On Some Vistas Discovered by Mathematics to the Russian Avant-Garde: Geometry, El Lissitzky and Gabo

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Diderot, great character of the Enlightenment, is one of the more remarkable scientific and humanistic symbols of the eighteenth century. His work, performed in collaboration with d'Alembert, Encyclopédie (1751–1765) [1] summarizes, although in hundreds of pages, the triumphs of the intellect and of the age—and of these two visionary men of genius. D'Alembert, the other half of that editorial adventure, in his article entitled “Dimension”, alluded to the fourth dimension, in contrast to traditional learning [2]. And in Diderot’s “Traité du Beau”, contained in Volume II, we find the first example of an analogy of beauty with mathematical discourse (although he does not refer to Diderot's work) is repeated: “beau comme un memoire sur la courbe que decrit un chien en courant apres son maitre” (as beautiful as the memory of the curve described by a dog running after its master) [4].

Note the view, especially popular among mathematicians, that beauty can be found in mathematical discourse. Of course, this idea could not occur to anyone without knowledge of mathematics, as Diderot and, to a lesser degree, Lautréamont had. However, the beauty of the visual image was not their business. Nevertheless, in the eighteenth and nineteenth centuries, the study of bizarre and mysterious curves became important.

It was only in the last half of the nineteenth century that an interest in visualizing contemporaneous mathematical ideas began. I shall make no attempt to describe that epoch called Modernism, whose genesis and strength are still matters under discussion [5]. However, there was at that time a general momentum that carried with it art, poetry, science, fashion, advertising and architecture. We can establish some interplay between these distinct types of activities. Consider the influence of mathematics upon the arts. It is my opinion that mathematics had three roles in the visual arts. First, it was a metaphor for progress. Second, it provided a language of forms and shapes. And third, mathematical concepts could enlighten, modify and penetrate art notions that were then reflected in the visual arts. My discussion of these roles will be best considered in the climate of the Russian avant-garde (Fig. 1), which constitutes one vigorous, optimistic and neat example of Modernism, and which perhaps served as a forerunner of some of the present attitudes in visual mathematics.

The Metaphor of Progress

In the 1920s Russian writer Iouri Tynianov, referring to Futurist poet Velimir Khlebnikov, wrote “the poet Khlebnikov becomes the Lobachevskii of words” [6]. In this sentence Tynianov is using Nicolai Lobachevskii as a metaphor for a founder of a new system or a novel theory—for this is what Lobachevskii had done. From Tynianov’s point of view, the old theory was Euclidean geometry, the deductive theory founded upon Euclid’s five postulates. The first four of Euclid’s postulates are self-evident, and the fifth can be paraphrased as “there exists only one parallel to a given straight line through a given point”. Tynianov’s metaphor refers to Lobachevskii’s idea about this postulate, which Lobachevskii introduced in his lecture delivered at the University of Kazan on 12 February 1826. That lecture replaced Euclid’s fifth postulate without affecting the coherence of the geometrical discourse. Also proposed independently by J. Bolyai, the new postulate allowed the existence of an infinite number of parallels to a line through a given point. However, the general acknowledgment of Lobachevskii’s ideas came many years later.

Meanwhile, another lecture questioned the dominant role of Euclidean Geometry. In fact, on 10 June 1854, Bernhard Riemann addressed the topic On the Hypotheses which Lie at the Basis of Geometry [7]. Indeed, this lecture, published 13 years later, introduced important mathematical concepts, such as the concept of manifolds, mentioned others (such as the fourth dimension), and contributed strongly to the philosophy of geometry [8]. Riemann also

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ABSTRACT

Generally speaking, all avant-garde movements have had one characteristic in common: belief in the new. It is also true that all of those movements were aware of changes, progress and advances in science. As a consequence, non-Euclidean geometry was considered a manifesto for revolution in the arts. This article discusses the visualization of mathematics—the process of transferring concepts from mathematics to works of art—with examples from the artworks and writings of El Lissitzky and Naum Gabo.
discussed the geometry of spherical surfaces on which straight lines correspond to great circles, called equators. As a consequence, there is a geometry (popularly known as Riemann’s Geometry) with no parallel lines, for two equators always meet at two points: the poles.

Riemann’s ideas together with Lobachevskii’s geometry have constituted the subject matter of philosophical and scientific debates since 1860. Thus, in the last half of the nineteenth century, the dominance of Euclidean geometry ended. A revolutionary change with respect to tradition had been accomplished.

Looking back on the turn of the century, we find signs of fundamental changes in science, in literature, in technology, in fashion—in short, everywhere. In the arts, the attacks on the role of representation followed one after another, from Impressionism to Cubism, which was the deepest criticism of the role of visual imagery as representations of reality. A never-ending story traverses all these ‘isms’. Most simply, we can regard them as the emblems of change and of denial of tradition.

For artists there was a widespread feeling that behind these changes, science was the ultimate cause of this transforming world. And progress legitimated the process. Thus, a Futurist manifesto says: “Comrades, we tell you now that the triumphant progress of science makes profound changes in humanity inevitable, changes which are hacking an abyss between those docile slaves of past tradition and us free moderns” [9].

In these lines we note the assured volume of confidence in progress implanted by the contemporaneous scientific avalanche. These manifestos are evidence of how science imposed itself on artistic thought.

This tendency was common to several avant-garde movements bearing different banners. Cubism and Futurism in the West were no less involved with science as a metaphor of progress than were the avant-garde artists in Russia. For example, I refer to a passage from Vladimir Markov’s Principles of New Art (1912): “It must be noticed that contemporary Europe which had done great conquests in the scientific and technological domains is very poor with respect to the evolution of plastic principles inherited from the past” [10].

This sentence could not have been written some years later. Indeed, the following years were years of increasing renewal in visual art, which achieved its chief goal in Constructivism.

Constructivism was the confluence of several diverse aspects of the avant-garde. As a collective ideology, it grew up in the years of 1917 to 1920. The Russian Revolution had provided the optimistic atmosphere sympathetic to new formulations in art. This was a period of discussion and of revolution extended to all spheres. Many explanations have been proposed for this historical avant-garde [11]. For our present purpose it suffices to scan the bundle of mathematical ideas involved in visual art practice. On the one hand, advance in mathematics contributed to support the metaphor of progress. On the other hand, mathematics represented a new way to approach visual arts problems and also created an appropriate place to look for non-naturalistic shapes according to the ideals of both Constructivism and Suprematism.
THE VISUALIZATION OF ABSTRACT MATHEMATICAL NOTIONS

El Lissitzky's essay *Art and Pangeometry* is an essential document for studying the mathematical issues discussed by the Russian avant-garde [12]. Although it was published in 1925 in Germany, from its first line we note it deals with our present situation: "In the period between 1918 and 1921 a lot of old rubbish was destroyed. In Russia we also dragged Art off its sacred throne" [13].

This tone, a typical denial of the past under Dadaist influence, is a rhetorical detour from the essay's aim of describing the parallel development of art and science—geometry—by means of analogies. For example, perspectival space, the representation of space that originated in the Renaissance, corresponds to the laws of three-dimensional Euclidean geometry. However, "in the meantime science undertook fundamental reconstructions" [14], for Euclid's laws had been destroyed by Lobachevski, Gauss and Riemann. And, in the arts, Cubism had replaced perspective [15]. So said Lissitzky, and so wrote Apollinaire, 12 years earlier [16].

The spatial conception of Suprematism is expressed by the phrase 'irrational space' [17]. In order to explain it, Lissitzky began with an inquiry into non-Euclidean geometries and Gaussian curvature (Fig. 2). Let us pause to sketch this point. Descartes' translation of geometry into algebra allows us to state geometrical properties in terms of functions involving the coordinates of the points concerned. In this way, the study of purely geometrical, and to some extent visual, properties of figures is reduced to the study of functions. Since the seventeenth century this approach has been to state and solve problems about curves. Consider the equality of shapes among figures. All will agree that for planar figures bounded by straight lines, equality of shape means equality of corresponding angles. But what if the borders are curved lines?

We can naively think that curvature at a point is, in some unprecise sense, like an angle. Thus, equality of shape would mean equality of curvature at the corresponding points. However much we stay on informal ground and understand curvature at a point as the index of the deviation of the curve from its tangent line in that point, or, in the case of surfaces, from its tangent plane, nevertheless a precise formal definition must refer to the functional translation of curves and surfaces.

For plane curves, curvature intuitively is the degree to which a curve is bent at each point. Consider the simplest plane curve, the circle. Because it is equally curved throughout, its curvature is constant and is measured by the reciprocal of the radius. So the smaller the radius, the larger the curvature. In all other curves the amount of curvature varies from point to point, therefore it must be measured with infinitesimals. Thus arises the necessity of using the functional translation of geometrical figures in order to deal with infinitesimals. Now consider a given plane curve and a point $P$ on it. Let $Q$ and $R$ be two neighboring points of the curve.

Therefore, there exists a circle through $P$, $Q$ and $R$. If $Q$, $R$ approach $P$, then the curvature of the circle approaches a limiting number. This number is defined as the curvature of the given curve at $P$ [18].

The measure of curvature for surfaces can now be reduced to the computation of the curvatures for plane curves. The method for determining the curvature of a surface is, briefly, as follows. Given a point on a surface, the lines tangent to this point lie on a plane. Draw the planes perpendicular to that plane through the point. Each of these planes will intersect the surface in a plane curve. As they are plane curves, their curvatures can be calculated in the way just shown. Thus we determine a set of real numbers, in which each number corresponds to the curvature of one of the plane sections. However this set has a minimum and a maximum, called the principal curvatures of the surface at the point.

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Lissitzky's use of sections of geometrical objects might well have descended from popular ideas that considered material objects as sections of four-dimensional entities.

It is by means of the Gaussian curvature of the surface at the point under consideration. This definition originated with the work of Gauss in 1827. It is by means of the Gaussian curvature, actually a number, that a very elegant characterization of non-Euclidean geometries can be formulated. In fact, if the sum of the internal angles of a triangle lying on a surface is less than two right angles (180°), then the surface has negative curvature. If the sum is greater than two right angles, then the surface has positive curvature. Euclid's law, which says that the sum of the internal angles of a triangle equals two right angles (Euclid, I. 32), holds in surfaces of zero curvature.

Now, let us come back to Lissitzky's essay. Its discussion refers to spaces of non-zero curvature. More precisely, it says that spaces in which Euclid's postulates hold are the only spaces we can visualize. For spaces of non-zero curvature "only a mirage can simulate this". This constitutes Lissitzky's criticism of irrational space, which, for him, was the spatial concept of Suprematism. What does irrational space mean? Lissitzky's explanation can be easily and accurately expressed in mathematical terms. For irrational space is a four-dimensional manifold. This was one of the fundamental notions introduced by Riemann's 1854 lecture, although Lissitzky does not mention it.

The definition of the concept of manifold is a difficult task. Briefly, an n-dimensional manifold is a space \( M \), which near each point is like the Euclidean space of dimension \( n \), i.e., the set of all \( n \)-ples of real numbers. Most geometrical forms whose points may be defined by \( n \) parameters are \( n \)-dimensional manifolds. Of course, Euclidean space of dimension \( n \) is the simplest \( n \)-dimensional manifold. Also, perceptible color qualities form a manifold of dimension three by virtue of the fact that all colors are produced by mixing three basic colors.

Riemann's 1854 lecture begins with the advice that manifolds are rare in ordinary life: "Color and the position of sensible objects are perhaps the only simple concepts whose instances form a multiply extended manifold". Compare Riemann's advice with Lissitzky's ideas about irrational space: "In this space the distances are measured only by the intensity and the position of the strictly defined color areas". He thus continues with several remarks that simply yield to the coincidence of Suprematist or irrational space with a four-dimensional manifold.

There is no evidence that Lissitzky had read Riemann's lecture or any other book that contains such kinds of ideas. However, these ideas had become part of the philosophical and scientific knowledge of the time. For instance, in The Foundations of Geometry (1897) Bertrand Russell wrote two passages dealing with color as an example of manifold. Another account of Riemann's ideas was given by H. V. Helmholtz in On the Origin and Significance of Geometrical Axioms. This was the first attempt to expose manifolds and curved spaces to an audience knowing only "the amount of geometry taught in our gymnasia".

Despite the impossibility of determining the exact origin upon which Lissitzky built his explanation of irrational space, it is still possible to make some comments. To start with, manifolds, even if they are not explicitly mentioned, form the underlying mathematical concept that gives meaning to Lissitzky's account of Suprematist space. Second, it seems reasonable that the concepts are of a mathematical kind, for the essay is full of advice to artists not to use 'advanced' scientific concepts without a deep understanding of the corresponding theories. Third, as they are mathematical spaces, "Our minds are incapable of visualizing this, but that is precisely the characteristic of mathematics—that it is independent of our powers of visualization".
cannot be represented; in short, it is impossible to give them material form" [37].

This last sentence contains the refutation of a then-current belief that had oversimplified and confused the discussion of the interplay between art and mathematics during the avant-garde period. I am referring to the belief that avant-garde visual art is, ultimately, nothing more than a transference from mathematics or from a mathematical approach to relativist space to the visual arts [38]. But, Lissitzky's lines touch upon a concept essential to an understanding of the role of mathematics in avant-garde visual arts—the concept of mathematical visualization. Thus, visual images with mathematical notions underlying them do not depend on models or representations of those notions. Visual works may be generated from purely abstract mathematical notions that, of course, have no three-dimensional representation.

Malevich's Black Square illustrates the effect of abstract mathematical notions upon the arts [39]. Insightful critics have shown that behind this artwork are thoughts about the fourth dimension [40]. But Black Square is a picture—it can be seen, it can be photographed. This is not possible with the fourth dimension—it is a mathematical concept, it cannot be photographed. At most we can obtain designs of representations of objects belonging to four-dimensional space in spaces of fewer dimensions, but not designs of the abstract concept of dimension itself [41].

When Lissitzky refers to Black Square, he says it "has now started to form a new space" [42]—indeed, irrational space [43]. He then points out the impossibility of visualizing those new spaces. Here Lissitzky approaches visualization in its more straightforward meaning, which, he says, neither Malevich's Black Square nor Suprematist painting had achieved. How to visualize irrational or Suprematist space? Or, to use mathematical terms, How to visualize four-dimensional manifolds? Or, to use direct, as in Gabo's sculptures.

In Lissitzky's opinion, he has answered these last questions in Proun Space (Fig. 3) [44]. The trick, influenced by motion pictures and advertisements, was to transform the surfaces of an almost-cubic room by displaying objects on them. The movement of the viewer and daylight changes in this environment produced the temporal coordinate. The objects were very simple—parallelepipeds, cubes and spheres—and established a relationship with the walking viewer. Clearly, all this comes from the mathematical approach to relativity [45]. Lissitzky designed a series of six lithographs in Proun [46]. One of the lithographs (Fig. 4) is a perspectival view of the Proun Space; another shows one of the objects (the object on the left wall in Fig. 3). Four other lithographs present planar sections of geometrical figures (Fig. 5). This probably comes from popular ideas that three-dimensional objects are sections of four-dimensional entities [47].

**Proun Space** was Lissitzky's visualization of a four-dimensional manifold—in his own words, the creation of 'imaginary space'. Whether or not he achieved it may be a subject of discussion. Yet his method of visualizing abstract mathematical notions was coherent, even if from these works alone it seems impossible to comprehend the mathematical concepts involved.

### The Language of Naum Gabo

Although Lissitzky's Suprematist ideas were in the sphere of a Constructivist tendency, his approach, similar to Malevich's, did not allow any kind of direct transference from mathematical concepts to visual artworks [48]. In order to discuss the analogical visualization of mathematical notions we must draw our attention outside Suprematism and appeal to Constructivism. A glance at Rodchenko's or Tatlin's works immediately reveals clear geometrical patterns. This does not mean that Constructivist works can be generally characterized by their resemblance to geometrical shapes.

Since the 1920s the controversy concerning who and what belongs to Constructivism has given rise to declarations, debates and writings. Rodchenko's spatial constructions, for example his Hanging Spatial Construction (see Fig. 1), and Tatlin's 'counter-reliefs' (a term coined by Tatlin in 1913 to describe assemblages of industrial materials) may have many elements in common with Pevsner's and Gabo's sculptures. Pevsner and Gabo declared their work to be Constructivist art; however, Gan called Rodchenko and Tatlin Constructivist artists at the same time that he excluded Pevsner and Gabo. It is a complex and many-sided topic. Art issues were, as always, confluent with social, ideological and political affairs [49]. However, for most artists, mathematics had an important role in the genesis of visual images. The role could be subtle, as was the case for Lissitzky and Malevich, or direct, as in Gabo's sculptures.

Gabo's proposal was stated in his Realistic Manifesto (1920), signed together with his brother Anton Pevsner [50]. The 1920s meant new days because of the Revolution, new knowledge thanks to science and technology and, as a result...
Physics and certainly had become acquainted with mathematics. Ellipses, just as the plane sections of the sphere are circles. The ellipsoid thus built fulfills one of the requirements of the Realistic Manifesto, the construction of volumes by means of planes. Gabo’s technique was founded upon this particular way of constructing surfaces. He had studied physics and certainly had become acquainted with mathematics. In those years the use of models was the standard method to illustrate properties of surfaces, for in visualizing them, one gained the intuitive background needed to discuss more abstract notions [52]. The model of the ellipsoid we have described is a particular case of a second-order surface constructed with cardboard circles.

Second-order surfaces, also called quadrics, are surfaces satisfying a second-order equation in three Cartesian coordinates. In the eighteenth century Euler classified them into nine different types according to their equations. Another criterion for classification of quadrics is whether they intersect a plane in a circle. In this way we get two classes. The first sort consists of quadrics that do not intersect any plane in a circle. They are the parabolic and hyperbolic cylinders and the hyperbolic paraboloid. The second sort includes the elliptical cylinder, the elliptical paraboloid, the hyperboloids of one and two sheets, the cone and the ellipsoid. In other terms, surfaces of this kind have circular plane sections that, in turn, allow the construction of models for these surfaces with the use of cardboard circles, as was the case for the ellipsoid [53].

As we see, the Constructivist principle of Gabo’s 1916–1917 sculptures rests upon free artistic variations of circle models for second-order surfaces. Gabo remained linked to the Moscow avant-garde and to Constructivism until he left Russia in 1922. Even if his subsequent works are not Constructivist in the historical sense, they are characterized by the use of geometrical forms. The basis of most of Gabo’s sculptures can be traced back to his Spheric Theme (1936) and Construction in Space: Crystal (1937) (Fig. 8). These sculptures have analogous mathematical representations. For example, Crystal was inspired by a model of the cubic ellipse. This is a space curve defined by a third-degree algebraic equation. Furthermore, it can be proved that it is obtained from the intersection of two quadrics [54]—indeed, from the intersection of an elliptical cylinder and a cone [55].

Spherical Theme is, by far, Gabo’s most popular sculpture and the basis upon which he constructed numerous works. Again, surprisingly, it corresponds to a mathematical representation of a surface, even if Gabo never made this claim. Indeed, its form coincides with the shape of the three-dimensional representation of Enneper’s Minimal Surface (Fig. 9) [56].

Gabo’s sculptures may be considered words in the artist’s language, an alphabet containing mathematical forms and shapes. Thus, in my mind, mathematics supplies visual arts with a marvelous and almost infinite catalogue of mysterious, enigmatic and unknown figures. And art gives pleasure to us.

THE TEACHINGS OF APOLLINAIRE

Gabo’s sculptures illustrate one of the roles mathematics plays in the visual arts. Visual mathematics supplied an alphabet of new forms that the epoch needed. For the sake of estimating the strength of scientific influence on Gabo’s sculptures, we can cite a precise remark by Herbert Read:

The creative construction which the artist presents to the world is not scientific, but poetic. It is the poetry of space, the poetry of time, of universal harmony, of physical unity. Art—it is its main function—accepts this universal manifold which science investigates and reveals, and reduces it to the concreteness of a plastic symbol [57].

This remark goes to the heart of our difficulty. How can we describe the process of visualizing mathematics? As we have seen there is not just one answer to this question. Gabo started from visual representations of mathematical elements and ended with sculptures. Lissitzky theorized concepts of art by means of mathematical notions that, in turn, resulted in visual works [58]. Both are attitudes resulting in visual mathematics. They are the more brilliant and extreme examples of an attitude common to the Russian avant-garde, and therefore to Modernism, for the Russian avant-garde was a compendium of Modernism.

Today’s equivalent of cardboard, plaster and wire models are computer-generated images. They propose to us, among other things, a catalogue of forms and shapes, as models did to Gabo. Nondeterminist ideas explain a whole world of phenomena and can be the source of fresh insights—in the
same way that Lissitzky reached an understanding of Suprematist space by appealing to Riemann’s ideas.

The effect of this convergence of art and science has a long history. Yet, in the age of Modernism a great period of direct mathematical influence upon the visual arts began. Mathematics provided the arts with an example of continuous progress—an encyclopedia of deep ideas, a cartography of forms. Apollinaire, perhaps the spokesperson of avant-garde movements, while meditating on art, concluded that “Geometry is to the plastic arts what grammar is to the art of the writer” [59]. Why not believe this idea to still be valid?

References and Notes

11. For the first systematic presentation of Constructivism, see C. Gray, The Great Experiment in Russian Art, 1862–1922 (London: Thames and Hudson, 1962) chapter VII. A more recent and comprehensive account is C. Lederer, Russian Constructivism, S. Barron and M. Tuchman, eds. (New Haven, CT: Yale Univ. Press, 1985). The complexities involved in studying the Russian avant-garde are many different definitions of curvature, but they are all equivalent. For a proof of this fact see Theorem 1 in Spivak [7] p. 2.
16. For the first systematic presentation of Constructivism, see C. Gray, The Great Experiment in Russian Art, 1862–1922 (London: Thames and Hudson, 1962) chapter VII. A more recent and comprehensive account is C. Lederer, Russian Constructivism, S. Barron and M. Tuchman, eds. (New Haven, CT: Yale Univ. Press, 1985). The complexities involved in studying the Russian avant-garde are many different definitions of curvature, but they are all equivalent. For a proof of this fact see Theorem 1 in Spivak [7] p. 2.
making speculations" (Russell [26] p. 63). Note that Herbert and Gauss were the only authors quoted in Riemann’s 1854 lecture (Riemann [7]).


35. Helmholtz [34] p. 648. Helmholtz was the most widely read writer on the foundations of geometry (see Russell [26] p. 29), and his article ‘On the Origin and Significance of Geometrical Axioms’, the most influential source for many philosophical debates on geometry from the 1870s onward (see Torretti [8] p. 155). Peter Nisbet has suggested that most features of Lissitzky’s writings in the early 1920s are drawn from the first volume of Spengler’s Decline and Fall (1924). See also Levinger’s ‘Kias Ppan als Esfnder. Nisbet comments on the fact that some of Lissitzky’s ideas are drawn from Spengler and Fracé (see Nisbet [35] pp. 28–30).


39. See the introduction to Liddler [11]. For example, according to E. Levinger (Levinger [25] p. 231), Lissitzky initiated to study the correspondences between art and mathematics as a way to understand the denial of absoluteness. Lissitzky’s ultimate aim was to instal a new social order, as art and mathematics had made.


42. At that time there were two big suppliers of mathematical models: Martin Schilling in Leipzig and B. G. Teubner in Leipzig and Berlin. They published catalogs: Catalog mathematischer Modelle für den Höheren mathematischen Unterricht (Leipzig: Martin Schilling, 1911) and Verzeichnis von H. Wieners und P. Treutleins mathematischen Modellen für Höhere Lehranstalten und Technische Fachschulen (Leipzig and Berlin: B. G. Teubner, 1912).


45. Space curves can be determined as the intersection of two surfaces given by equations in homogeneous coordinates F(x, y, z, w) = 0 and G(x, y, z, w) = 0. In case of the cubical ellipse the cone F(x, y, z, w) = w^2 + 2z = 0 and the elliptical cylinder G(x, y, z, w) = w^2 + y^2 = 0.

46. Enneper’s surface is the surface given in parametrized form as: x = -u - u^2/3 + v^3, y = v - u^2, z = -u^2 + v. The principal curvatures are:

\[ k_1 = 2/(1 + u^2 + v^2) \] and \[ k_2 = -2/(1 + u^2 + v^2) \].

Therefore H = (k_1 + k_2)/2 = 0. H is called the mean curvature, and H = 0 is the condition for minimal surfaces. See J. C. C. Nitsche, Vorlesungen über Minimalflächen (Berlin: Springer, 1975) pp. 75–81. Since H = 0, it follows that \[ k_1 = -k_2 \]. Hence the Gaussian curvature of the surface is, i.e. the product of the principal curvatures is a negative. This point adds a difference between Gabo’s Sphere, Sphere and Enneper’s Minimal Surface, because Gabo’s form is made with two flat circles (see Fig. 9), and, as a consequence, its Gaussian curvature is zero. However, from a purely visual point of view, both look very similar because their shapes are identical. For an account of minimal surfaces in art, see M. Emmer, “Soap Bubbles in Art and Science: From the Past to the Future of Mathematical Art”, Leonardo 20, No. 4, 327–334 (1987). A computer-generated image of Enneper’s surface appears in D. Hoffman, ”The Computer-Aided Discovery of New Embedded Minimal Surfaces”, The Mathematical Intelligencer 9, No. 3, 8–21 (1987).

47. Read [51] p. 11.

48. See Levinger [25]. Levinger argues that Lissitzky’s analogy between pictorial space and mathematical concepts led to the foundations of the theoretical bases for nonobjective art, different from both Suprematism and Constructivism. Nonobjective art is discussed in Levinger [25] pp. 228–230.
