1871

Ada Lovelace (1815–1852)

Einführung

Joasia Krysa

Es gab nie eine Anmerkung G

In einem Brief an Ada Augusta Lovelace vom 2. Juli 1843 schreibt Charles Babbage: »Die verbesserte Form der Bernoulli-Anmerkung gefällt mir sehr gut, aber ich kann sie besser beurteilen, wenn ich das Diagramm und die Notation habe.«[1] Was Babbage hier als Bernoulli-Anmerkung bezeichnet, ist die letzte in einer Reihe von Anmerkungen, in denen Lovelace die *Analytical Engine* interpretierte, den ersten vollautomatischen, universellen Computer, den Babbage 1834 erfunden hatte, der zu seinen Lebzeiten jedoch niemals ganz fertiggestellt wurde. Sie fügte diese Anmerkungen ihrer Übersetzung eines Artikels von Luigi Federico Menabrea hinzu, den dieser verfasste, nachdem er einen Vortrag von Babbage über die Maschine gehört hatte. Lovelaces Übersetzung erschien 1843 mit ihren ausführlichen Anmerkungen (die drei Mal so lang waren wie der Aufsatz) und war mit A.A.L. unterzeichnet.[2]

Im gleichen Brief wiederholt Babbage die Reihenfolge der Anmerkungen, die zur Übermittlung an den Verleger in Vorbereitung waren:

A An Lady L. geschickt
B Bei C.B.
C Dito
D An Lady L. geschickt
E Bei C.B.
F Von Lady L. einbehalten
G Wo ist sie geblieben?
H Bei C.B.

In ihrer Antwort auf seine Frage nach der fehlenden Anmerkung G schreibt Lovelace: »Es gab nie eine Anmerkung G. Ich weiß nicht, warum ich H anstelle von G wählte & dadurch letzteren ebenbürtigen Buchstaben kränkte.«[3]

1 | Brief von Charles Babbage an Ada Augusta Lovelace, 2. Juli 1843, Schreibmaschinendurchschlag, British Library, Additional ADD 54089.

2 | Luigi Federico Menabrea, »Notions sur la machine analytique de Charles Babbage«, in: *Bibliothèque Universelle de Genève*, 82, Oktober 1842. Der übersetzte Artikel mit Lovelaces Anmerkungen erschien unter dem Titel »Sketch of the Analytical Engine invented by Charles Babbage, Esq. By L. F. Menabrea, of Turin, Officer of the Military Engineers. With Notes by The Translator«, in: *Scientific Memoirs*, 3, 1843, S. 666–731.

3 | Handschriftlicher Brief von Ada Augusta Lovelace an Charles Babbage, 2. Juli 1843, British Library, Additional ADD 37192.

In der finalen publizierten Fassung dieser Arbeit wird Anmerkung H zu Anmerkung G und in der Folgezeit, zusammen mit Babbages Maschine und Lovelaces übrigen Anmerkungen, zu einem der wichtigsten Bezugspunkte in der Geschichte der modernen Computertechnologie.[4]

Anmerkung G ist bedeutsam, weil sie eine Beschreibung der Methode und ein Diagramm eines Algorithmus enthält, um eine Maschine auf die Berechnung der Bernoulli-Zahlen zu programmieren.[5] Das Diagramm gilt weithin als das erste Computerprogramm, und die Reihe der Anmerkungen ist ein erster Ausdruck der Computertheorie; zusammen bilden sie, was man heute als die Software bezeichnen würde, um die damals noch nicht existierende Hardware von Babbages Maschine zu betreiben.

Doch damals herrschte eine gewisse Unsicherheit, was die Bedeutung von Babbages Erfindung sowie die Grenzen und zukünftigen Potenziale der Computertechnologie als solcher betraf. In Anmerkung G, die in diesem Notizbuch in der ersten veröffentlichten Fassung von 1843 vollständig wieder abgedruckt wird, formuliert Lovelace ihre diesbezüglichen Einschränkungen:

> Was die Fähigkeiten der Analytical Engine [Analytischen Maschine] angeht, so ist es erstrebenswert, sich vor möglicherweise aufkommenden überzogenen Vorstellungen zu hüten. […] Die Analytical Engine ist nicht in der Lage, irgendetwas zu *erschaffen*. Sie kann alles tun, *von dem wir wissen, wie man sie anweisen muss,* um es auszuführen. Sie kann eine Analyse *durchführen;* aber sie hat nicht die Fähigkeit, irgendwelche analytischen Beziehungen oder Wahrheiten *vorwegzunehmen*. Ihre Domäne besteht darin, uns zu helfen, etwas *zur Verfügung zu stellen,* mit dem wir bereits vertraut sind.[6]

Obwohl Anmerkung G zur Vorsicht mahnt, was das Potenzial dieser Maschine und von mechanischen Computern im Allgemeinen betrifft, ein eigenständiges Denken (Künstliche Intelligenz) hervorzubringen, erkennt Lovelace die besondere Bedeutung der Maschine insofern, als diese den Übergang vom Rechnen zu allgemeinen Computer-Anwendungen markiert – von einer Maschine, die Zahlen nur tabellieren kann, zu einer programmierbaren universellen Maschine, die in der Lage ist, Symbole nach bestimmten Regeln zu verarbeiten und auf diese Weise die unterschiedlichsten Dinge, wie etwa Musik, Dichtung oder Bilder, zu generieren.[7] In Anmerkung A schreibt sie:

> Die Analytical Engine hingegen ist nicht nur dazu geeignet, die Ergebnisse einer bestimmten Funktion und keiner anderen *tabellarisch darzustellen,* sondern auch, irgendeine beliebige Funktion *zu entwickeln und tabellarisch darzustellen*. Man könnte diese Maschine tatsächlich als den materiellen Ausdruck jeder unbestimmten Funktion eines beliebigen Grades von Allgemeinheit und Komplexität beschreiben. […] Man kann den Betriebsmechanismus sogar unabhängig von irgendeinem zu bearbeitenden Gegenstand in Gang setzen.[8]

Die Besonderheit der Maschine ist der Einsatz von Lochkarten, die zu ihrer Programmierung verwendet wurden – ein Verfahren, das Babbage nach der Einführung der Lochkarten durch Joseph Jacquard im Jahr 1804 übernahm, mit denen Webstühle programmiert wurden, um die Herstellung von Webmustern zu automatisieren und zu steuern. Lovelace bemerkte, dass die Maschine »*algebraische Muster webt,* so wie der Jacquard-Webstuhl Blumen und Blätter webt«.[9] In ausdrucksvollen Worten, die Poesie und Mathematik miteinander verbinden (wie es der Tochter des Dichters Lord Byron entsprach), stellt sie sich zudem vor, wie die Maschine »kunstvolle und wissenschaftliche Musikstücke von beliebiger Komplexität oder Länge komponiert«;[10] damit nahm sie Donald Knuths Bemerkung aus dem Jahr 1968 vorweg, dass man das Programmieren von Computern nicht nur rein funktional, sondern auch als »ästhetische Erfahrung, ganz ähnlich wie das Komponieren von Gedichten oder Musik« betrachten könne.[11] Lovelace verweist auf die poetische und metaphysische Dimension einer technologischen Erfindung – etwas, worauf auch Geoffrey Batchen in einem Artikel aufmerksam macht, in dem er im Zusammenhang mit William Henry Fox Talbots Kontaktkopien von Spitze [*lace*] von 1835 die parallele Geschichte der Analytical Engine und Spitze beschreibt.[12]

Der spekulative Charakter ihrer Arbeiten, der in Einklang mit Babbages Beschreibung von Lovelace als »Zahlenzauberin« steht, wirkt bis heute faszinierend, beispielsweise ihr Kindheitstraum, eine Flugmethode zu erfinden und ein Buch über *Flyology* zu schreiben (der Hensons Entwurf einer Dampf-Flugmaschine von 1842 antizipiert):

> Sobald ich das Fliegen perfektioniert habe, verfolge ich den Plan für eine […] Dampfmaschine, die, wenn ich sie jemals zustande bringe, wunderbarer sein wird als Dampfschiffe oder Dampfwagen; es geht darum, ein Ding in der Form eines Pferdes zu bauen, in dessen Innerem sich eine Dampfmaschine befindet, die so konstruiert ist, dass sie ein riesiges Flügelpaar bewegen kann, das an der Außenseite des Pferdes befestigt ist und dieses in die Lüfte heben kann, während ein Mensch auf seinem Rücken sitzt.[13]

Es gelang ihr, in ihrem Werk und ihrem Leben den wissenschaftlichen Rationalismus mit einer subjektiven Vorstellungskraft zu verbinden, die von den Erfahrungen der Industriellen Revolution und den zahlreichen technologischen Neuerungen der damaligen Zeit geprägt war.[14] Doch abweichend von einer Denkweise und einem Wissenserwerb, die auf einzelnen Disziplinen beruhen und durch das Industriezeitalter verstärkt wurden, war Lovelace zutiefst überzeugt von der Notwendigkeit, *alle* Disziplinen miteinander zu verbinden. Der Versuch, die Trennung der Disziplinen zu überwinden, ist seither ein durchgängiger Aspekt des zeitgenössischen Denkens, wie etwa im Werk des Kybernetikers Heinz von Foerster, der argumentierte, dass es in einer immer komplexer werdenden Welt nicht mehr möglich sei, die traditionellen Wissenschaften als vorherrschende Denkstruktur aufrechtzuerhalten. Daraus folgt eine Verschiebung zu einer Herangehensweise, die er als »systemics« beschrieb und die in komplexen Verbindungen und Wechselbeziehungen *Dinge zusammen sieht*.[15] Lovelaces Begriff hierfür war »Poetische Wissenschaft«, und ihre Anmerkung G nimmt das unbegrenzte Potenzial von Maschinen, Komplexität auszudrücken, vorweg.[16]

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4 | Bis zur Entstehung der Arbeiten von Konrad Zuse, Alan Turing, Howard Aiken oder Grace Hopper und zur Erfindung der Maschinen, die zu Symbolen des modernen Computerzeitalters wurden (Z1, Baby Manchester Computer, Harvard Mark I und ENIAC), sollten noch fast hundert Jahre vergehen.

5 | In Anmerkung G erklärt Lovelace, dass die von ihr gewählten Bernoulli-Zahlen »ein ziemlich kompliziertes Beispiel« seien, um die Rechenleistung der Maschine zu erklären (»Sketch of the Analytical Engine invented by Charles Babbage« [wie Anm. 2], Note G, S. 724), sich jedoch besonders gut eigneten, um den Unterschied zwischen Babbages früherer sogenannter *Difference Engine* [Differenzmaschine] als reiner *Rechen*-Maschine und seiner fortschrittlicheren *Analytical Engine* als einem Computer für universelle Anwendungen zu veranschaulichen.

6 | Ebd., S. 722.

7 | Mit der Zeit wird auch dies anfechtbar. Entgegen den Einschränkungen, die Lovelace etwa hundert Jahre zuvor formulierte, verfasst der Mathematiker und Computerwissenschafter Alan Turing 1950 seinen einflussreichen Aufsatz über »Computing Machinery and Intelligence«, um die wahrgenommenen Grenzen von Maschinen, selbst zu denken (»Künstliche Intelligenz«), in Zweifel zu ziehen. Der legendäre Anfangssatz lautet: »Ich schlage vor, die Frage zu betrachten: ›Können Maschinen denken?‹«

8 | »Sketch of the Analytical Engine invented by Charles Babbage« [wie Anm. 2], Note A, S. 691, 694.

9 | Ebd., S. 696.

10 | Ebd., S. 696, 694.

11 | Donald E. Knuth, *The Art of Computer Programming*, Reading, Mass.: Addison-Wesley Publishing Company 1981 [Orig. 1968]. Interessant ist in diesem Zusammenhang auch, dass in Anerkennung ihres Beitrags zum Programmieren von Computern eine objektorientierte Programmiersprache nach Lovelace benannt wurde.

12 | Geoffrey Batchen, »Electricity Made Visible«, in: *New Media Art: Practice and Context in the UK 1994–2001,* hrsg. v. Lucy Kimbell, Arts Council England und Cornerhouse Publications 2004, S. 27–44. Es ist interessant zu beobachten, dass Hüseyin Alptekin 1999 eine Arbeit aus Gobelinstickerei über einer bestickten Leinwand herstellt und diese mit »Lovelace« betitelt, wodurch viele dieser Fäden zusammenlaufen. Er erklärt: »Lace, das denotativ das Ende einer Liebe bezeichnet, die mit Liebe erinnert wird, kann sowohl bedeuten, nach etwas verrückt zu sein, als auch auf Stickerei verweisen. Überdies zitiert *Lovelace* verschiedene Quellen und deutet auf verschiedene Kontexte hin. Es macht nicht nur Anleihen bei dem berühmten Pornostar Linda Lovelace, sondern auch bei der berühmten Mathematikerin Ada Byron Lovelace, nach der ein Infobiogen-Server benannt wurde. Tüll, Verhängung, Verschleierung, Enthüllung, Stickerei … Bei Fantasien geht es um den Aufschub des Genießens, weil die Wahrheit in den Tanz der sieben Schleier eingewoben ist.« (zit. nach: www.clubmedia.de/clubmedia99/hba_html/hba_set_12.html, abgerufen im Oktober 2011).

13 | Zitat aus einem Brief von Lovelace an ihre Mutter, Lady Byron, datiert Montag, 7. April 1828, in: Betty A. Toole, *Ada, the Enchantress of Numbers*, Mill Valley, Cal.: Strawberry Press 1992, S. 34.

14 | Die wichtigsten waren Charles Wheatstones Telegraf, Joseph Jacquards Webstuhl und Lochkarten, Sir David Brewsters Kaleidoskop, Michael Faradays frühe Untersuchungen zur Elektrizität sowie Mary Somervilles wissenschaftliche Veröffentlichung *On the Connexion of the Physical Science*, 1934.

15 | Interview mit Heinz von Foerster in dem Dokumentarfilm *Das Netz* (2003) von Lutz Dammbeck.

16 | Die Autorin dankt allen, die sie während ihrer Recherchen zur Vorbereitung dieses Notizbuchs auf unterschiedliche Weise unterstützt haben: Colin Harris, Mary Clapinson and Alan Brown, Bodleian Library, University of Oxford; Adrian Shindler and Justin Clegg, The British Library, London; Matthew Connell und Paul Wilson, Powerhouse Museum, Sydney; Nicola Thwaite, The National Trust, London; Jo Francis und John Fuegi; und Lucia Pietroiusti.

SKETCH

OF THE

ANALYTICAL ENGINE

INVENTED BY

CHARLES BABBAGE, Esq.

By L. F. MENABREA,

of Turin,

OFFICER OF THE MILITARY ENGINEERS.

WITH NOTES BY THE TRANSLATOR.

[Extracted from the 'Scientific Memoirs,' vol. iii.]

LONDON:

PRINTED BY RICHARD AND JOHN E. TAYLOR,

RED LION COURT, FLEET STREET.

1843.

highly important for some of the future wants of science in its manifold, complicated and rapidly-developing fields of inquiry, to arrive at.

Without, however, stepping into the region of conjecture, we will mention a particular problem which occurs to us at this moment as being an apt illustration of the use to which such an engine may be turned for determining that which human brains find it difficult or impossible to work out unerringly. In the solution of the famous problem of the Three Bodies, there are, out of about 295 coefficients of lunar perturbations given by M. Clausen (Astro^e. Nachrichten, No. 406) as the result of the calculations by Burg, of two by Damoiseau, and of one by Burckhardt, fourteen coefficients that differ in the nature of their algebraic sign; and out of the remainder there are only 101 (or about one-third) that agree precisely both in signs and in amount. These discordances, which are generally small in individual magnitude, may arise either from an erroneous determination of the abstract coefficients in the development of the problem, or from discrepancies in the data deduced from observation, or from both causes combined. The former is the most ordinary source of error in astronomical computations, and this the engine would entirely obviate.

We might even invent laws for series or formulæ in an arbitrary manner, and set the engine to work upon them, and thus deduce numerical results which we might not otherwise have thought of obtaining. But this would hardly perhaps in any instance be productive of any great practical utility, or calculated to rank higher than as a kind of philosophical amusement. A. A. L.

Note G.—Page 689.

It is desirable to guard against the possibility of exaggerated ideas that might arise as to the powers of the Analytical Engine. In considering any new subject, there is frequently a tendency, first, to *overrate* what we find to be already interesting or remarkable; and, secondly, by a sort of natural reaction, to *undervalue* the true state of the case, when we do discover that our notions have surpassed those that were really tenable.

The Analytical Engine has no pretensions whatever to *originate* any thing. It can do whatever we *know how to order it* to perform. It can *follow* analysis; but it has no power of *anticipating* any analytical relations or truths. Its province is to assist us in making *available* what we are already acquainted with. This it is calculated to effect primarily and chiefly of course, through its executive faculties; but it is likely to exert an *indirect* and reciprocal influence on science itself in another manner. For, in so distributing and combining the truths and the formulæ of analysis, that they may become most easily and rapidly amenable to the mechanical combinations of the engine, the relations and the nature of many subjects in that science are necessarily thrown into new lights, and more profoundly investigated. This is a decidedly indirect, and a somewhat *speculative*, consequence of such an invention. It is however pretty evident, on general principles, that in devising for mathematical truths a new form in which to record and throw themselves out for actual use, views are likely to be induced, which should again react on the more theoretical phase of the subject. There are in all extensions of human power, or additions to human

knowledge, various *collateral* influences, besides the main and primary object attained.

To return to the executive faculties of this engine: the question must arise in every mind, are they *really* even able to *follow* analysis in its whole extent? No reply, entirely satisfactory to all minds, can be given to this query, excepting the actual existence of the engine, and actual experience of its practical results. We will however sum up for each reader's consideration the chief elements with which the engine works:—

1. It performs the four operations of simple arithmetic upon any numbers whatever.

2. By means of certain artifices and arrangements (upon which we cannot enter within the restricted space which such a publication as the present may admit of), there is no limit either to the *magnitude* of the *numbers* used, or to the *number* of *quantities* (either variables or constants) that may be employed.

3. It can combine these numbers and these quantities either algebraically or arithmetically, in relations unlimited as to variety, extent, or complexity.

4. It uses algebraic *signs* according to their proper laws, and developes the logical consequences of these laws.

5. It can arbitrarily substitute any formula for any other; effacing the first from the columns on which it is represented, and making the second appear in its stead.

6. It can provide for singular values. Its power of doing this is referred to in M. Menabrea's memoir, page 685, where he mentions the passage of values through zero and infinity. The practicability of causing it arbitrarily to change its processes at any moment, on the occurrence of any specified contingency (of which its substitution of $(\frac{1}{2}\cos.\overline{n+1}\,\theta + \frac{1}{2}\cos.\overline{n-1}\,\theta)$ for $(\cos.n\theta.\cos.\theta)$ explained in Note E., is in some degree an illustration), at once secures this point.

The subject of integration and of differentiation demands some notice. The engine can effect these processes in either of two ways:—

First. We may order it, by means of the Operation and of the Variable-cards, to go through the various steps by which the required *limit* can be worked out for whatever function is under consideration.

Secondly. It may (if we know the form of the limit for the function in question) effect the integration or differentiation by direct* substitu-

* The engine cannot of course compute limits for perfectly *simple* and *uncompounded* functions, except in this manner. It is obvious that it has no power of representing or of manipulating with any but *finite* increments or decrements; and consequently that wherever the computation of limits (or of any other functions) depends upon the *direct* introduction of quantities which either increase or decrease *indefinitely*, we are absolutely beyond the sphere of its powers. Its nature and arrangements are remarkably adapted for taking into account all *finite* increments or decrements (however small or large), and for developing the true and logical modifications of form or value dependent upon differences of this nature. The engine may indeed be considered as including the whole Calculus of Finite Differences; many of whose theorems would be especially and beautifully fitted for development by its processes, and would offer peculiarly interesting considerations. We may mention, as an example, the calculation of the Numbers of Bernoulli by means of the *Differences of Nothing*.

tion. We remarked in Note B., that any *set* of columns on which numbers are inscribed, represents merely a *general* function of the several quantities, until the special function have been impressed by means of the Operation and Variable-cards. Consequently, if instead of requiring the value of the function, we require that of its integral, or of its differential coefficient, we have merely to order whatever particular combination of the ingredient quantities may constitute that integral or that coefficient. In $a\,x^n$, for instance, instead of the quantities

V_0	V_1	V_2	V_3
a	n	x	ax^n

$a\,x^n$

being ordered to appear on V_3 in the combination $a\,x^n$, they would be ordered to appear in that of

$$a\,n\,x^{n-1}.$$

They would then stand thus:—

V_0	V_1	V_2	V_3
a	n	x	$a\,n\,x^{n-1}$

$a\,n\,x^{n-1}$

Similarly, we might have $\frac{a}{n}\,x^{(n+1)}$, the integral of $a\,x_n$.

An interesting example for following out the processes of the engine would be such a form as

$$\int \frac{x^n\,d\,x}{\sqrt{a^2 - x^2}},$$

or any other cases of integration by successive reductions, where an integral which contains an operation repeated n times can be made to depend upon another which contains the same $n-1$ or $n-2$ times, and so on until by continued reduction we arrive at a certain *ultimate* form, whose value has then to be determined.

The methods in Arbogast's *Calcul des Dérivations* are peculiarly fitted for the notation and the processes of the engine. Likewise the whole of the Combinatorial Analysis, which consists first in a purely numerical calculation of indices, and secondly in the distribution and combination of the quantities according to laws prescribed by these indices.

We will terminate these Notes by following up in detail the steps through which the engine could compute the Numbers of Bernoulli, this being (in the form in which we shall deduce it) a rather complicated example of its powers. The simplest manner of computing these numbers would be from the direct expansion of

$$\frac{x}{\epsilon^x - 1} = \frac{1}{1 + \frac{x}{2} + \frac{x^2}{2.3} + \frac{x^3}{2.3.4} + \&c.} \qquad (1.)$$

which is in fact a particular case of the development of

$$\frac{a + b\,x + c\,x^2 + \&c.}{a' + b'x + c'x^2 + \&c.}$$

mentioned in Note E. Or again, we might compute them from the well-known form

$$B_{2n-1} = 2 \,.\, \frac{1.2.3....2n}{(2\,\pi)^{2n}} \,.\, \left\{ 1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + ... \right\} \qquad (2.)$$

or from the form

$$B_{2n-1} = \frac{\pm 2\,n}{(2^{2n}-1)\,2^{n-1}} \left\{ \begin{array}{l} \frac{1}{2} n^{2n-1} \\ -(n-1)^{2n-1}\left\{1+\frac{1}{2}.\frac{2n}{1}\right\} \\ +(n-2)^{2n-1}\left\{1+\frac{2n}{1}+\frac{1}{2}.\frac{2n.(2n-1)}{1.2}\right\} \\ -(n-3)^{2n-1}\left\{\begin{array}{l}1+\frac{2n}{1}+\frac{2n.2n-1}{1.2}+ \\ +\frac{1}{2}.\frac{2n.(2n-1).(2n-2)}{1.2.3}\end{array}\right\} \\ + \ldots \quad \ldots \quad \ldots \quad \ldots \end{array} \right\} \qquad (3.)$$

or from many others. As however our object is not simplicity or facility of computation, but the illustration of the powers of the engine, we prefer selecting the formula below, marked (8.). This is derived in the following manner:—

If in the equation

$$\frac{x}{\epsilon^x - 1} = 1 - \frac{x}{2} + B_1\frac{x^2}{2} + B_3\frac{x^4}{2.3.4} + B_5\frac{x^6}{2.3.4.5.6} + \ldots. \qquad (4.)$$

(in which B_1, B_3 ...., &c. are the Numbers of Bernoulli), we expand the denominator of the first side in powers of x, and then divide both numerator and denominator by x, we shall derive

$$1 = \left(1 - \frac{x}{2} + B_1\frac{x^2}{2} + B_3\frac{x^4}{2.3.4} + \ldots\right)\left(1 + \frac{x}{2} + \frac{x^2}{2.3} + \frac{x^3}{2.3.4} \ldots\right) \qquad (5.)$$

If this latter multiplication be actually performed, we shall have a series of the general form

$$1 + D_1\,x + D_2\,x^2 + D_3\,x^3 + \ldots\ldots\ldots\ldots\ldots\ldots \qquad (6.)$$

in which we see, first, that all the coefficients of the powers of x are severally equal to zero; and secondly, that the general form for D_{2n} the co-efficient of the $2n+1$th *term*, (that is of x^{2n} any *even* power of x), is the following:—

$$\left.\begin{array}{l} \frac{1}{2.3...2n+1} - \frac{1}{2}.\frac{1}{2.3..2n} + \frac{B_1}{2}.\frac{1}{2.3..2n-1} + \frac{B_3}{2.3.4}.\frac{1}{2.3..2n-3} + \\ \qquad + \frac{B_5}{2.3.4.5.6}.\frac{1}{2.3...2n-5} + \ldots + \frac{B_{2n-1}}{2.3...2n}.1 = 0 \end{array}\right\} \qquad (7.)$$

Multiplying every term by $(2.3\ldots 2n)$, we have

$$\left.\begin{array}{l} 0=-\frac{1}{2}\cdot\frac{2n-1}{2n+1}+B_1\left(\frac{2n}{2}\right)+B_3\left(\frac{2n.2n-1.2n-2}{2.3.4}\right)+ \\ \quad +B_5\left(\frac{2n.2n-1\ldots\ldots\ldots 2n-4}{2.3.4.5.6}\right)+\ldots\ldots+B_{2n-1} \end{array}\right\} \quad (8.)$$

which it may be convenient to write under the general form:—

$$0=A_0+A_1B_1+A_3B_3+A_5B_5+\ldots+B_{2n-1}\ldots\ldots\ldots(9.)$$

A_1, A_3, &c. being those functions of n which respectively belong to B_1, B_3, &c.

We might have derived a form nearly similar to (8.), from D_{2n-1} the coefficient of any *odd* power of x in (6.); but the general form is a little different for the coefficients of the *odd* powers, and not quite so convenient.

On examining (7.) and (8.), we perceive that, when these formulæ are isolated from (6.) whence they are derived, and considered in themselves separately and independently, n may be any whole number whatever; although when (7.) occurs *as one of the* D's in (6.), it is obvious that n is then not arbitrary, but is always a certain function of the *distance of that* D *from the beginning*. If that distance be $= d$, then

$$2n+1=d, \text{ and } n=\frac{d-1}{2} \text{ (for any } even \text{ power of } x.)$$

$$2n=d, \text{ and } n=\frac{d}{2} \text{ (for any } odd \text{ power of } x.)$$

It is with the *independent* formula (8.) that we have to do. Therefore it must be remembered that the conditions for the value of n are now modified, and that n is a perfectly *arbitrary* whole number. This circumstance, combined with the fact (which we may easily perceive) that whatever n is, every term of (8.) after the $(n+1)$th is $= 0$, and that the $(n+1)$th term itself is always $= B_{2n-1}\cdot\frac{1}{1} = B_{2n-1}$, enables us to find the value (either numerical or algebraical) of any nth Number of Bernoulli B_{2n-1}, *in terms of all the preceding ones*, if we but know the values of $B_1, B_3 \ldots . B_{2n-3}$. We append to this Note a Diagram and Table, containing the details of the computation for B_7, (B_1, B_3, B_5 being supposed given).

On attentively considering (8.), we shall likewise perceive that we may derive from it the numerical value of *every* Number of Bernoulli in succession, from the very beginning, *ad infinitum*, by the following series of computations:—

1st Series.—Let $n = 1$, and calculate (8.) for this value of n. The result is B_1.

2nd Series.—Let $n = 2$. Calculate (8.) for this value of n, substituting the value of B_1 just obtained. The result is B_3.

3rd Series.—Let $n = 3$. Calculate (8.) for this value of n, substituting the values of B_1, B_3 before obtained. The result is B_5. And so on, to any extent.

The diagram* represents the columns of the engine when just prepared

* See the diagram at the end of these Notes.

for computing B_{2n-1}, (in the case of $n = 4$); while the table beneath them presents a complete simultaneous view of all the successive changes which these columns then severally pass through in order to perform the computation. (The reader is referred to Note D, for explanations respecting the nature and notation of such tables.)

Six numerical *data* are in this case necessary for making the requisite combinations. These data are 1, 2, n $(=4)$, B_1, B_3, B_5. Were $n = 5$, the additional datum B_7 would be needed. Were $n = 6$, the datum B_9 would be needed; and so on. Thus the actual *number of data* needed will always be $n + 2$, for $n = n$; and out of these $n + 2$ data, $(\overline{n+2-3})$ of them are successive Numbers of Bernoulli. The reason why the Bernoulli Numbers used as data, are nevertheless placed on *Result*-columns in the diagram, is because they may properly be supposed to have been previously computed in succession by the *engine* itself; under which circumstances each B will appear as a *result*, previous to being used as a *datum* for computing the succeeding B. Here then is an instance (of the kind alluded to in Note D.) of the same Variables filling more than one office in turn. It is true that if we consider our computation of B_7 as a perfectly *isolated* calculation, we may conclude B_1, B_3, B_5 to have been arbitrarily placed on the columns; and it would then perhaps be more consistent to put them on V_{21}, V_{22}, V_{23} as data and not results. But we are not taking this view. On the contrary, we suppose the engine to be *in the course of* computing the Numbers to an indefinite extent, from the very beginning; and that we merely single out, by way of example, *one amongst* the successive but distinct series' of computations it is thus performing. Where the B's are fractional, it must be understood that they are computed and appear in the notation of *decimal* fractions. Indeed this is a circumstance that should be noticed with reference to all calculations. In any of the examples already given in the translation and in the Notes, some of the *data*, or of the temporary or permanent results, might be fractional, quite as probably as whole numbers. But the arrangements are so made, that the nature of the processes would be the same as for whole numbers.

In the above table and diagram we are not considering the *signs* of any of the B's, merely their numerical magnitude. The engine would bring out the sign for each of them correctly of course, but we cannot enter on *every* additional detail of this kind, as we might wish to do. The circles for the signs are therefore intentionally left blank in the diagram.

Operation-cards 1, 2, 3, 4, 5, 6 prepare $-\frac{1}{2}\cdot\frac{2n-1}{2n+1}$. Thus, Card 1 multiplies *two* into n, and the three *Receiving* Variable-cards belonging respectively to V_4, V_5, V_6, allow the result $2n$ to be placed on each of these latter columns (this being a case in which a triple receipt of the result is needed for subsequent purposes); we see that the upper indices of the two Variables used, during Operation 1, remain unaltered.

We shall not go through the details of every operation singly, since the table and diagram sufficiently indicate them; we shall merely notice some few peculiar cases.

By Operation 6, a *positive* quantity is turned into a *negative* quantity, by simply subtracting the quantity from a column which has only zero upon it. (The sign at the top of V_4 would become − during this process.)

Operation 7 will be unintelligible, unless it be remembered that if we were calculating for $n = 1$ instead of $n = 4$, Operation 6 would have completed the computation of B_1 itself; in which case the engine, instead of continuing its processes, would have to put B_1 on V_{21}; and then either to stop altogether, or to begin Operations 1, 2 . . . . 7 all over again for value of n (= 2), in order to enter on the computation of B_3; (having however taken care, previous to this recommencement, to make the number on V_3 equal to *two*, by the addition of unity to the former $n = 1$ on that column). Now Operation 7 must either bring out a result equal to zero (if $n = 1$); or a result *greater* than *zero*, as in the present case; and the engine follows the one or the other of the two courses just explained, contingently on the one or the other result of Operation 7. In order fully to perceive the necessity of this *experimental* operation, it is important to keep in mind what was pointed out, that we are not treating a perfectly isolated and independent computation, but one out of a series of antecedent and prospective computations.

Cards 8, 9, 10 produce $-\frac{1}{2}\cdot\frac{2n-1}{2n+1} + B_1\frac{2n}{2}$. In Operation 9 we see an example of an upper index which again becomes a value after having passed from preceding values to zero. V_{11} has successively been ${}^0V_{11}$, ${}^1V_{11}$, ${}^2V_{11}$, ${}^0V_{11}$, ${}^3V_{11}$; and, from the nature of the office which V_{11} performs in the calculation, its index will continue to go through further changes of the same description, which, if examined, will be found to be regular and periodic.

Card 12 has to perform the same office as Card 7 did in the preceding section; since, if n had been = 2, the 11th operation would have completed the computation of B_3.

Cards 13 to 20 make A_3. Since A_{2n-1} always consists of $2n - 1$ factors, A_3 has three factors; and it will be seen that Cards 13, 14, 15, 16 make the second of these factors, and then multiply it with the first; and that 17, 18, 19, 20 make the third factor, and then multiply this with the product of the two former factors.

Card 23 has the office of Cards 11 and 7 to perform, since if n were = 3, the 21st and 22nd operations would complete the computation of B_5. As our case is B_7, the computation will continue one more stage; and we must now direct attention to the fact, that in order to compute A_5 it is merely necessary precisely to repeat the group of Operations 13 to 20; and then, in order to complete the computation of B_7, to repeat Operations 21, 22.

It will be perceived that every unit added to n in B_{2n-1}, entails an additional repetition of operations (13...23) for the computation of B_{2n-1}. Not only are all the *operations* precisely the same however for every such repetition, but they require to be respectively supplied with numbers from the very *same pairs of columns*; with only the one exception of Operation 21, which will of course need B_5 (from V_{23}) instead of B_3 (from V_{22}). This identity in the *columns* which supply the requisite numbers, must not be confounded with identity in the *values* those columns have upon them and give out to the mill. Most of those

values undergo alterations during a performance of the operations (13 . . . . 23), and consequently the columns present a new set of values for the *next* performance of (13 . . . . 23) to work on.

At the termination of the *repetition* of operations (13...23) in computing B_7, the alterations in the values on the Variables are, that

$V_6 = 2n - 4$ instead of $2n - 2$.
$V_7 = 6$ . . . . 4.
$V_{10} = 0$ . . . . 1.
$V_{13} = A_0 + A_1B_1 + A_3B_3 + A_5B_5$ instead of $A_0 + A_1B_1 + A_3B_3$

In this state the only remaining processes are first: to transfer the value which is on V_{13}, to V_{24}; and secondly to reduce V_6, V_7, V_{13} to zero, and to add* *one* to V_3, in order that the engine may be ready to commence computing B_9. Operations 24 and 25 accomplish these purposes. It may be thought anomalous that Operation 25 is represented as leaving the upper index of V_3 still = unity. But it must be remembered that these indices always begin anew for a separate calculation, and that Operation 25 places upon V_3 the *first* value *for the new calculation*.

It should be remarked, that when the group (13...23) is *repeated*, changes occur in some of the *upper* indices during the course of the repetition: for example, ${}^3V_6$ would become ${}^4V_6$ and ${}^5V_6$.

We thus see that when $n = 1$, nine Operation-cards are used; that when $n = 2$, fourteen Operation-cards are used; and that when $n > 2$, twenty-five Operation-cards are used; but that no *more* are needed, however great n may be; and not only this, but that these same twenty-five cards suffice for the successive computation of all the Numbers from B_1 to B_{2n-1} inclusive. With respect to the number of *Variable*-cards, it will be remembered, from the explanations in previous Notes, that an average of three such cards to each *operation* (not however to each Operation-*card*) is the estimate. According to this the computation of B_1 will require twenty-seven Variable-cards; B_3 forty-two such cards; B_5 seventy-five; and for every succeeding B after B_5, there would be thirty-three additional Variable-cards (since each repetition of the group (13...23) adds eleven to the number of operations required for computing the previous B). But we must now explain, that whenever there is a *cycle of operations*, and if these merely require to be supplied with numbers from the *same pairs of columns* and likewise each operation to place its *result* on the *same* column for every repetition of the whole group, the process then admits of a *cycle of Variable-cards* for effecting its purposes. There is obviously much more symmetry and simplicity in the arrangements, when cases do admit of repeating the Variable as well as the Operation-cards. Our present example is of this nature. The only exception to a *perfect identity* in *all* the processes and columns used, for every repetition of Operations (13...23) is, that Operation 21 always requires one of its factors from a new column, and Operation 24 always puts its result

* It is interesting to observe, that so complicated a case as this calculation of the Bernoullian Numbers, nevertheless, presents a remarkable simplicity in one respect; viz., that during the processes for the computation of *millions* of these Numbers, no other arbitrary modification would be requisite in the arrangements, excepting the above simple and uniform provision for causing one of the data periodically to receive the finite increment unity.

on a new column. But as these variations follow the same law at each repetition, (Operation 21 always requiring its factor from a column *one* in advance of that which it used the previous time, and Operation 24 always putting its result on the column *one* in advance of that which received the previous result), they are easily provided for in arranging the recurring group (or cycle) of Variable-cards.

We may here remark that the average estimate of three Variable-cards coming into use to each operation, is not to be taken as an absolutely and literally correct amount for all cases and circumstances. Many special circumstances, either in the nature of a problem, or in the arrangements of the engine under certain contingencies, influence and modify this average to a greater or less extent. But it is a very safe and correct *general* rule to go upon. In the preceding case it will give us seventy-five Variable-cards as the total number which will be necessary for computing any B after B_3. This is very nearly the precise amount really used, but we cannot here enter into the minutiæ of the few particular circumstances which occur in this example (as indeed at some one stage or other of probably most computations) to modify slightly this number.

It will be obvious that the very *same* seventy-five Variable-cards may be repeated for the computation of every succeeding Number, just on the same principle as admits of the repetition of the thirty-three Variable-cards of Operations (13...23) in the computation of any *one* Number. Thus there will be a *cycle of a cycle* of Variable-cards.

If we now apply the notation for cycles, as explained in Note E, we may express the operations for computing the Numbers of Bernoulli in the following manner:—

(1...7), (24, 25) .... gives B_1 = 1st number; (n being = 1).
(1...7), (8...12), (24, 25) .... B_3 = 2nd .....; (n .... = 2).
(1...7), (8...12), (13...23), (24, 25) .... B_5 = 3rd .....; (n .... = 3).
(1...7), (8...12), 2 (13...23), (24, 25) .... B_7 = 4th .....; (n .... = 4).
....
....
(1...7), (8...12), $\Sigma(+1)^{n-2}$ (13...23), (24, 25)... B_{2n-1} = nth .....; (n .... = n).

Again,

$$(1...7),(24,25),\underset{\text{limits 1 to } n}{\Sigma(+1)^n}\left\{(1...7),(8...12),\underset{\text{limits 0 to }(n+2)}{\Sigma(n+2)}(13...23),(24,25)\right\}$$

represents the total operations for computing every number in succession, from B_1 to B_{2n-1} inclusive.

In this formula we see a *varying cycle* of the *first* order, and an ordinary cycle of the *second* order. The latter cycle in this case includes in it the varying cycle.

On inspecting the ten Working-Variables of the diagram, it will be perceived, that although the *value* on any one of them (excepting V_4 and V_5) goes through a series of changes, the *office* which each performs is in this calculation *fixed* and *invariable*. Thus V_6 always prepares the *numerators* of the factors of any A; V_7 the *denominators*. V_8 always receives the $(2n-3)$th factor of A_{2n-1}, and V_9 the $(2n-1)$th. V_{10} always decides which of two courses the succeeding processes are to follow, by feeling for the value of n through

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means of a subtraction; and so on; but we shall not enumerate further. It is desirable in all calculations, so to arrange the processes, that the *offices* performed by the Variables may be as uniform and fixed as possible.

Supposing that it was desired not only to tabulate B_1, B_3, &c., but A_0, A_1, &c.; we have only then to appoint another series of Variables, V_{41}, V_{42}, &c., for receiving these latter results as they are successively produced upon V_{11}. Or again, we may, instead of this, or in addition to this second series of results, wish to tabulate the value of each successive *total* term of the series (8), viz: A_0, $A_1 B_1$, $A_3 B_3$, &c. We have then merely to multiply each B with each corresponding A, as produced; and to place these successive products on Result-columns appointed for the purpose.

The formula (8.) is interesting in another point of view. It is one particular case of the general Integral of the following Equation of Mixed Differences:—

$$\frac{d^2}{dx^2}\left(z_{n+1}\,x^{2n+2}\right) = (2n+1)(2n+2)\,z^n\,x^{2n}$$

for certain special suppositions respecting z, x and n.

The *general* integral itself is of the form,

$$z_n = f(n).x + f_1(n) + f_2(n).x^{-1} + f_3(n).x^{-3} + \ldots$$

and it is worthy of remark, that the engine might (in a manner more or less similar to the preceding) calculate the value of this formula upon most *other* hypotheses for the functions in the integral, with as much, or (in many cases) with more, ease than it can formula (8.).

A. L. L.

Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 *et seq.*)

Number of Operation.	Nature of Operation.	Variables acted upon.	Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.
1	×	${}^1V_2 \times {}^1V_3$	${}^1V_4, {}^1V_5, {}^1V_6$	$\left\{ {}^1V_2 = {}^1V_2 \atop {}^1V_3 = {}^1V_3 \right\}$	$= 2n$
2	−	${}^1V_4 - {}^1V_1$	${}^2V_4$	$\left\{ {}^1V_4 = {}^2V_4 \atop {}^1V_1 = {}^1V_1 \right\}$	$= 2n - 1$
3	+	${}^1V_5 + {}^1V_1$	${}^2V_5$	$\left\{ {}^1V_5 = {}^2V_5 \atop {}^1V_1 = {}^1V_1 \right\}$	$= 2n + 1$
4	÷	${}^2V_5 \div {}^2V_4$	${}^1V_{11}$	$\left\{ {}^2V_5 = {}^0V_5 \atop {}^2V_4 = {}^0V_4 \right\}$	$= \frac{2n-1}{2n+1}$
5	÷	${}^1V_{11} \div {}^1V_2$	${}^2V_{11}$	$\left\{ {}^1V_{11} = {}^2V_{11} \atop {}^1V_2 = {}^1V_2 \right\}$	$= \frac{1}{2} \cdot \frac{2n-1}{2n+1}$
6	−	${}^0V_{13} - {}^2V_{11}$	${}^1V_{13}$	$\left\{ {}^2V_{11} = {}^0V_{11} \atop {}^0V_{13} = {}^1V_{13} \right\}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$
7	−	${}^1V_3 - {}^1V_1$	${}^1V_{10}$	$\left\{ {}^1V_3 = {}^1V_3 \atop {}^1V_1 = {}^1V_1 \right\}$	$= n - 1 \ (= 3)$
8	+	${}^1V_2 + {}^0V_7$	${}^1V_7$	$\left\{ {}^1V_2 = {}^1V_2 \atop {}^0V_7 = {}^1V_7 \right\}$	$= 2 + 0 = 2$
9	÷	${}^1V_6 \div {}^1V_7$	${}^3V_{11}$	$\left\{ {}^1V_6 = {}^1V_6 \atop {}^0V_{11} = {}^3V_{11} \right\}$	$= \frac{2n}{2} = A_1$
10	×	${}^1V_{21} \times {}^3V_{11}$	${}^1V_{12}$	$\left\{ {}^1V_{21} = {}^1V_{21} \atop {}^3V_{11} = {}^3V_{11} \right\}$	$= B_1 \cdot \frac{2n}{2} = B_1 A_1$
11	+	${}^1V_{12} + {}^1V_{13}$	${}^2V_{13}$	$\left\{ {}^1V_{12} = {}^0V_{12} \atop {}^1V_{13} = {}^2V_{13} \right\}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$
12	−	${}^1V_{10} - {}^1V_1$	${}^2V_{10}$	$\left\{ {}^1V_{10} = {}^2V_{10} \atop {}^1V_1 = {}^1V_1 \right\}$	$= n - 2 \ (= 2)$
13	−	${}^1V_6 - {}^1V_1$	${}^2V_6$	$\left\{ {}^1V_6 = {}^2V_6 \atop {}^1V_1 = {}^1V_1 \right\}$	$= 2n - 1$
14	+	${}^1V_1 + {}^1V_7$	${}^2V_7$	$\left\{ {}^1V_1 = {}^1V_1 \atop {}^1V_7 = {}^2V_7 \right\}$	$= 2 + 1 = 3$
15	÷	${}^2V_6 \div {}^2V_7$	${}^1V_8$	$\left\{ {}^2V_6 = {}^2V_6 \atop {}^2V_7 = {}^2V_7 \right\}$	$= \frac{2n-1}{3}$
16	×	${}^1V_8 \times {}^3V_{11}$	${}^4V_{11}$	$\left\{ {}^1V_8 = {}^0V_8 \atop {}^3V_{11} = {}^4V_{11} \right\}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3}$
17	−	${}^2V_6 - {}^1V_1$	${}^3V_6$	$\left\{ {}^2V_6 = {}^3V_6 \atop {}^1V_1 = {}^1V_1 \right\}$	$= 2n - 2$
18	+	${}^1V_1 + {}^2V_7$	${}^3V_7$	$\left\{ {}^2V_7 = {}^3V_7 \atop {}^1V_1 = {}^1V_1 \right\}$	$= 3 + 1 = 4$
19	÷	${}^3V_6 \div {}^3V_7$	${}^1V_9$	$\left\{ {}^3V_6 = {}^3V_6 \atop {}^3V_7 = {}^3V_7 \right\}$	$= \frac{2n-2}{4}$
20	×	${}^1V_9 \times {}^4V_{11}$	${}^5V_{11}$	$\left\{ {}^1V_9 = {}^0V_9 \atop {}^4V_{11} = {}^5V_{11} \right\}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = A_3$
21	×	${}^1V_{22} \times {}^5V_{11}$	${}^0V_{12}$	$\left\{ {}^1V_{22} = {}^1V_{22} \atop {}^0V_{12} = {}^2V_{12} \right\}$	$= B_3 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{3} = B_3 A_3$
22	+	${}^2V_{12} + {}^2V_{13}$	${}^3V_{13}$	$\left\{ {}^2V_{12} = {}^0V_{12} \atop {}^2V_{13} = {}^3V_{13} \right\}$	$= A_0 + B_1 A_1 + B_3 A_3$
23	−	${}^2V_{10} - {}^1V_1$	${}^3V_{10}$	$\left\{ {}^2V_{10} = {}^3V_{10} \atop {}^1V_1 = {}^1V_1 \right\}$	$= n - 3 \ (= 1)$
					Here follows a repetition of Operations thirteen to twenty-three.
24	+	${}^4V_{13} + {}^0V_{24}$	${}^1V_{24}$	$\left\{ {}^4V_{13} = {}^0V_{13} \atop {}^0V_{24} = {}^1V_{24} \right\}$	$= B_7$
25	+	${}^1V_1 + {}^1V_3$	${}^1V_3$	$\left\{ {}^1V_1 = {}^1V_1 \atop {}^1V_3 = {}^1V_3 \right\}$	$= n + 1 = 4 + 1 = 5$; by a Variable-card

Number of Operation.	Data. ${}^1V_1$ ○ 0 0 0 1 [1]	Data. ${}^1V_2$ ○ 0 0 0 2 [2]	Data. ${}^1V_3$ ○ 0 0 0 4 [n]	Working Variables. ${}^0V_4$ ○ 0 0 0 0	${}^0V_5$ ○ 0 0 0 0	${}^0V_6$ ○ 0 0 0 0	${}^0V_7$ ○ 0 0 0 0	${}^0V_8$ ○ 0 0 0 0	${}^0V_9$ ○ 0 0 0 0	${}^0V_{10}$ ○ 0 0 0 0	${}^0V_{11}$ ○ 0 0 0 0	${}^0V_{12}$ ○ 0 0 0 0	${}^0V_{13}$ ○ 0 0 0 0	Result Variables. ${}^1V_{21}$ ○ B_1 in a decimal fraction. [B_1]	${}^1V_{22}$ ○ B_3 in a decimal fraction. [B_3]	${}^1V_{23}$ ○ B_5 in a decimal fraction. [B_5]	${}^0V_{24}$... ○ 0 0 0 [B_7]
1	...	2	n	$2n$	$2n$	$2n$											
2	1	...	...	$2n-1$													
3	1	...	...	...	$2n+1$												
4	...	...	...	0	0	...	...	...	...	...	$\frac{2n-1}{2n+1}$						
5	...	2	...	...	...	...	...	...	...	...	$\frac{1}{2} \cdot \frac{2n-1}{2n+1}$						
6	...	...	...	...	...	...	...	...	...	...	0	....	$-\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$				
7	1	...	n	...	...	...	...	...	...	$n-1$							
8	...	2	...	...	...	...	2										
9	...	...	...	...	...	$2n$	2	...	...	...	$\frac{2n}{2} = A_1$						
10	...	...	...	...	...	...	...	...	...	...	$\frac{2n}{2} = A_1$	$B_1 \cdot \frac{2n}{2} = B_1 A_1$	....	B_1			
11	...	...	...	...	...	...	...	...	...	...	....	0	$\left\{ -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2} \right\}$				
12	1	...	...	...	...	...	...	...	...	$n-2$							
13	1	...	...	...	...	$2n-1$											
14	1	...	...	...	...	...	3										
15	...	...	...	...	...	$2n-1$	3	$\frac{2n-1}{3}$									
16	...	...	...	...	...	...	...	0	...	...	$\frac{2n}{2} \cdot \frac{2n-1}{3}$						
17	1	...	...	...	...	$2n-2$											
18	1	...	...	...	...	...	4										
19	...	...	...	...	...	$2n-2$	4	...	$\frac{2n-2}{4}$	...	$\left\{ \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{3} = A_3 \right\}$						
20	...	...	...	...	...	...	...	...	0								
21	...	...	...	...	...	...	...	...	...	...	0	$B_3 A_3$	....	......	B_3		
22	...	...	...	...	...	...	...	...	...	...	....	0	$\left\{ A_3 + B_1 A_1 + B_3 A_3 \right\}$				
23	1	...	...	...	...	...	...	...	...	$n-3$							
	Here follows a repetition of Operations thirteen to twenty-three.																
24	...	...	...	...	...	...	...	...	...	...	....	....	....	......	......	......	B_7
25	1	...	$n+1$	...	...	0	0										

Letter from Mr. C. Babbage to Augusta, Ada, Countess of Lovelace

My dear Lady Lovelace,

If you are as fastidious about acts of your friends as you are about those of your pen, I much fear I shall equally lose your friendship and your notes. I like much the improved form of the Bernouille* Note but can judge of it better when I have the diagram and Notation. [*Bernouelli?]

I am very reluctant to return the admirable and philosophic view of the Abral Engine contained in Note A. Pray do not alter it and do let me have it returned on Monday. I send also the rest of Note D. There is still one trifling misapprehension about the Variable cards ___ A Variable card may order any number of Variables to receive the same number upon theirs at the same instant of time – But a Variable card never can be directed to order more than one Variable to be given off at once because the mill could not receive it the mechanism would not permit it. All this it was impossible for you to know by intuition and the more I read your notes the more surprised I am at them and regret not having earlier explored so rich a vein of the noblest metal.

The account of them stands thus:

A Sent to Lady L.
B With C.B.
C Ditto
D Sent to Lady L.
E With C.B.
F Retained by Lady L.
G Where is it gone?
H With C.B.

I have not seen Mr. Wheatstone and am ashamed to write until I can positively put the whole of the notes into his hands.

I will attend your commands tomorrow ___

And am,
Ever most truly yours,
C. BABBAGE

1 Dorset St.
Manchr Sq.
2nd July 1843

Transcript of a typed copy of the letter, British Library.

Letter from Augusta, Ada, Countess of Lovelace to Mr. C. Babbage

Sunday, 2 July 1843

Ockham, Sunday 6 o'clock, 2 July

I have worked incessantly, & most successfully, all day. You will admire the Table & Diagram extremely.

They have been made out with extreme care, & all the indices most minutely & scrupulously attended to. Lord L___ is at this moment kindly inking it all over for me.

I had to do it in pencil.

You must bring all the Notes with you tomorrow; as I have observations to make on each one; & especially on this final one H.

There never was a Note G. I do not know why I chose H instead of G, & thus insulted the latter worthy letter.

I cannot imagine what you mean about the Variable-cards; since I never either supposed in my own mind, that one Variable-card could give off more than one Variable at a time; nor have (as far as I can make out) expressed such an idea in any passage whatever.

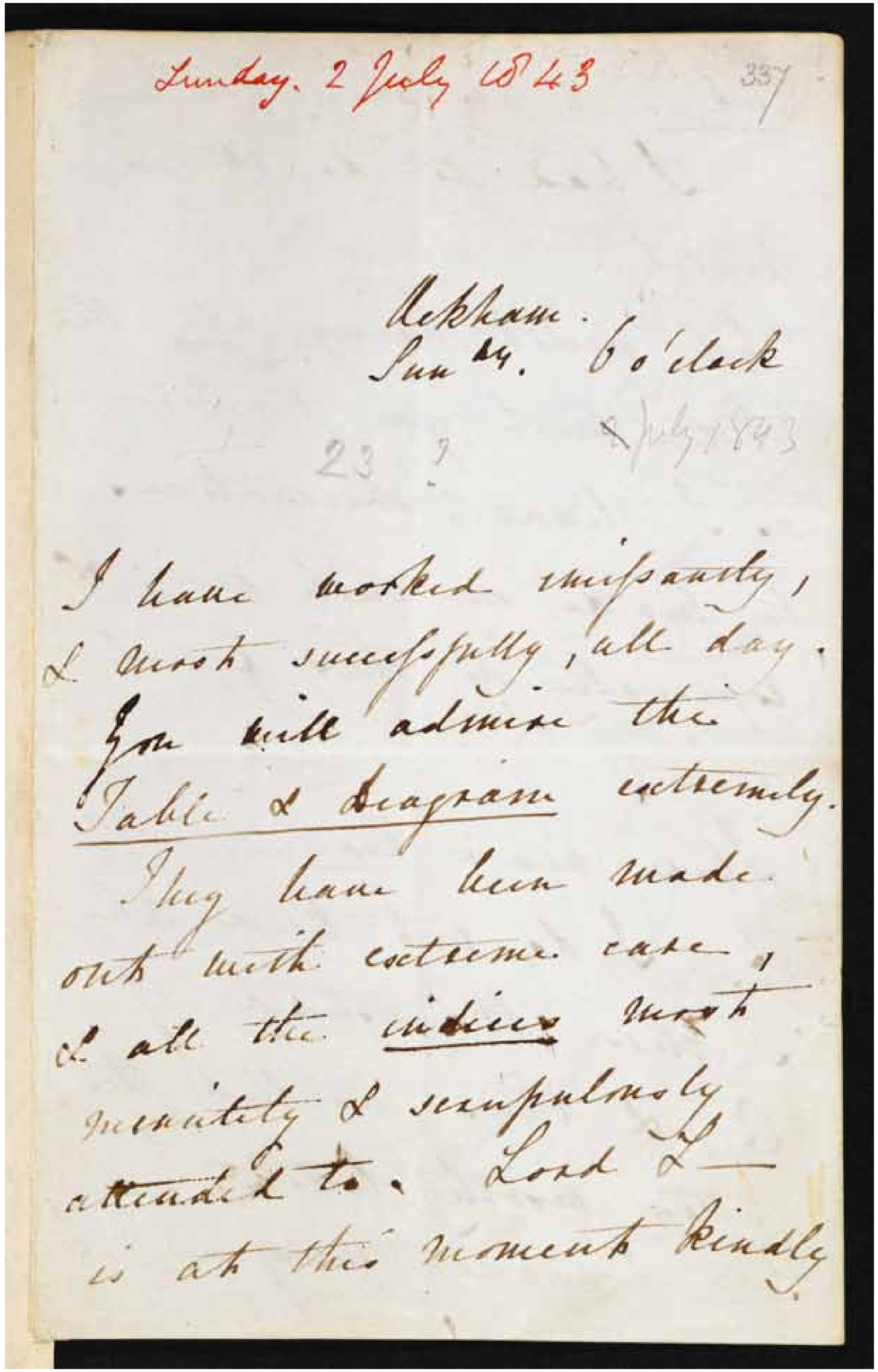
I cannot find what I fancied I had put in Note A; so I return it whole & sound, for your speedy relief.

I send back note D. You will find the only alteration I wished to make pinned over; in the upper part of Sheet 2.

So I now retain nothing but Note F, which I shall give you tomorrow.

Lord L___ has put up, I find, in a separate cover, all that belongs to Note H. (He is quite enchanted with the beauty & symmetry of the Table & Diagram). No – I find I can put in Note D with H.

Transcript from original handwritten letter, British Library



Sunday. 2 July 1843

337

Ockham.
Sunday. 6 o'clock

2 July 1843

23

I have worked incessantly, & most successfully, all day. You will admire the Table & Diagram extremely. They have been made out with extreme care, & all the indices most minutely & scrupulously attended to. Lord L— is at this moment kindly *inking* *it all over* for me.

I had to do it in pencil.

You must bring *all* the Notes with you tomorrow; as I have observations to make on each one; & especially on this final one H.

There never *was* a Note *G*. I do not know why I chose H instead of G, & thus insulted the latter worthy letter.

I cannot imagine what you mean about the Variable-Cards; since I never either supposed in my own mind, that one Variable-card *could give* off more than one Variable at a time; nor have (as far as I can make out) supposed ~~[illegible]~~ such an idea in any passage whatever.

I cannot find what I fancied I had put in Note A; so I return it whole & sound, for your speedy relief.

I send back Note D.
You will find the only alteration
~~[illegible]~~ I wished to make
pinned over; in the
upper part of Sheet 2.

So I now retain
nothing but Note F, which
I shall give you tomorrow.

Lord L— has
put up, I find, in a
separate cover, all that
belongs to Note H. (He is
quite enchanted with the
beauty & symmetry of the Table
& Diagram). (He— I find I
can put in Note D with H.

Letter from Augusta, Ada, Countess of Lovelace to Mr. C. Babbage

Thursday, 4 July 1843, Ockham

My Dear Babbage. I now write to you expressly on three points; which I have very fully & leisurely considered during the last 18 hours; & think of sufficient importance to induce me to send a servant up so that you may have this letter by half after six this evening. The servant will leave Town tomorrow morning early, but will call for anything you may have for me, at eight o'clock in the morning before he goes.

Firstly: The few lines I enclosed you last night about the connection of (8) with the famous Integral, I by no means intend you to insert, unless you fully approve the doing so.

It is perhaps very dubious whether there is any sufficient pertinence in noticing at all that (8) is an Integral. Is not every formula an Integral of something? (. . . we may not always be able to determine the form after something as in this case); and is this consequently, any way palpable an important purpose announced in noticing that (8) is an Integral? Such notice would rather seem to imply that this formulae mean not Integral, & that (8) is peculiar, either in being an Integral at all, or else in being the Integral of a very important & peculiar formula. In short the pros & cons from the insertion, seem to me to depend mainly upon the two following questions:

Is (8) more pre-eminently an Integral rather than formulae?

If not, then is the form of which it is an Integral, in any way a very remarkable form?

Should the first question be answered in the negative, still the second question may justify the assertion, if answered affirmatively. But if both are negative, I think the insertion irrelevant.

Secondly: Lord L___ suggests my signing the translation & the Notes; by which he means, simply putting at the end of the former: 'translated by A.A.L.'; & adding to each note the initials A.A.L.

It is not my wish to proclaim who has written it; at the same time that I rather wish to append anything that may tend hereafter to individualise, & identify it, with other productions of the said A.A.L.

My third topic, tho my last, is our most anxious & important: I have yesterday evening & this morning very amply analysed the question of the number of Variable-Cards, as mentioned in the final Note H (or G?). And I find that you & I between us made a mess of it (for which I can perfectly account, an a very natural manner). I enclose what I wish to insert instead of that which is now there. I think the present wrong passage is only about eight or ten lines, & is I believe on the second of the three great sheets which are to follow the Diagram.

The fact is that if my own exposition about the Variable-Cards on Note D, had been strictly followed by myself, in Note H; this error would not have occurred. The confusion has arisen simply from the circumstance of applying to the Variable-Cards, facts which relate to the Operation-Cards.

In Note D, it is very well & lucidly demonstrated that every single Operation, demands the use of at least three Variable-Cards. It does not signify whether the operations be in cycles or not. A million successive additions +, +, +, &c, &c, &c, would each demand the use of three new Variable-Cards, under ordinary circumstances. In Note H, the erroneous lines are found on the hasty supposition that the cycle, or recurring group, of Operation-Cards (13 . . . 23) will be fed by a cycle, or recurring group, of Variable-Cards.

I enclose what I believe it ought to be.

If already gone to the printer, we must alter that passage in the proofs, unless you could call at the printers & there paste over the amendment. –

I can scarcely describe to you how very ill & harassed I felt yesterday. Pray excuse any abruptness or other unpleasantness of manner, if there were any.

I am breathing well again today, & am much better in all respects; owing to Dr L's remedies. He certainly does seem to understand the case, I mean the treatment of it, which is the main thing.

As for the theory of it, he says truly that time & Providence alone can develop that. It is so anomalous an affair altogether. A Singular Function, in very deed!

Think of my having to walk, (or rather run), to the Station, in half an hour last evening; while I suppose you were feasting & flirting in luxury & ease at your dinner.

It must be a very pleasant merry sort of thing to have a Fairy in one's service, mind & limbs! – I envy you! – I, poor little Fairy, can only get dull heavy mortals, to wait on me! – Ever Yours A. L.

Transcript from handwritten original, British Library

Sonnet, the Rainbow, 1851

Sonnet
The Rainbow.

Bow down in hope, in thanks, all ye who mourn; –
Where'in that peerless arch of radiant hues
Surpassing earthly tints, – the storm subdues!
Of nature's strife and tears 'tis heaven-born,
To soothe the sad, the sinning and the forlorn; –
A lovely loving token; to infuse;
The hope, the faith, that pow'r divine endures
With latent good, the woes by which we're torn.

'Tis like a sweet repentance of the skies;
To beckon all those by sense of sin opprest,
And prove what loveliness may spring from sighs!
A pledge: – thats deep implanted in the breast
A hidden light may burn that never dies,
But bursts thro' clouds in purest hues exprest!

'Tis like a sweet repentance of the skies,
To beckon all by sense of sin opprest, –
Proclaiming loveliness from tears and sighs!
A pledge: – thats deep implanted in the breast
A hidden light may burn that never dies,
But bursts thro' clouds in purest hues exprest!

100 Notes – 100 Thoughts / 100 Notizen – 100 Gedanken

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Introduction / Einführung: Joasia Krysa

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